Magnetized Core-Collapse with Resistivity

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Formations of Compact Objects: from the cradle to the grave   Mar. 7 - 9, 2012

1. Intorduction

Magnetized Supernova

✔SN supported by B-field (and Rotation) (LeBlanc & Wilson ‘70)

$E_{grv} \rightarrow (E_{rot} \rightarrow) E_{mag} \rightarrow E_{kin}$

One of the scenarios of a core-collapse supernova

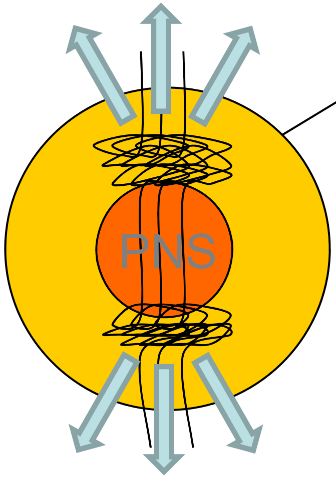
✔Requires a magnetic flux of a magnetar class

→ Magnetar candidates possibly originate from magnetized SNe

✔has been studied well for past several years

Magnetar candidates

$B_{surface} \sim 10^{14} - 10^{15}\,G$



Iron core

r～1000 km

The resistivity in a magnetized SN

Previous works ignores the resistivity

Magnetic Reynolds number in PNS (with the Spitzer resistivity)

$$R_m = \tau_{dif} / \tau_{dyn} \sim 8\times10^{15}\left(\frac{Z}{26}\right)\left(\frac{L}{4\times10^{5}\,cm}\right)\left(\frac{T}{5\times10^{10}\,K}\right)^{3/2}\left(\frac{v}{2\times10^{8}\,cm/s}\right)$$

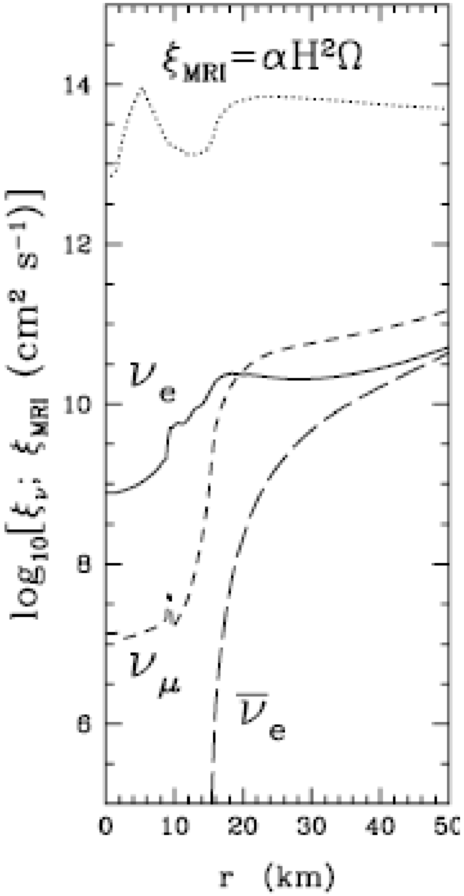
But,

A Turbulent Resistivity may be important.

Due to a convection

→ $\eta \sim 10^{13}$ - 10^{14} $cm^2\,s^{-1}$ （T. A. Thompson+‘05）

→ $R_{m,max} \sim 1 – 10$



This work

We study roles of a turbulent resistivity
in a magnetized core-collapse by carrying out
2D-axisymmetric resistive MHD simulations.

2. Numerical Code and Models

Numerical code

2D Resistive MHD code

“yamazakura　山桜”

- Time explicit Finite volume method

→ High resolution central scheme

(Kurganov & Tadmor ‘00)

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大和心を
人問わば
朝日に匂ふ
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本居宣長

- 3rd ord. in time、2nd ord. in space
- Numerical viscosity: $\sim (\Delta x)^3 (a(u) u_{xxx})_x$.
- Constraint Transport scheme（for divB=0 and divJ=0）
- Introduction of a resistive term
- Poisson solver: MICCG(1,2)

Yamazakura has passed several test problems

Resistive Magnetohydrodynamic (MHD) Equations

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$

Mass conservation

$$\frac{\partial}{\partial t}(\rho\mathbf{v}) + \nabla\cdot\left(\rho\mathbf{vv} - \frac{\mathbf{BB}}{4\pi}\right) = -\nabla\left(p + \frac{B^2}{8\pi}\right) - \rho\nabla\Phi$$

Momentum conservation

$$\frac{\partial}{\partial t}\left(e + \frac{\rho v^2}{2} + \frac{B^2}{8\pi}\right)$$

$$+\nabla\cdot\left[\left(e + p + \frac{\rho v^2}{2} + \frac{B^2}{4\pi}\right)\mathbf{v} - \frac{(\mathbf{v}\cdot\mathbf{B})\mathbf{B}}{4\pi} + \underline{\frac{\eta}{c}\mathbf{j}\times\mathbf{B}}\right] = -\rho(\nabla\Phi)\cdot\mathbf{v}$$

Energy conservation

$$\frac{\partial\mathbf{B}}{\partial t} + \nabla\times\left(-\mathbf{v}\times\mathbf{B} + \underline{\frac{4\pi\eta}{c}}\mathbf{j}\right) = 0$$

Induction Eq.

$$\Delta\Phi = 4\pi G\rho$$

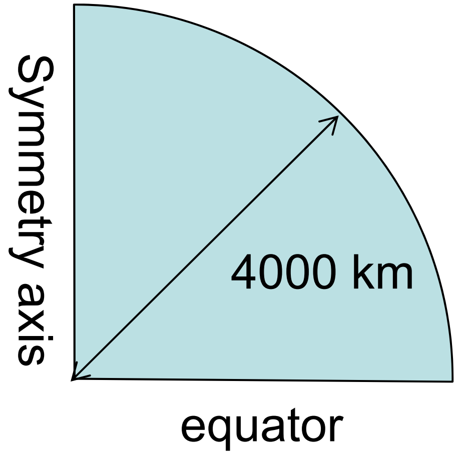
Poisson Eq.

