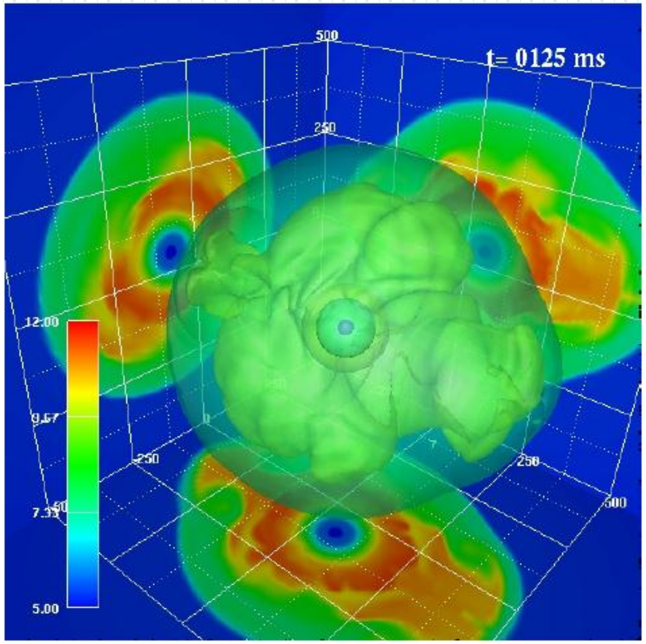
2012/3/9 Formations of Compact Objects : from the cradle to the grave
@Waseda University

Three dimensional simulations of supernovae

Takiwaki, T., Kotake, K., & Suwa, Y. 2012,accepted to ApJ; arXiv:1108.3989
and recent result

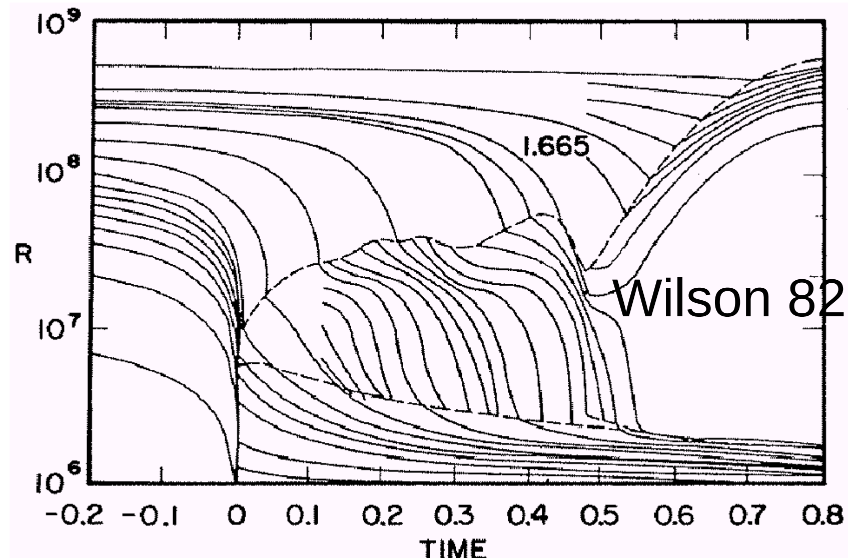


Tomoya Takiwaki
National Astronomical
Observatory of Japan

History of studies of supernovae

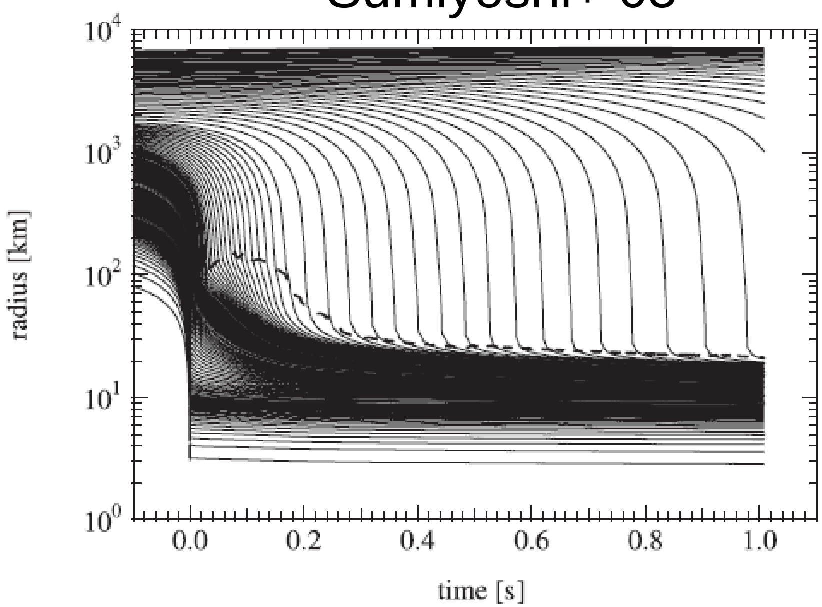
- Colgate & White '66
  began simulation
- Wilson' 82
  delayed explosion
- Very sophisticated simulations under 1D could not reproduce supernovae explosions(e.g. Sumiyoshi et al. '05).

Supernovae modelers devote about 50 years to solve neutrino transport with adequate accuracy.

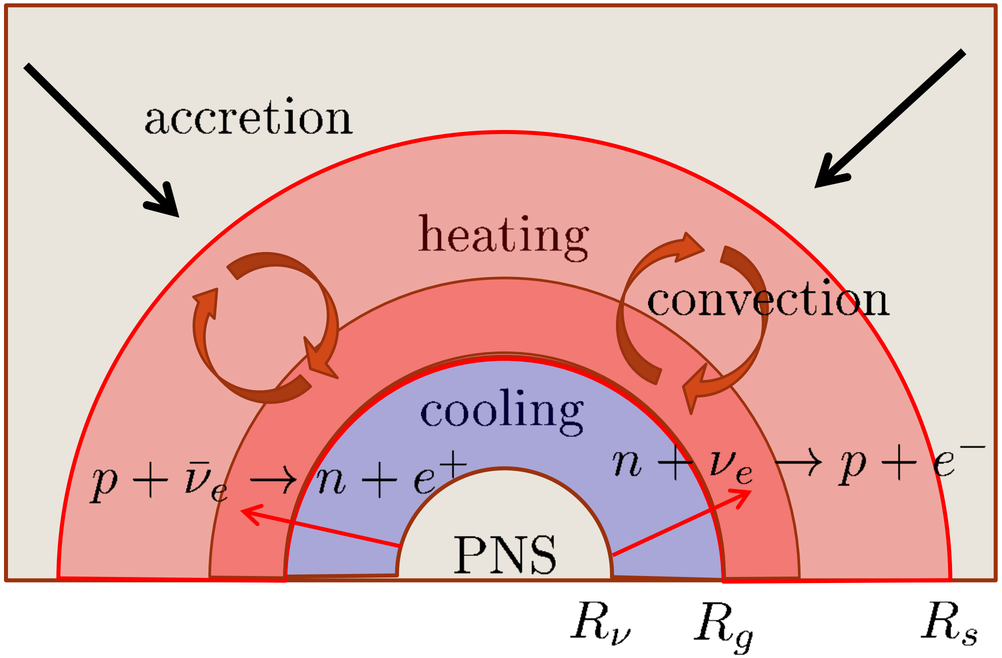


Wilson 82

Sumiyoshi+ 05



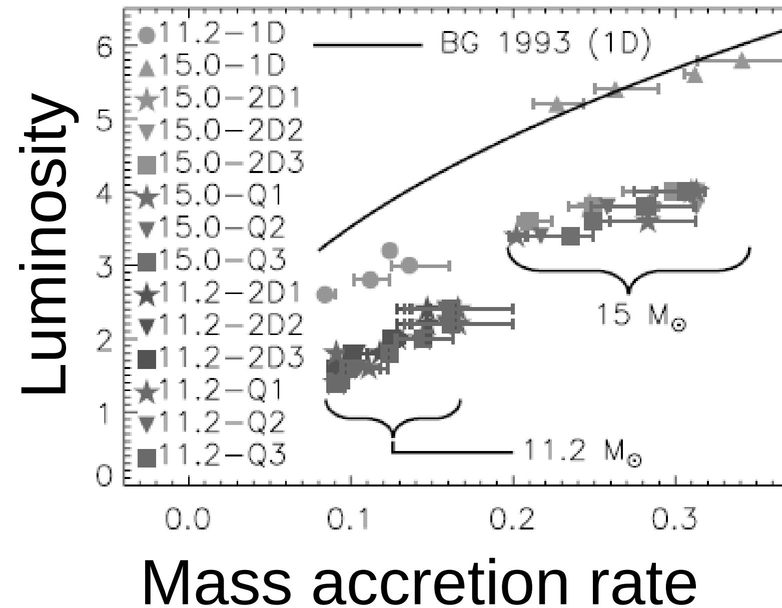
Breakthrough by 2D simulations



$$\mathcal{C} \propto 1/r^9 \quad p + e^- \rightarrow n + \nu_e$$
$$\mathcal{H} \propto 1/r^5 \quad n + \nu_e \rightarrow p + e^-$$

By convection, heated material is transported to the place, just behind the shock.