-Shen’s EOS

-Ye: a function of density (Liebendorfer ‘05)

✔Numerical domain：

- Progenitor: 15 $M_{\odot}$ (r~10^{13} cm) (Woosley ‘95)
- a quarter of the meridian plane of a core of a 4000 km radius
- Axisymmetric, equatorially-symmetric



✔Numerical grids：

- cylindrical coordinate
  $N_{\varpi} \times N_z = 720 \times 720$（nonuniform）
- resolution in the center：400 m

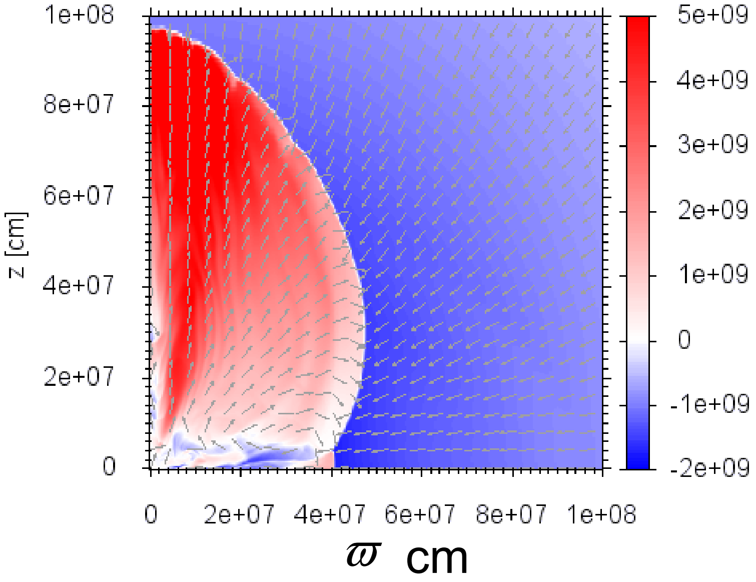
✔Numerical model

Dipole B-field
+
（differential
rotation）

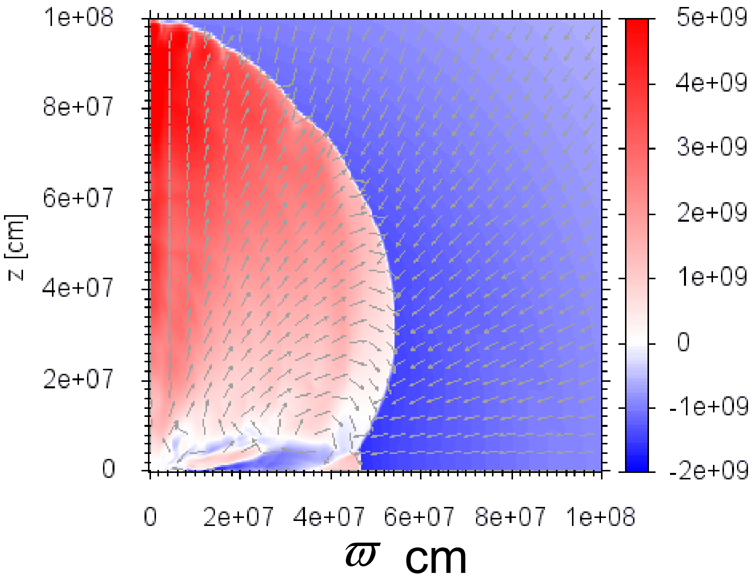
	Rotation			No-Rotation		
Magnetic energy M/W (%)	0.5			5.0		
Rotational energy T/W (%)	0.5			0.0		
resistivity η (cm^2/s)	0	10^{13}	10^{14}	0	10^{13}	10^{14}

3. Results

Dipole M/W=0.5% Rotation

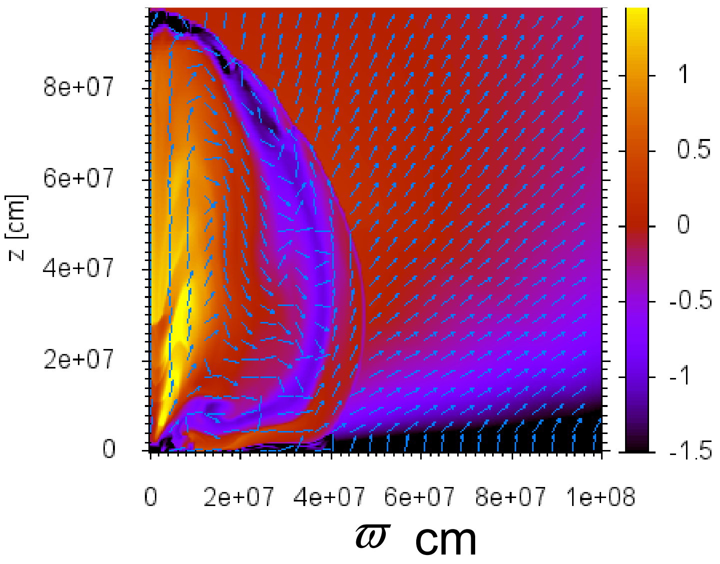


Velocity direction & Density

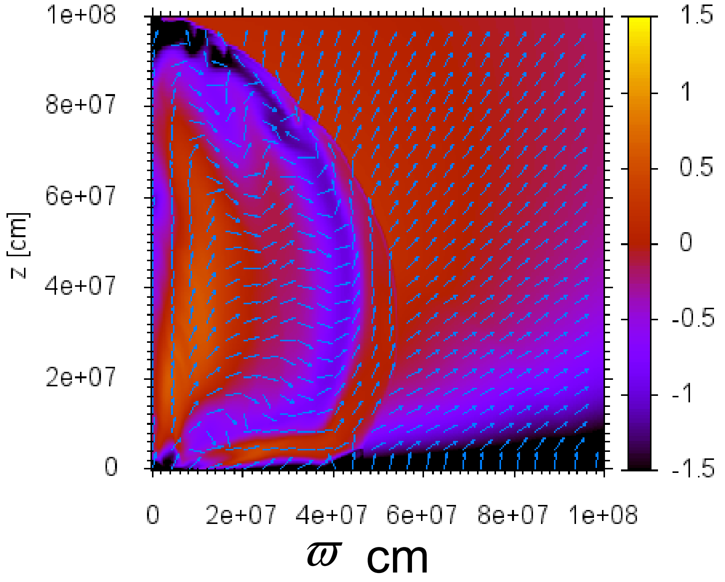


$\eta = 0$ $t = 164$ ms

$\eta = 10^{14}$ $t = 170$ ms

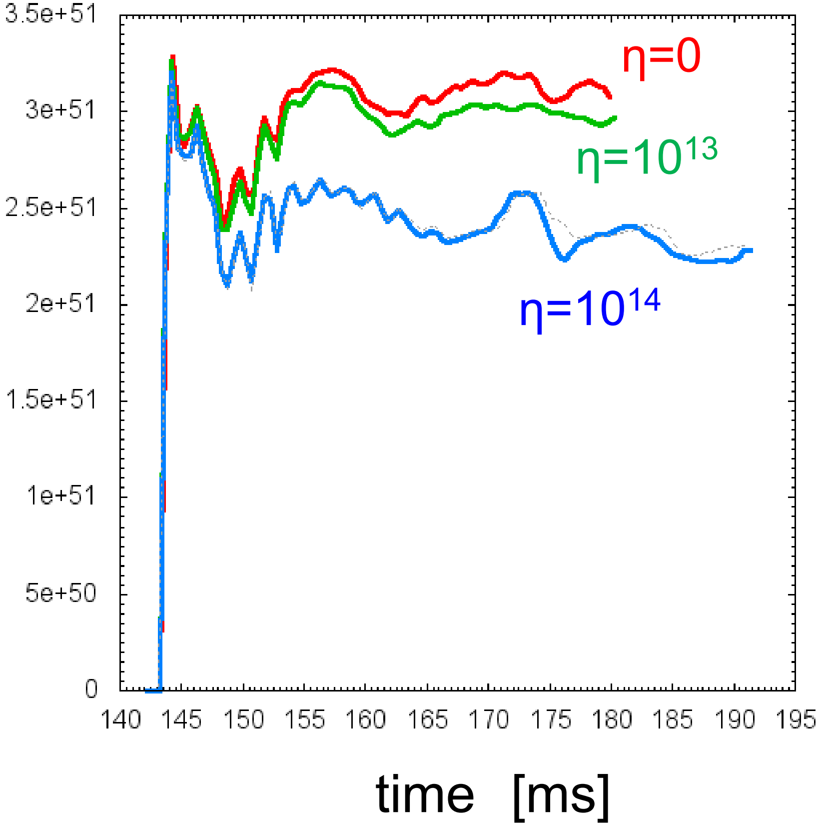


B-field direction & Pm/P

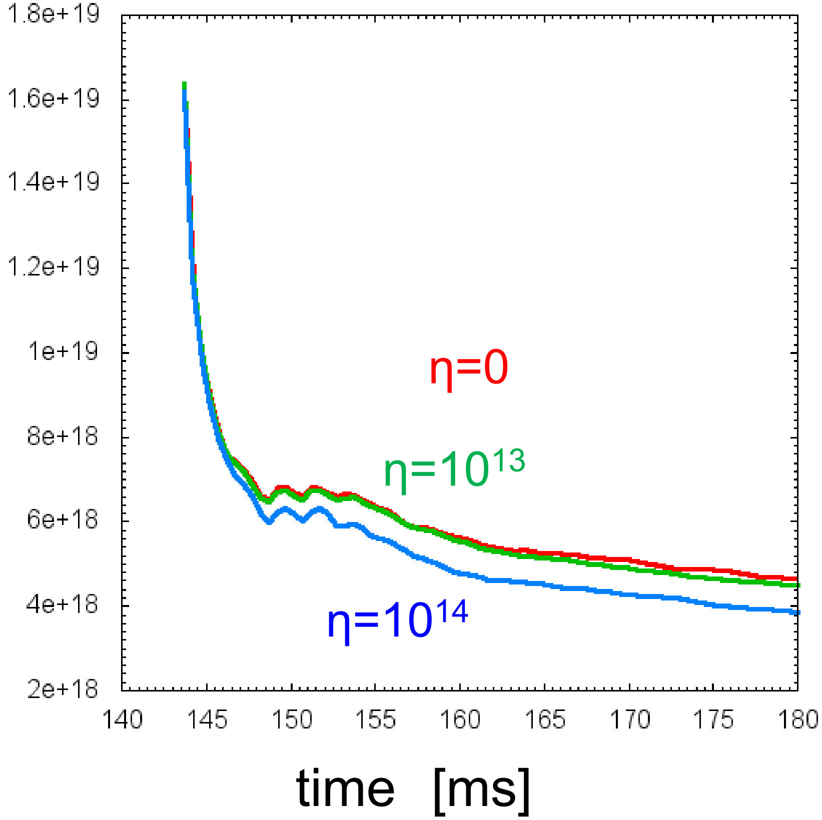


The Evolution of The Explosion Energy

Explosion energy [erg]