Murphy & Burrows 2008

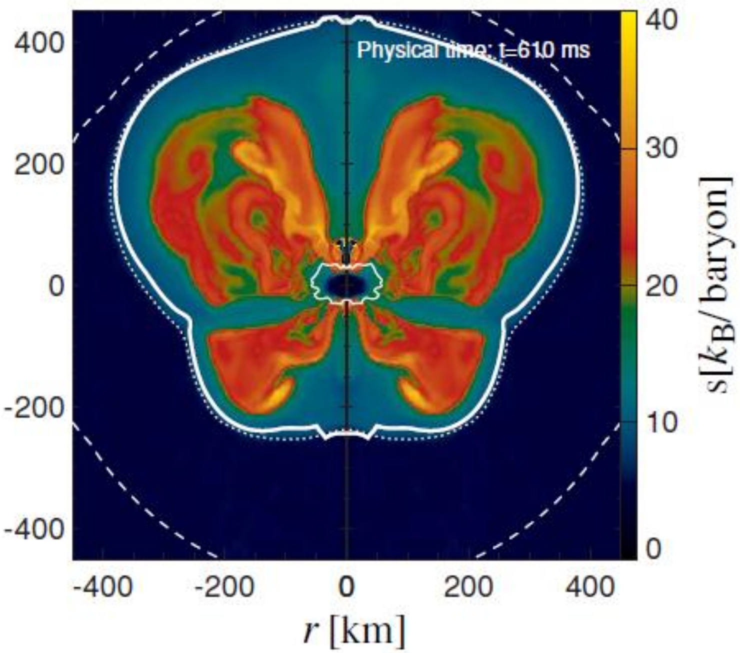


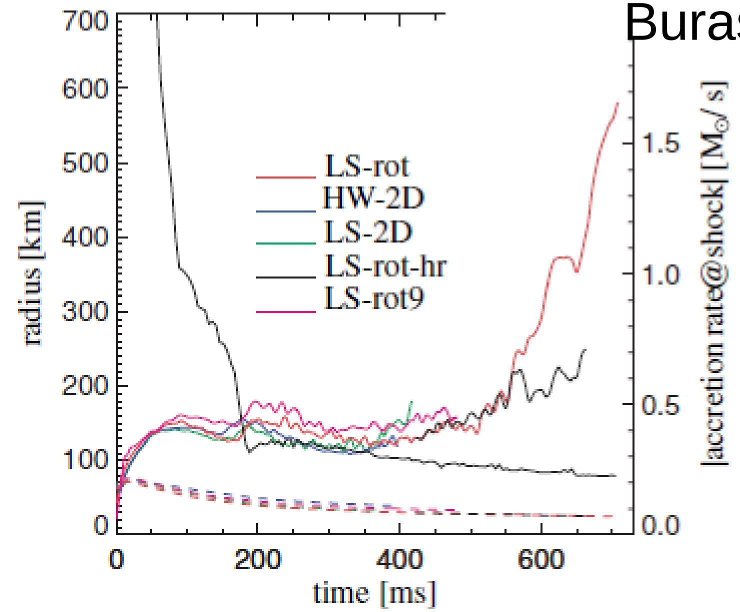
2 D lessen luminosity necessary to revive the shock.

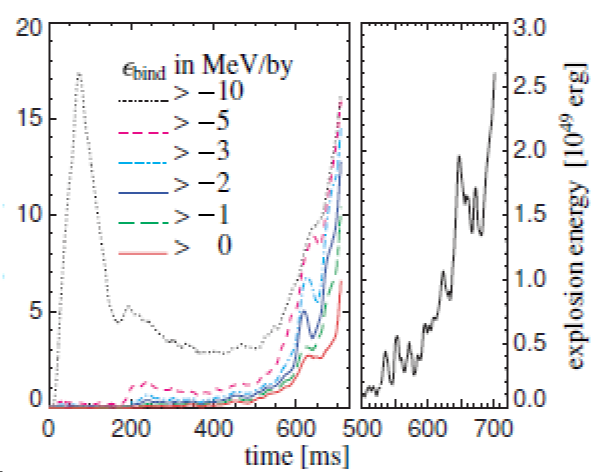
Sophisticated simulation in 2D

Marek and Janka 2009

Buras et al 2006

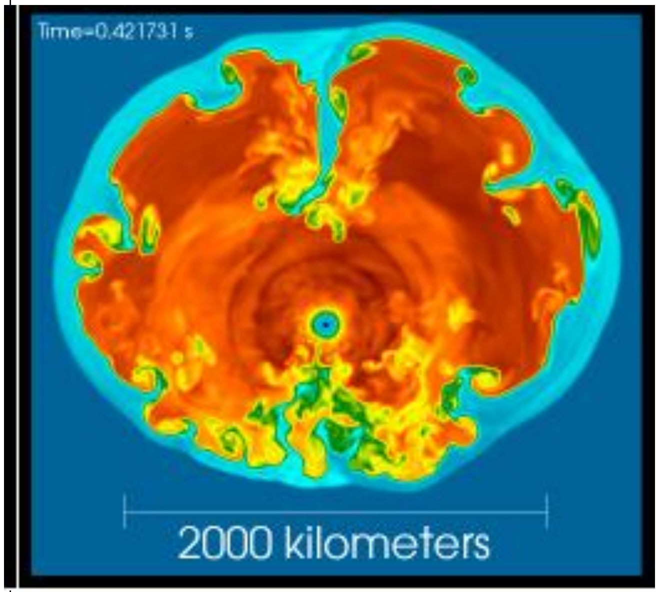


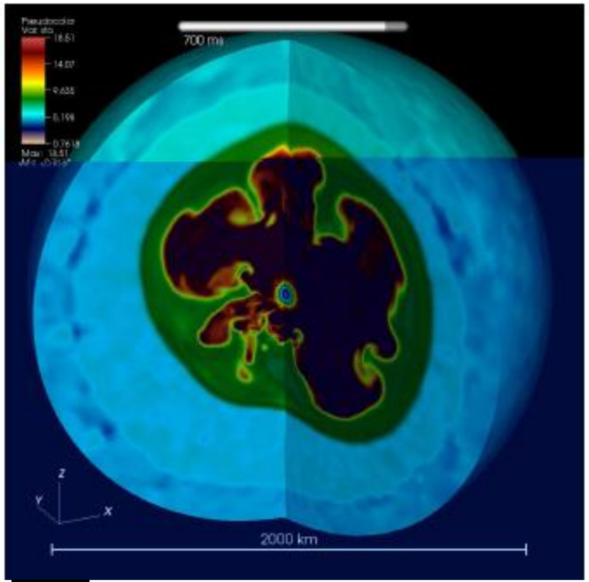


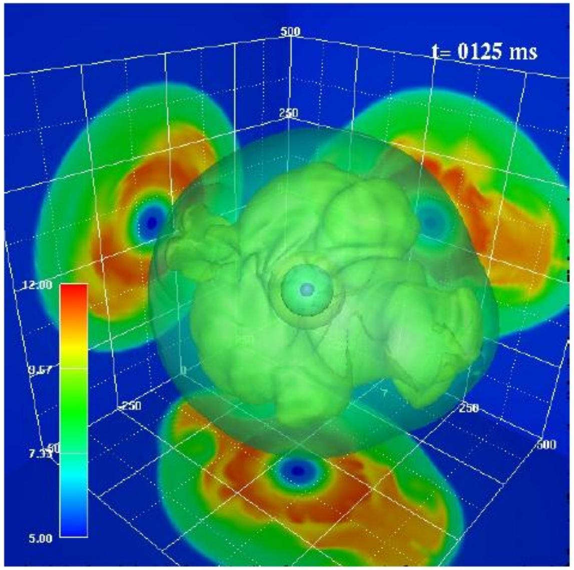


With Ray-by-Ray Boltzmann transport, marginally successful model was proposed.
Note that the explosion energy of that is very small.

Open problem:
3 dimensional supernovae modeling







Nordhaus et al. 2010

Hanke et al. 2012

Takiwaki et al. 2012

Gray, light bulb ~ **Gray, light bulb**

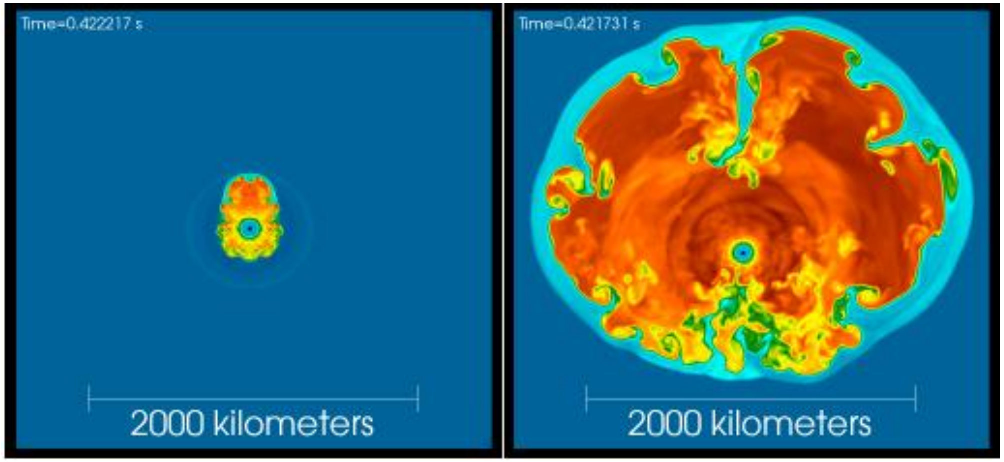
Spectral transport

AMR
minimum dx~0.5km

High resolution
At 200km
dr~3km,
r dθ~rdΦ~5km
(my estimation)

Low resolution
At 200km
dr~3.5km
r dθ~10km
r dΦ~ 40km

Princeton group said ....