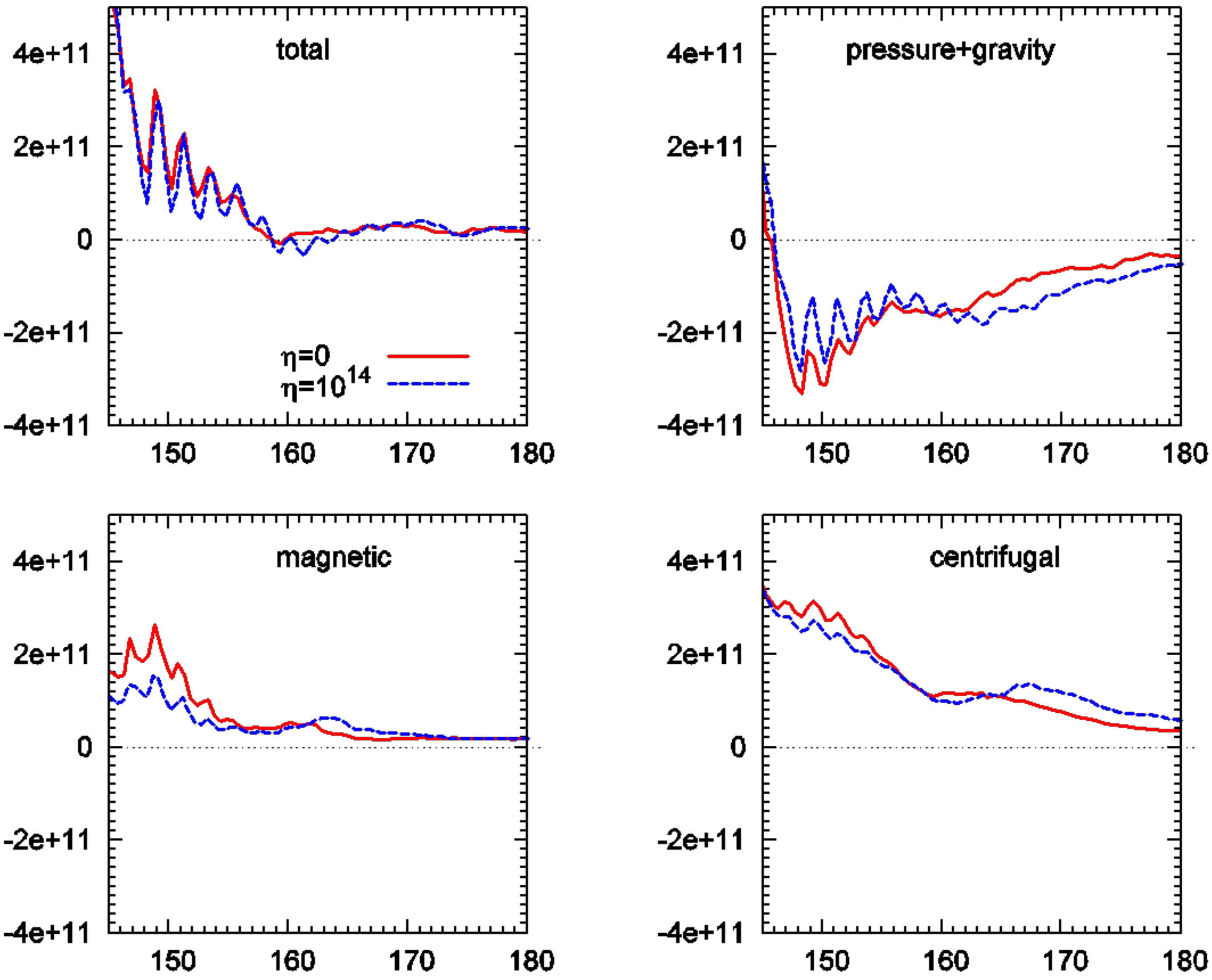


Explosion energy per unit mass [erg/g]



✔Resistivity weaken the explosion.

Force per unit mass (dyne/g)

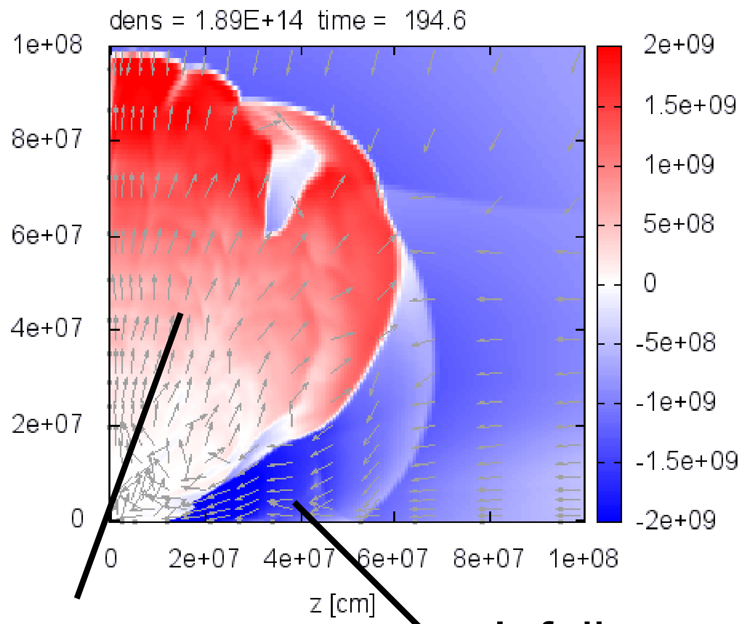


Dipole M/W=5.0% No-Rotation

Velocity direction & Density

38 ms after bounce

η = 0



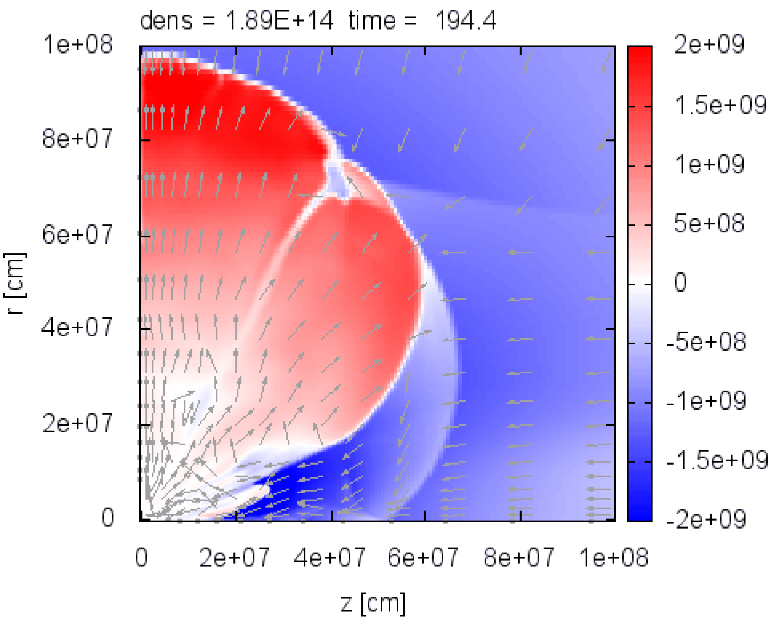
z [cm]

ϖ [cm]

Expulsion region

Infall region

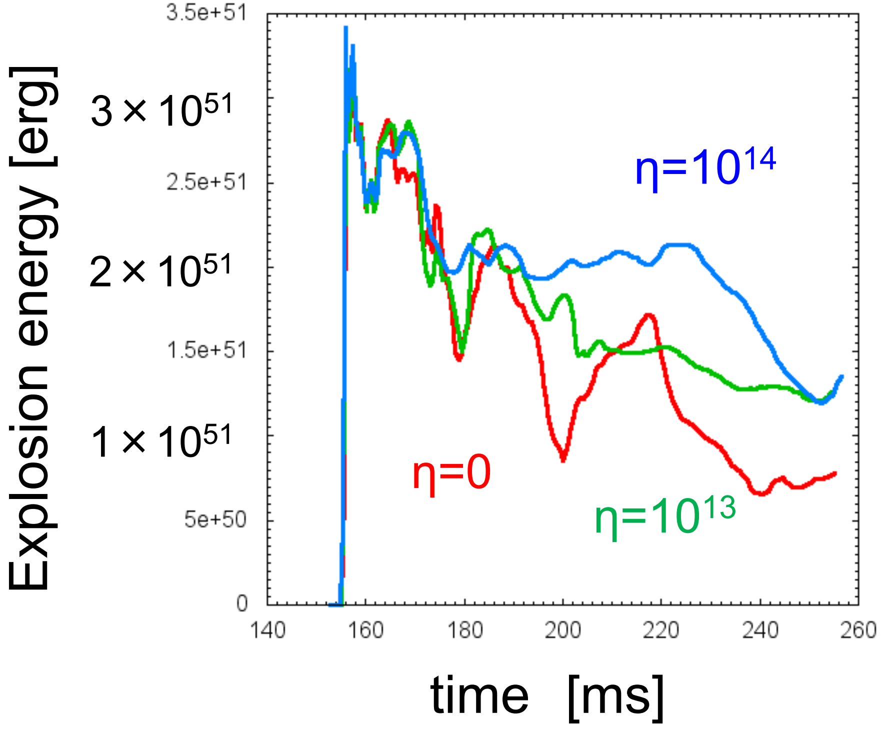
η = 10^{14}



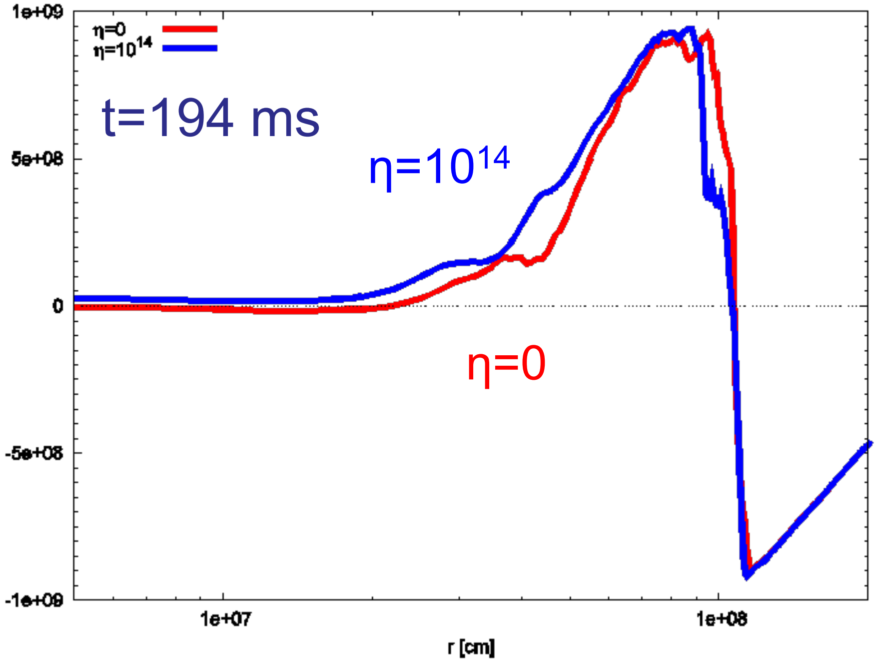
z [cm]

ϖ [cm]