Nordhaus et al. 2010

2D Cylindrical(Left) vs

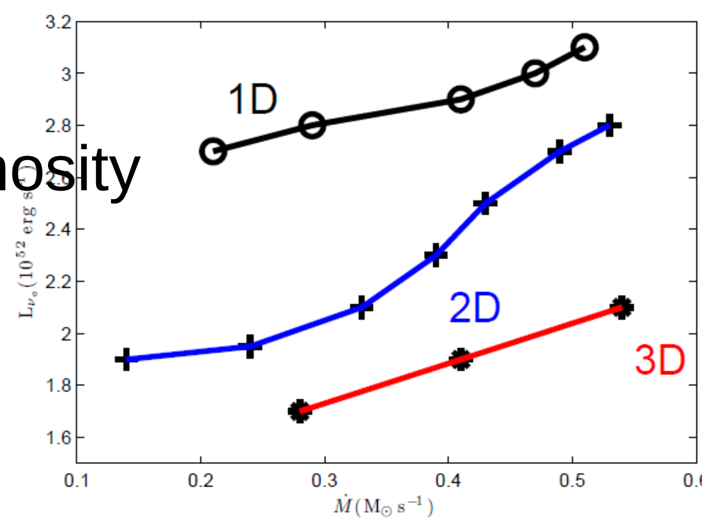
3D Cartesian(Right)

Gray Luminosity

AMR

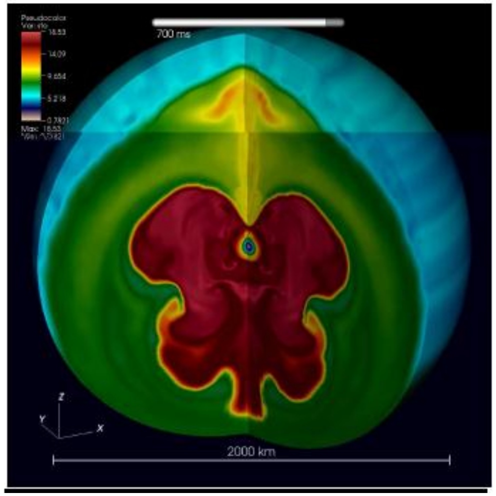
2D < 3D

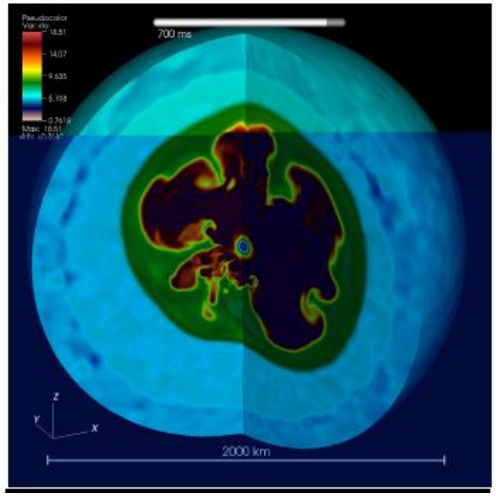
Luminosity



Mass accretion rate

MPA group said ....





Hanke et al. 2011

2D spherical(Left) vs

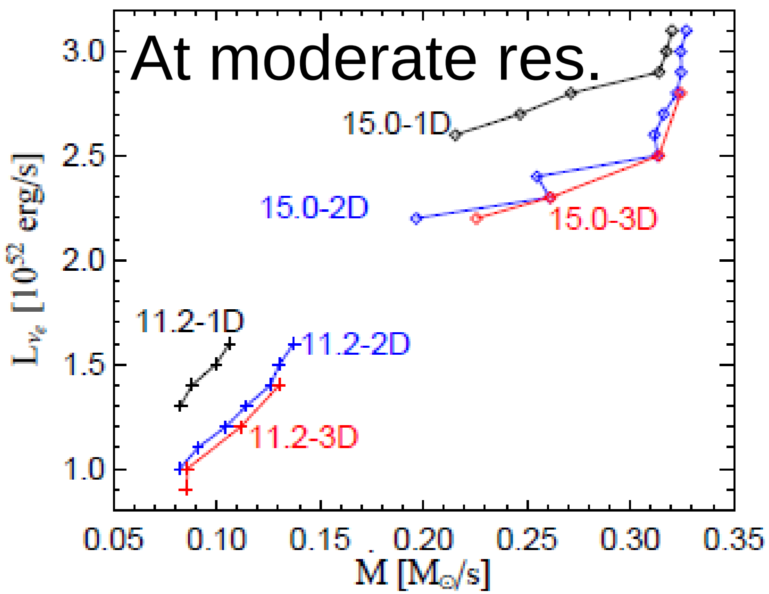
3D spherical (Right)

Gray Luminosity

Resolution study

At Moderate res. : 2D<3D (a little)

At High res. : **2D > 3D**



Motivation of this study

The effect of three dimensional convection is open question!

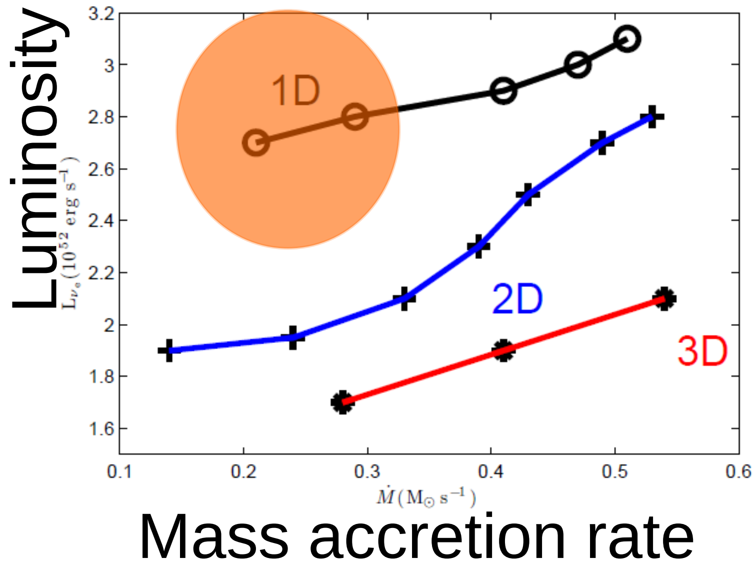
Here I want to discuss a "realistic" situation.

–L=3x10^52erg/s with diminishing trend

–Average energy of neutrino changes

–The cooling of ν _X

Nordhaus et al. 2010



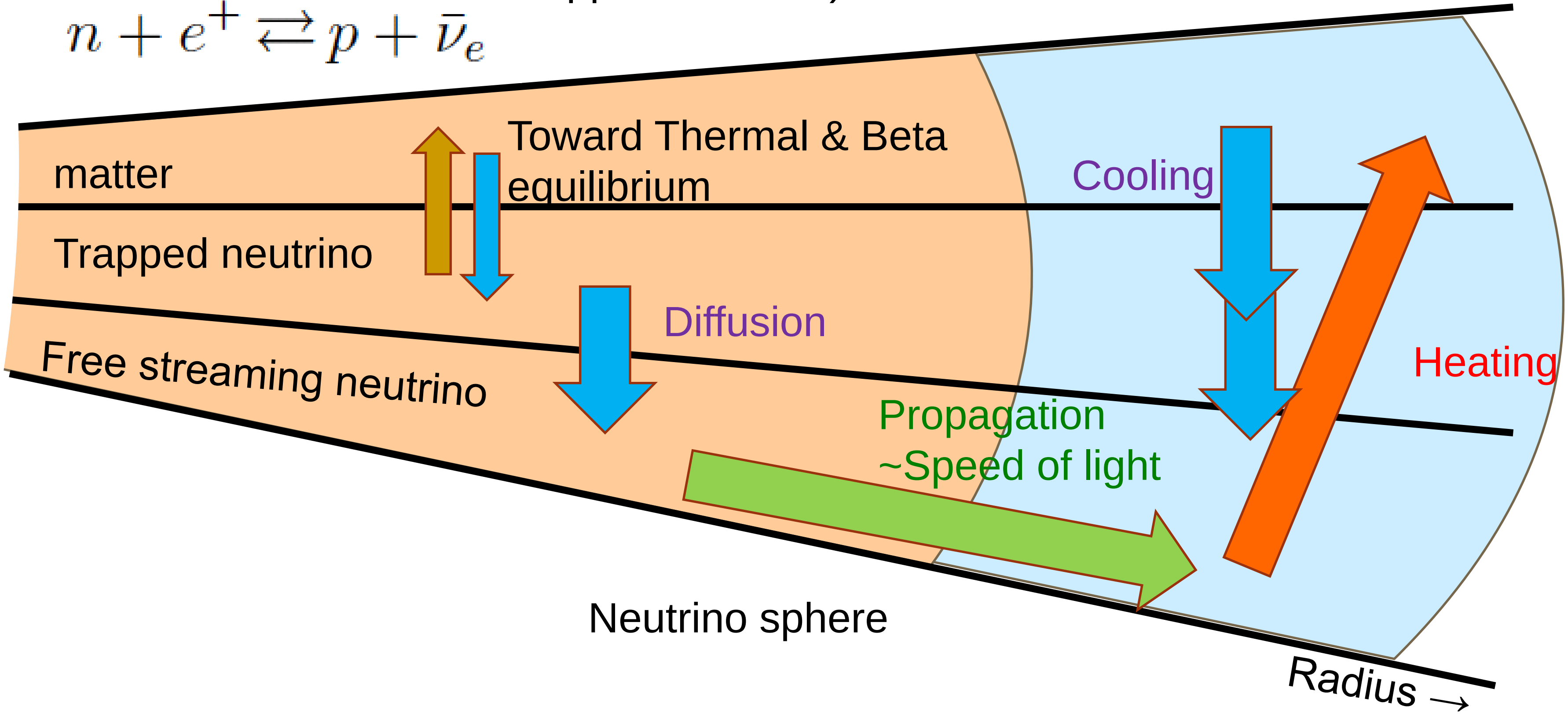
With IDSA neutrino transport scheme, we performed 3 dimensional simulations under the situation.