The Explosion Energy



Average radial velocity in the expulsion region

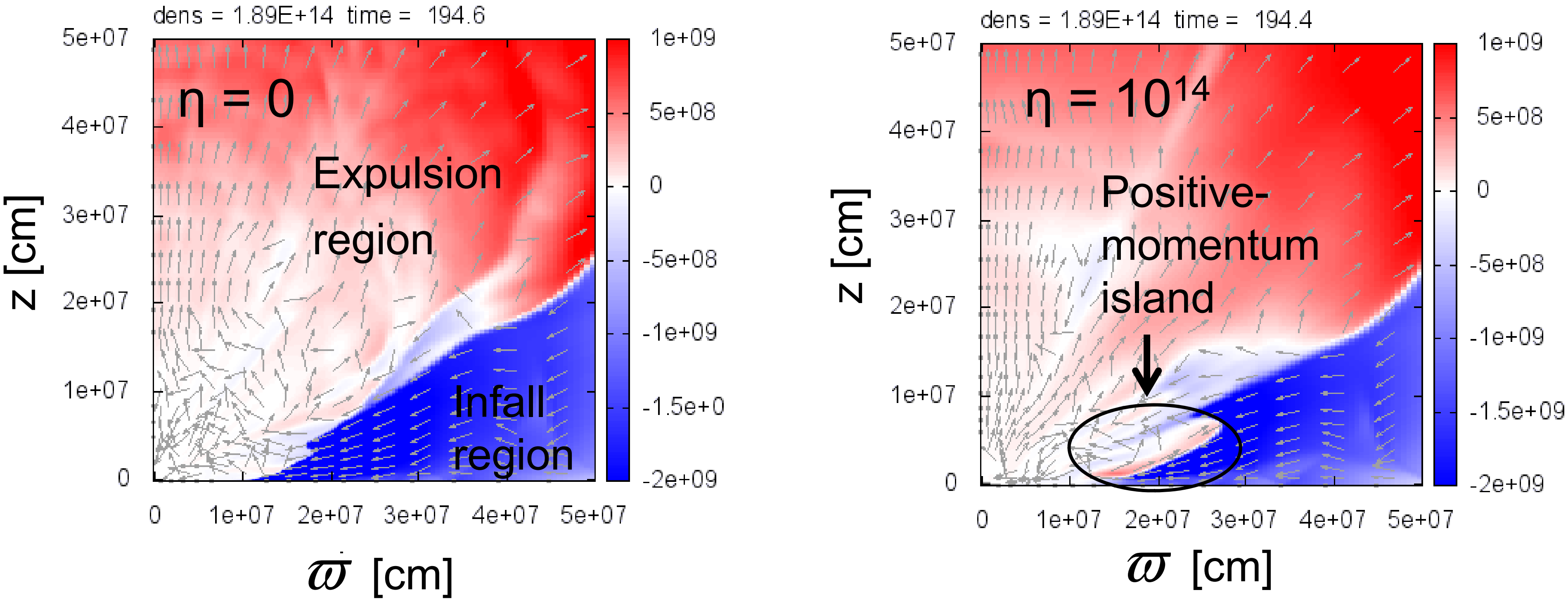


✔Resistivity enhances the explosion.