IDSA Neutrino transport, concept

$$p + e^- \rightleftarrows n + \nu_e$$

$$n + e^+ \rightleftarrows p + \bar{\nu}_e$$

IDSA(isotropic diffusion source approximations)



Dividing neutrino into two parts. Trapped and free streaming.
For v_X, simple leakage scheme

IDSA: trapped part

$$\frac{df}{cdt} + \mu\frac{\partial f}{\partial r} + \left[\mu\left(\frac{d\ln\rho}{cdt} + \frac{3v}{cr}\right) + \frac{1}{r}\right](1-\mu^2)\frac{\partial f}{\partial \mu}$$
$$+ \left[\mu^2\left(\frac{d\ln\rho}{cdt} + \frac{3v}{cr}\right) - \frac{v}{cr}\right]E\frac{\partial f}{\partial E}$$
$$= j(1-f) - \chi f + \frac{E^2}{c(hc)^3}$$
$$\times \left[(1-f)\int Rf'd\mu' - f\int R(1-f')d\mu'\right].$$

f(x,y,z,E,theta,phi)
6 dimensional variable

Trapped Particle

Angular integration

$$\frac{df^{\mathrm{t}}}{cdt} + \frac{1}{3}\frac{d\ln\rho}{cdt}E\frac{\partial f^{\mathrm{t}}}{\partial E} = j - (j+\chi)f^{\mathrm{t}} - \Sigma.$$

Diffusion term
(To free streaming part)

$$\Sigma = \min\left\{\max\left[\alpha + (j+\chi)\frac{1}{2}\int f^{\mathrm{s}}d\mu, 0\right], j\right\}$$
$$\alpha = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{-r^2}{3(j+\chi+\phi)}\frac{\partial f^{\mathrm{t}}}{\partial r}\right).$$

Energy integration

$$Y^{\mathrm{t}} = \frac{m_{\mathrm{b}}}{\rho}\frac{4\pi}{(hc)^3}\int f^{\mathrm{t}}E^2dEd\mu$$
$$Z^{\mathrm{t}} = \frac{m_{\mathrm{b}}}{\rho}\frac{4\pi}{(hc)^3}\int f^{\mathrm{t}}E^3dEd\mu,$$

Determine temperature and chemical potential for Fermi-Dirac distribution by Y and Z

$$\frac{\partial}{\partial t}(\rho Y^{\mathrm{t}}) + \frac{\partial}{r^2\partial r}(r^2 v\rho Y^{\mathrm{t}})$$
$$= m_{\mathrm{b}}\frac{4\pi c}{(hc)^3}\int [j - (j+\chi)f^{\mathrm{t}} - \Sigma]E^3dE.$$

$$f_l^{\mathrm{t}}(E) = \{\exp[\beta_l(E - \mu_l)] + 1\}^{-1},$$

IDSA: free streaming part

$$\frac{df}{cdt} + \mu\frac{\partial f}{\partial r} + \left[\mu\left(\frac{d\ln\rho}{cdt} + \frac{3v}{cr}\right) + \frac{1}{r}\right]\left(1-\mu^2\right)\frac{\partial f}{\partial\mu}$$
$$+ \left[\mu^2\left(\frac{d\ln\rho}{cdt} + \frac{3v}{cr}\right) - \frac{v}{cr}\right]E\frac{\partial f}{\partial E}$$
$$= j\left(1-f\right) - \chi f + \frac{E^2}{c\left(hc\right)^3}$$
$$\times\left[\left(1-f\right)\int Rf'd\mu' - f\int R\left(1-f'\right)d\mu'\right].$$

f(x,y,z,E,theta,phi)
6 dimensional variable

Weak coupling

$$\frac{\partial\hat{f}^s}{c\partial\hat{t}} + \hat{\mu}\frac{\partial\hat{f}^s}{\partial r} + \frac{1}{r}\left(1-\hat{\mu}^2\right)\frac{\partial\hat{f}^s}{\partial\hat{\mu}} = -\left(\hat{j}+\hat{\chi}\right)\hat{f}^s + \hat{\Sigma}.$$

Angular integration

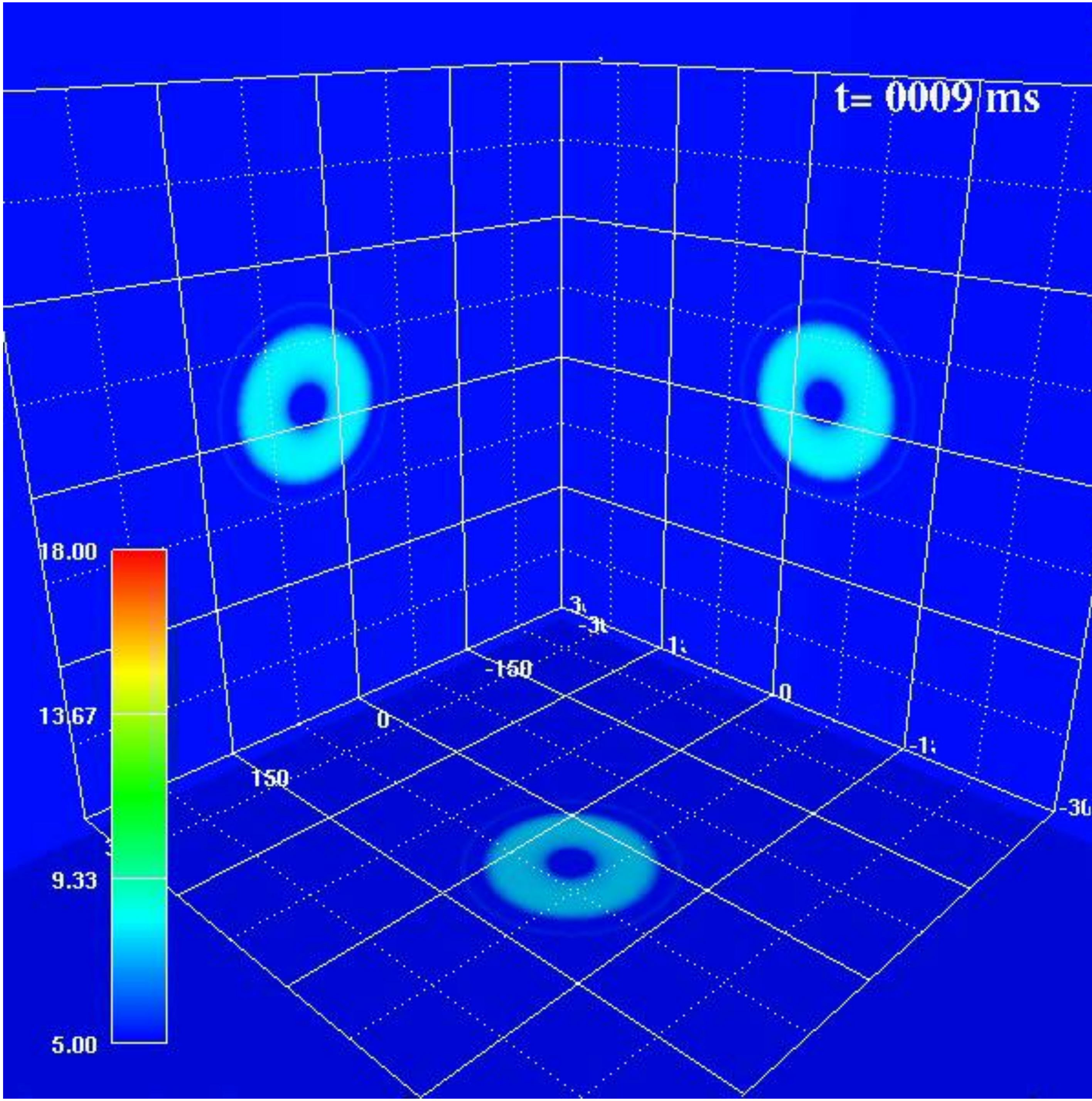
↓Ray-by-Ray approximation is used

$$\frac{\partial}{c\partial\hat{t}}\int \mathrm{d}\hat{\mu}\hat{f}^s + \frac{1}{r^2}\frac{\partial}{\partial r}r^2\int \mathrm{d}\hat{\mu}\hat{\mu}\hat{f}^s = \int \mathrm{d}\hat{\mu} - \left(\hat{j}+\hat{\chi}\right)\hat{f}^s + \hat{\Sigma}$$

Different from the original IDSA,
We treat the LHS explicitly and the RHS implicitly.

f(x,y,z,E)
4 dimensional variable

Newton Method is used for solving RHS.
No message passing during the iteration.