An enlarged view

Velocity direction & Density

t=194 ms

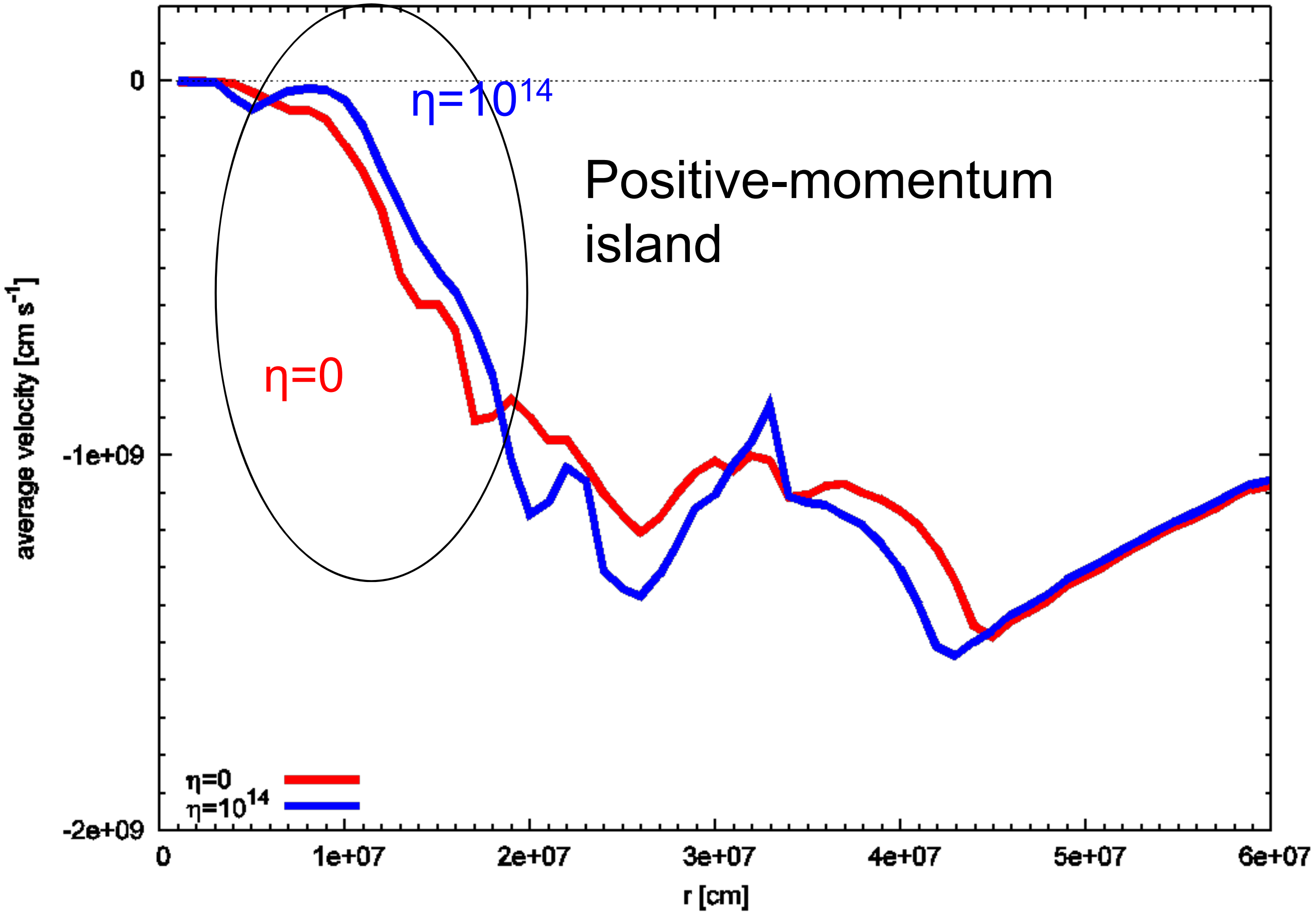


η=0 : An Inflow of a negative momentum from the infall region to expulsion region damages the matter eruption.

η=10^{14} : A positive-momentum island protect the expulsion region from the inflow.

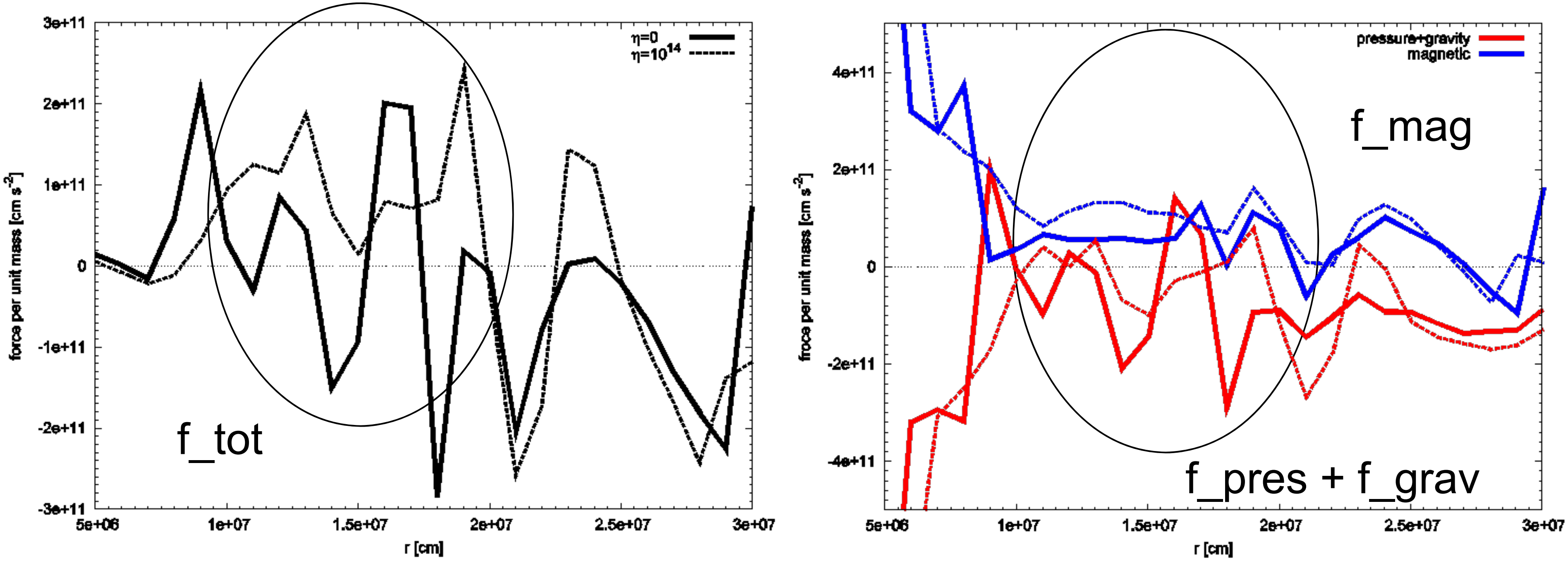
◆The radial-distribution of a radial velocity （infall region）