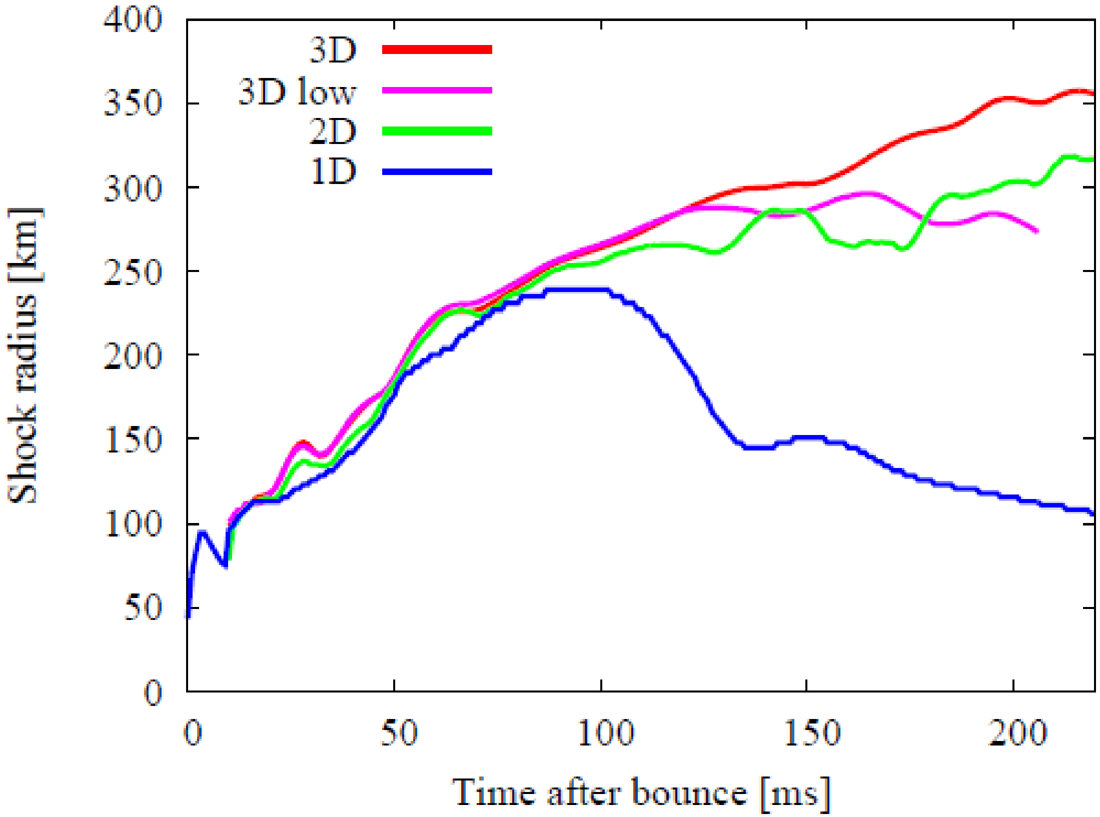


320x64x32
(Takiwaki+12)
->
320x64x128
r:0-5000km

3D sim. begins from 10ms after bounce.

11.2M_s
LS EOS
(K=180MeV)

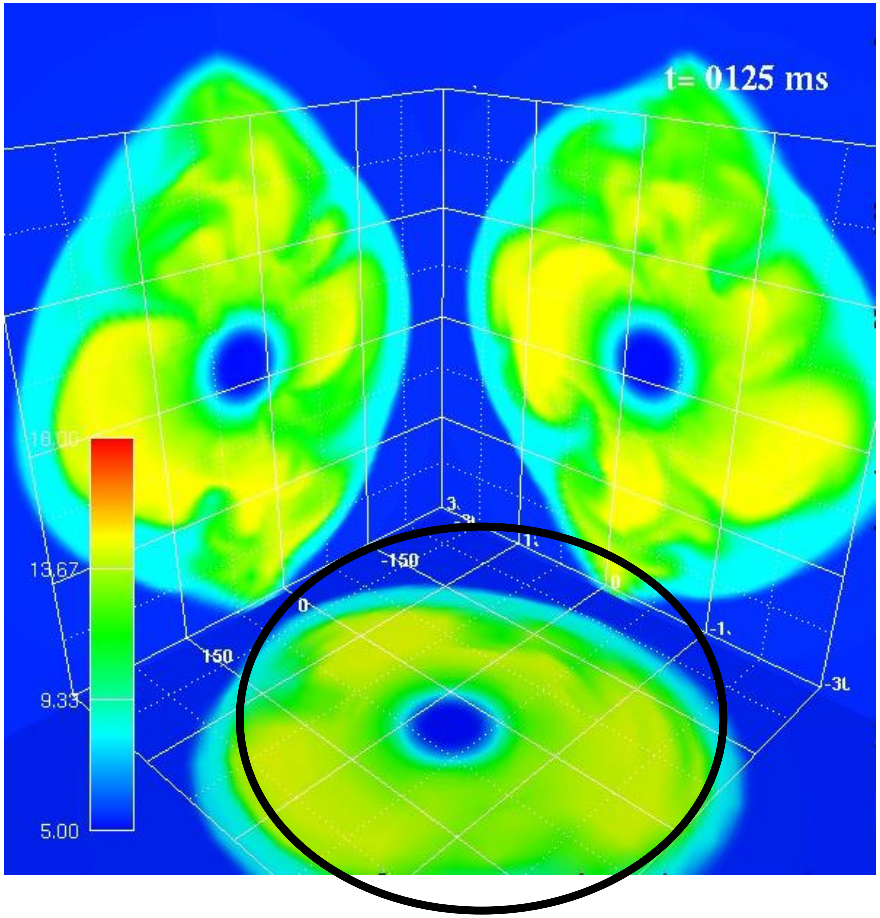
Evolution of Shock



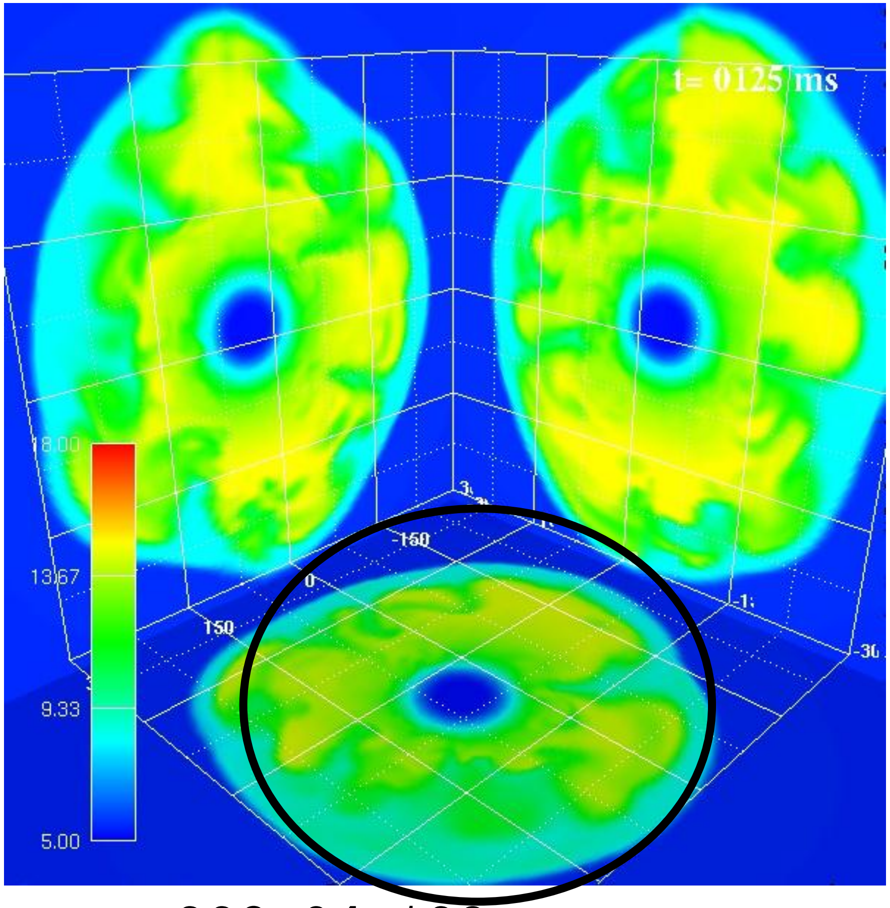
1D : 320
2D : 320x64
3D : 320x64x128
3D low: 200x32x 64

High-resolution-3D model is the best!