t=175 ms



◆The radial-distribution of a radial force（infall region）

Solid lines： η=0
Dashed lines： η=10^{14}

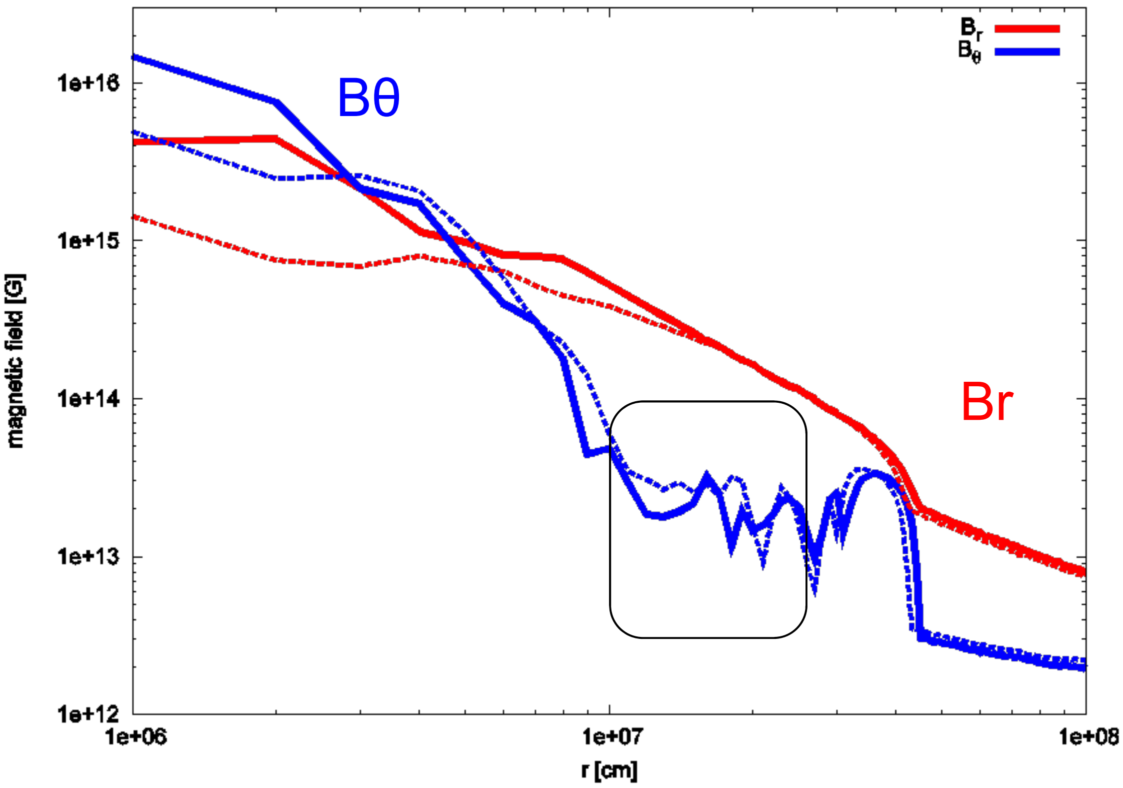

t=175 ms

A force is larger in η=10^{14} around100 – 200 km
→ due to a magnetic force.

◆The radial-distribution of a B-field（infall region, log）

Solid lines： η=0
Dashed lines： η=10^{14}

t=175 ms



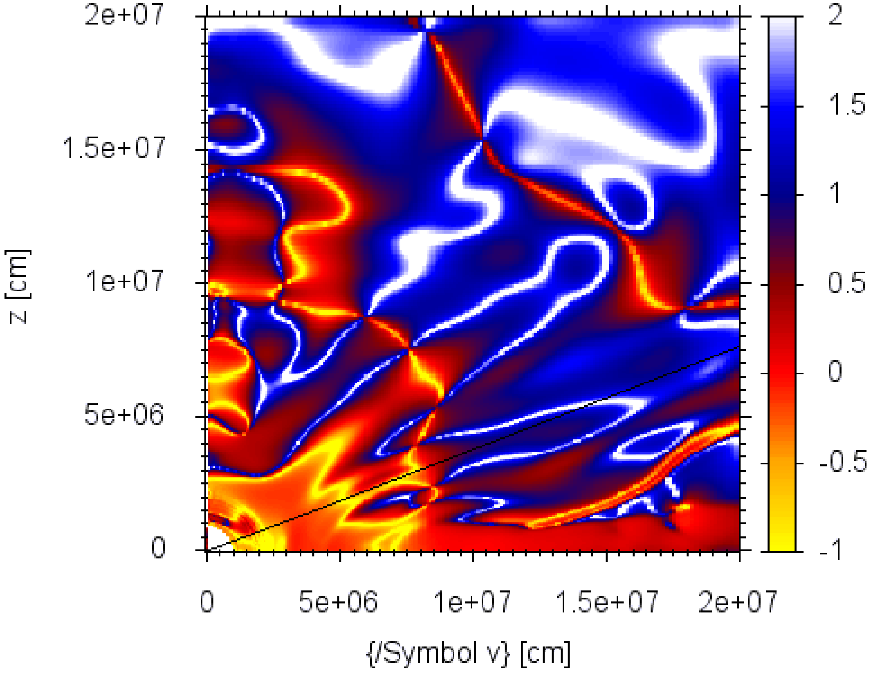
A radial magnetic force

$$[\mathbf{j}\times\mathbf{B}]_r = \frac{B_\theta}{4\pi}\left[-\frac{\partial B_\theta}{\partial r} \boxed{+\frac{1}{r}\frac{\partial B_r}{\partial \theta}} - \frac{B_\theta}{r}\right]$$

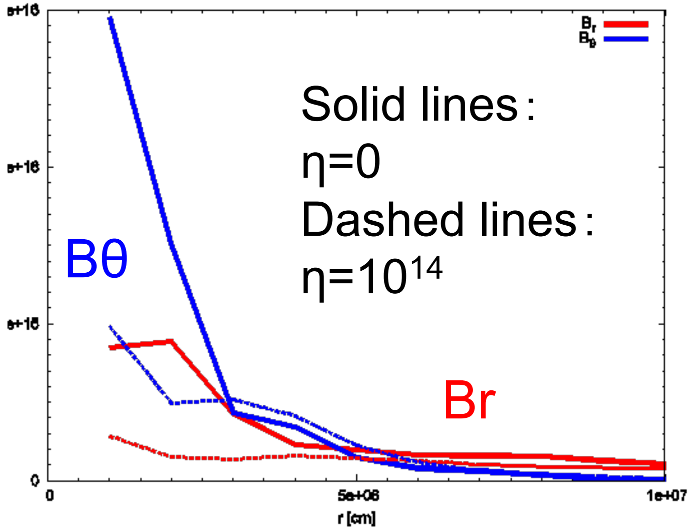
◆A diffusion of B-field

t=175 ms

Rm $\eta=10^{14}$



The radial-distribution of a B-field (infall region, linear)



Solid lines : $\eta=0$

Dashed lines : $\eta=10^{14}$

✔A strong B_θ around the center diffuses outward

4. Summary

We have done 2D-MHD simulations of strongly-magnetized core-collapses under low magnetic Reynolds numbers.

✔A turbulent resistivity possibly affect the dynamics of a core-collapse.

-In a rotating case: The explosion is weakened by a resistivity

→ An amplification of B-field is suppressed.

-In a non-rotating case: The explosion is enhanced

→ A strong B-field around the center diffuses outward and prevent a matter falling around the equator, which otherwise damages a a matter eruption in the expulsion region.

Future works

✔ 3D simulations

✔ Numerical simulations of a collapse of a weakly-magnetized core.