Difference of Resolution



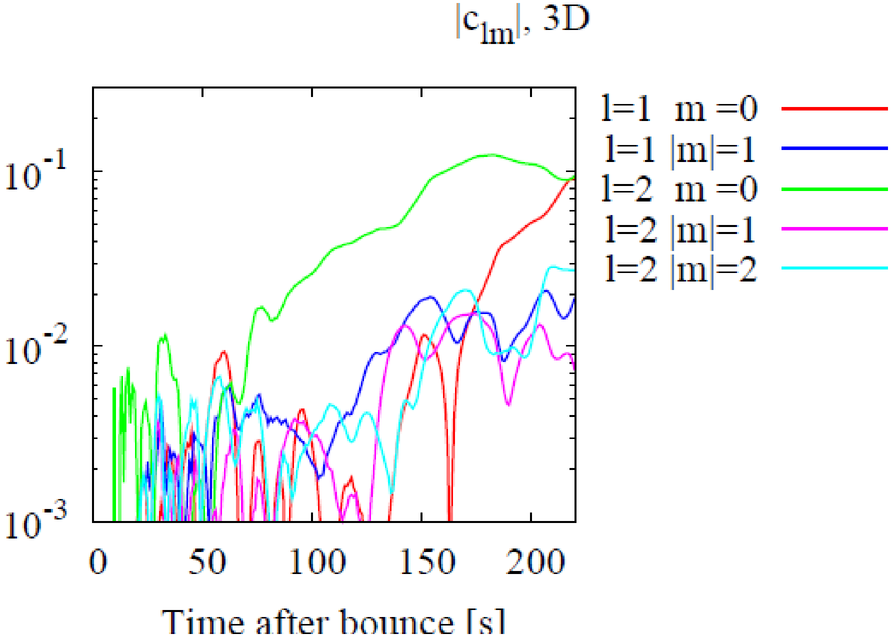
300x64x32

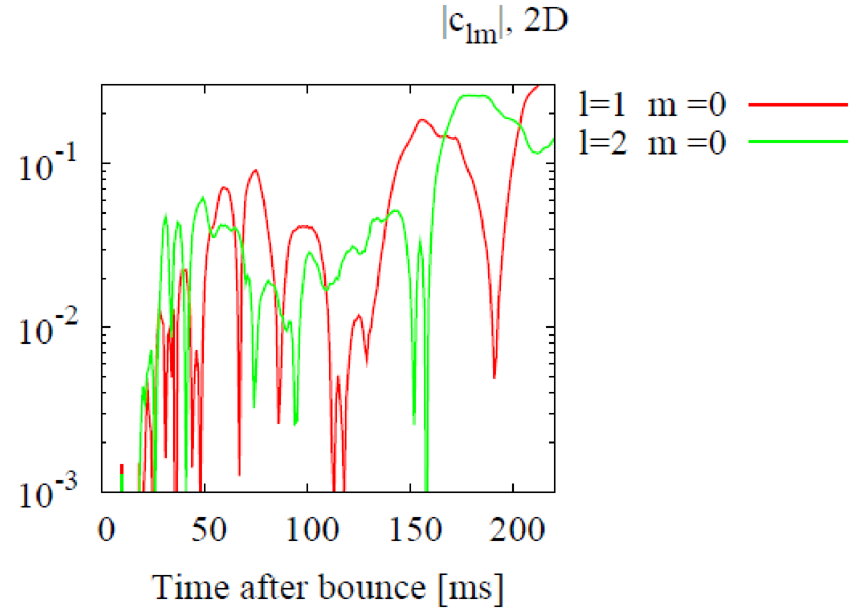


320x64x128

Fine structure inside the shock is found in high-resolution model!

Anisotropy of shock, SASI activity





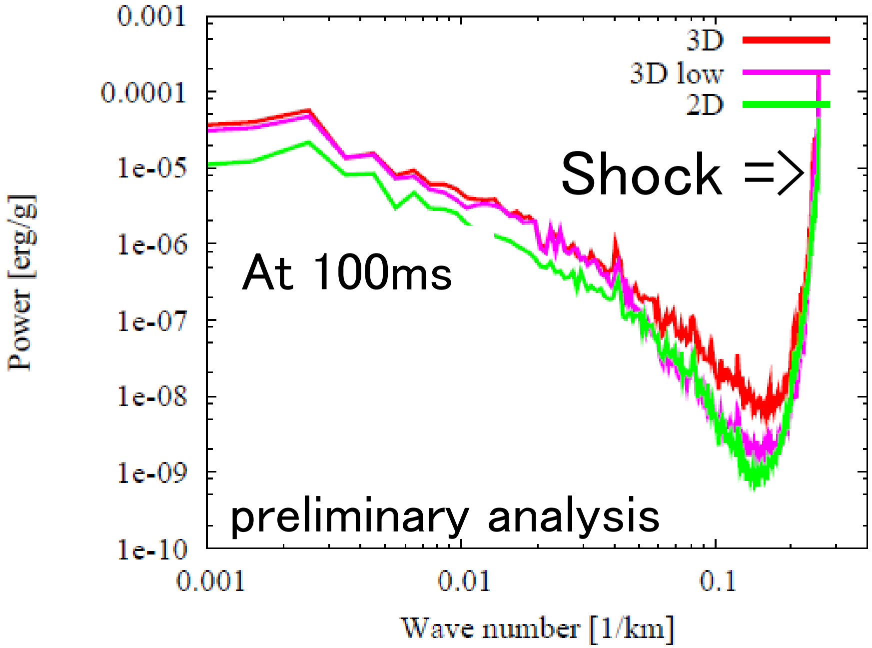
$$R_S(\theta,\phi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l} c_{lm}\,Y_{lm}(\theta,\phi),$$

Spherical harmonics of the shock radius

50ms–150ms: The amplitude is decreased, during the expansion of the shock.

In 3D, l=1, m=0 mode is relatively small.

Fourier Analysis



Fourier analysis is given by these step.

1. Cal. angle averaged radial velocity
2. Cal. Deviation of the velocity from the averaged velocity
3. Fourier transform the velocity got in the step2.

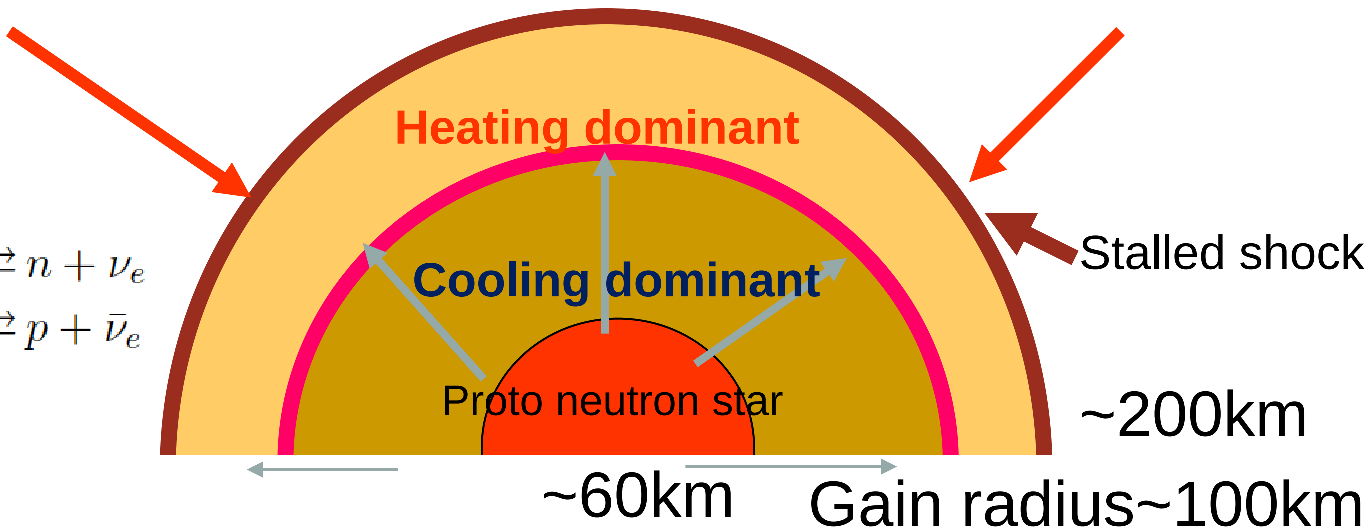
The power law index is steeper than −5/3.

Effective resolution is coarse for 2D that might suppress the growth of the turbulence. In the small scale(large wave number), power of 3D is bigger than 2D.

Coarse grid gives weaker power especially in the small scale.

Advection time vs Heating time

$$p + e^- \rightleftarrows n + \nu_e$$
$$n + e^+ \rightleftarrows p + \bar{\nu}_e$$



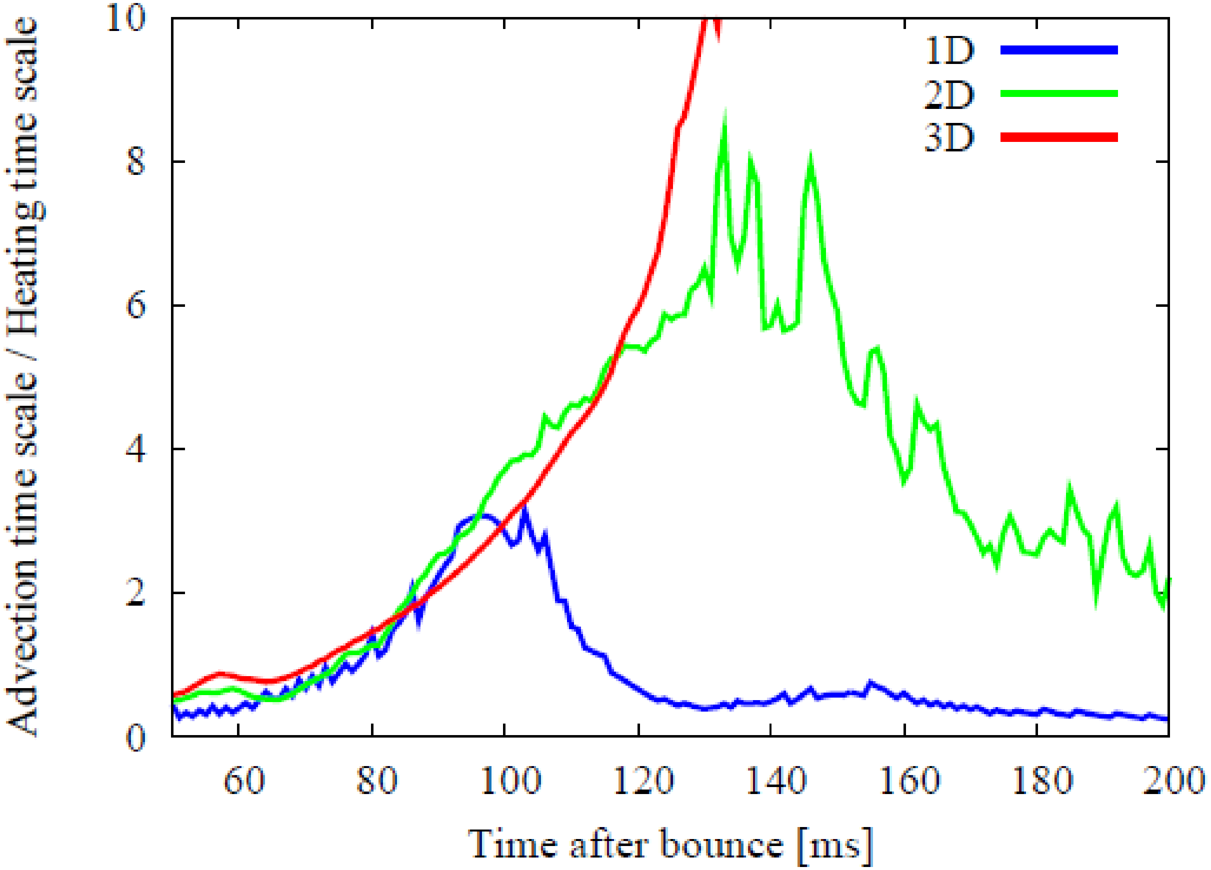
Advection time scale:
Accreted matter passes heating dominant region in this time scale

Heating time scale:
After this time scale, the matter become unbound from gravitational potential

$$\tau_{\mathrm{adv}}(t) = -\int_{r_{\mathrm{gain}}(t)}^{r_{\mathrm{sh}}(t)} \frac{1}{v_r(r,t)} \mathrm{d}r \quad > \quad \tau_{\mathrm{heat}}(t) = \frac{4\pi \int_{r_{\mathrm{gain}}(t)}^{r_{\mathrm{sh}}(t)} \varepsilon_{\mathrm{bind}}^{\mathrm{shell}}(r,t)\rho(r,t) r^2 \mathrm{d}r}{4\pi \int_{r_{\mathrm{gain}}(t)}^{r_{\mathrm{sh}}(t)} Q(r,t) r^2 \mathrm{d}r}$$

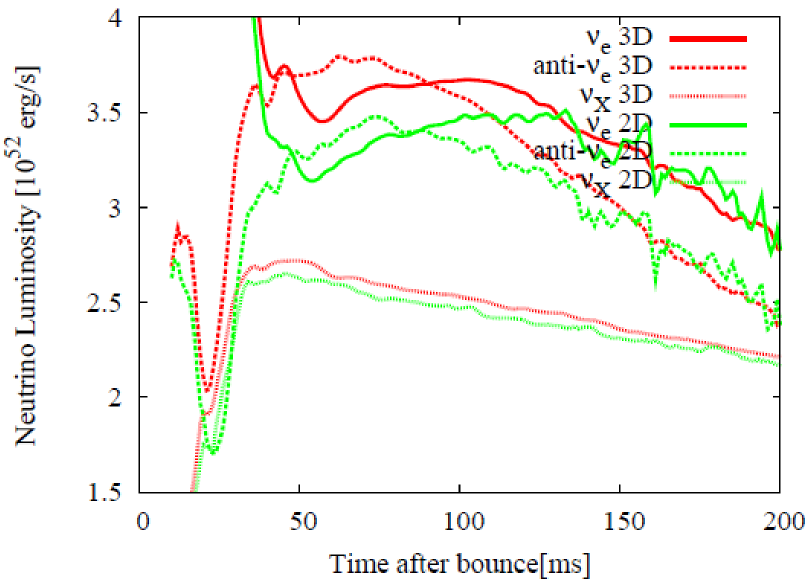
The ratio of the two time scale is important probe to judge success of supernovae.

Advection time vs Heating time



The ratio of the two timescale of 3D is actually bigger than the others.

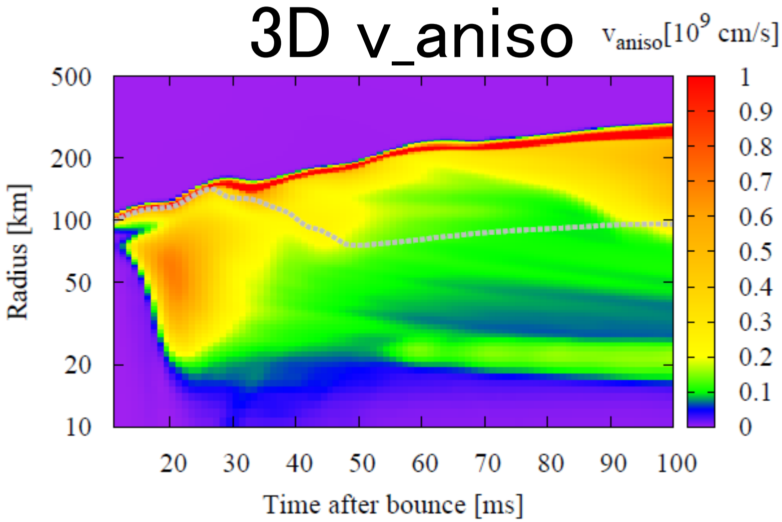
Neutrino Heating 3D vs 2D



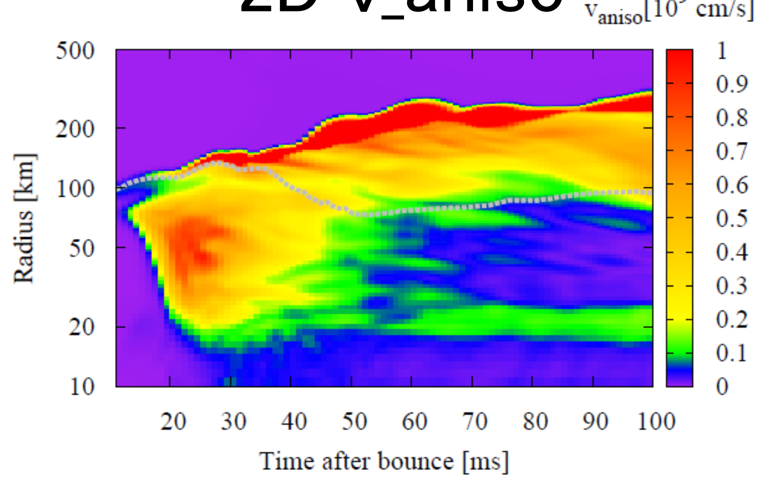
Luminosity of 3D is larger than that of 2D.

Convection below the neutrino sphere(~70km) of 3D is stronger than that of 2D.

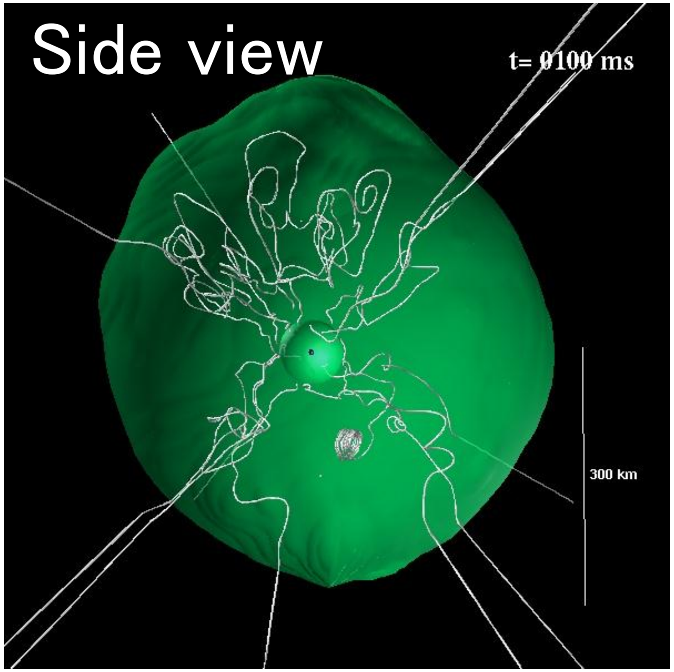
3D v_aniso

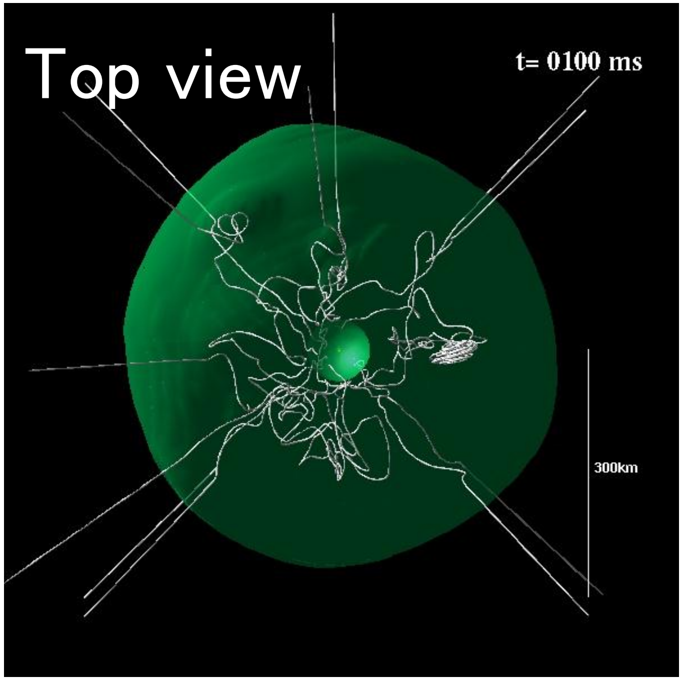


2D v_aniso



Tracer Particle analysis



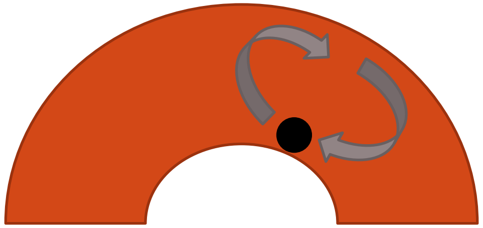


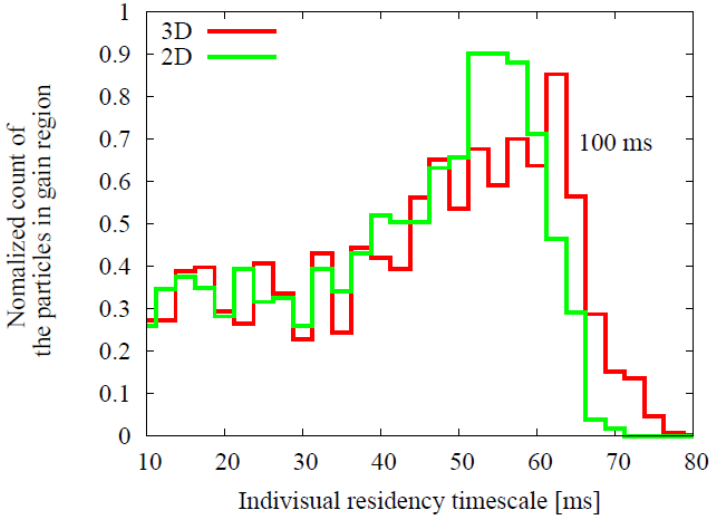
We performed tracer particle analysis.

Deposit particles everywhere and follow their advection.

Advection timescale: detailed comparison 3D vs 2D

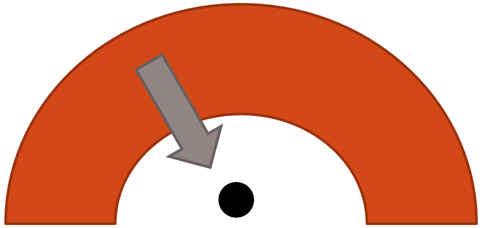
Particles remaining inside the gain region

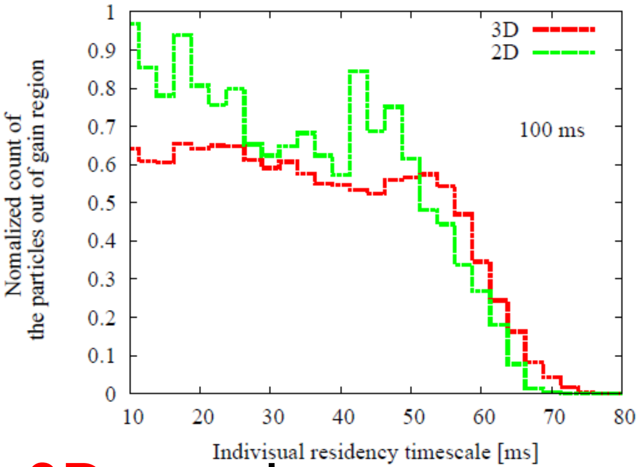




3D > 2D

Particle escaped from the gain region

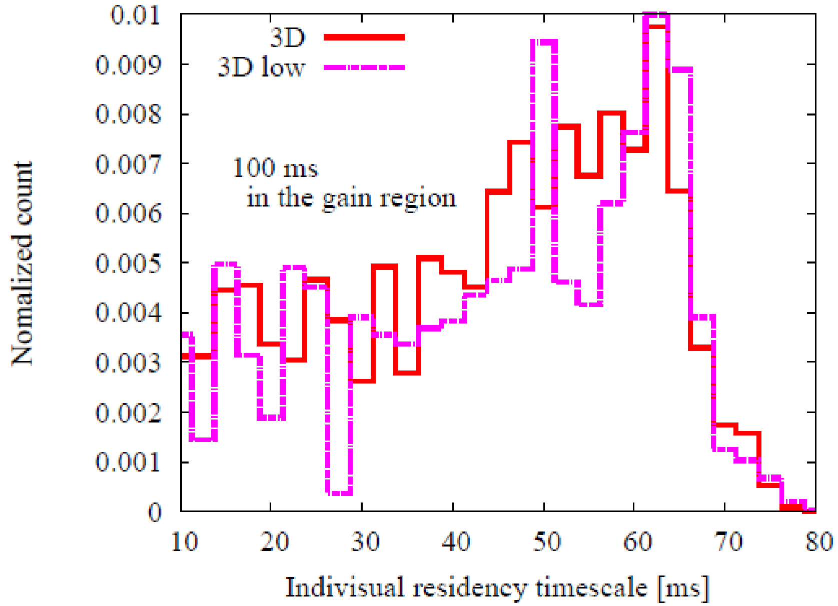




3D can keep more particles inside the gain region than 2D.

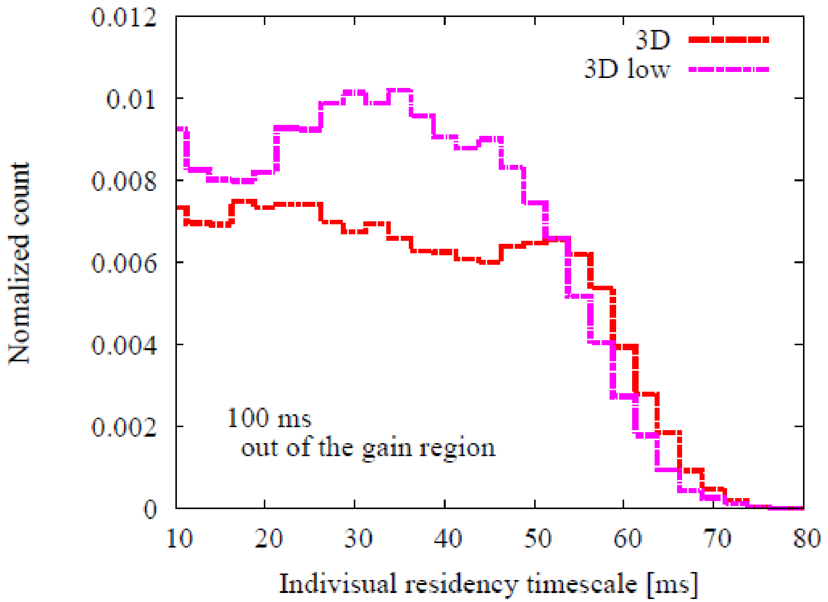
Advection timescale: detailed comparison Resolution dependence

Particles remaining inside the gain region



3D high > 3D low

Particle escaped from the gain region



3D high can keep more particles inside the gain region than 3D low.

Summary

We performed 3D simulations that begins with core-collapse of 11.2 M_s progenitor with spectral neutrino transport.

We find the average shock radius of 3D high resolution model go faster than the other models.

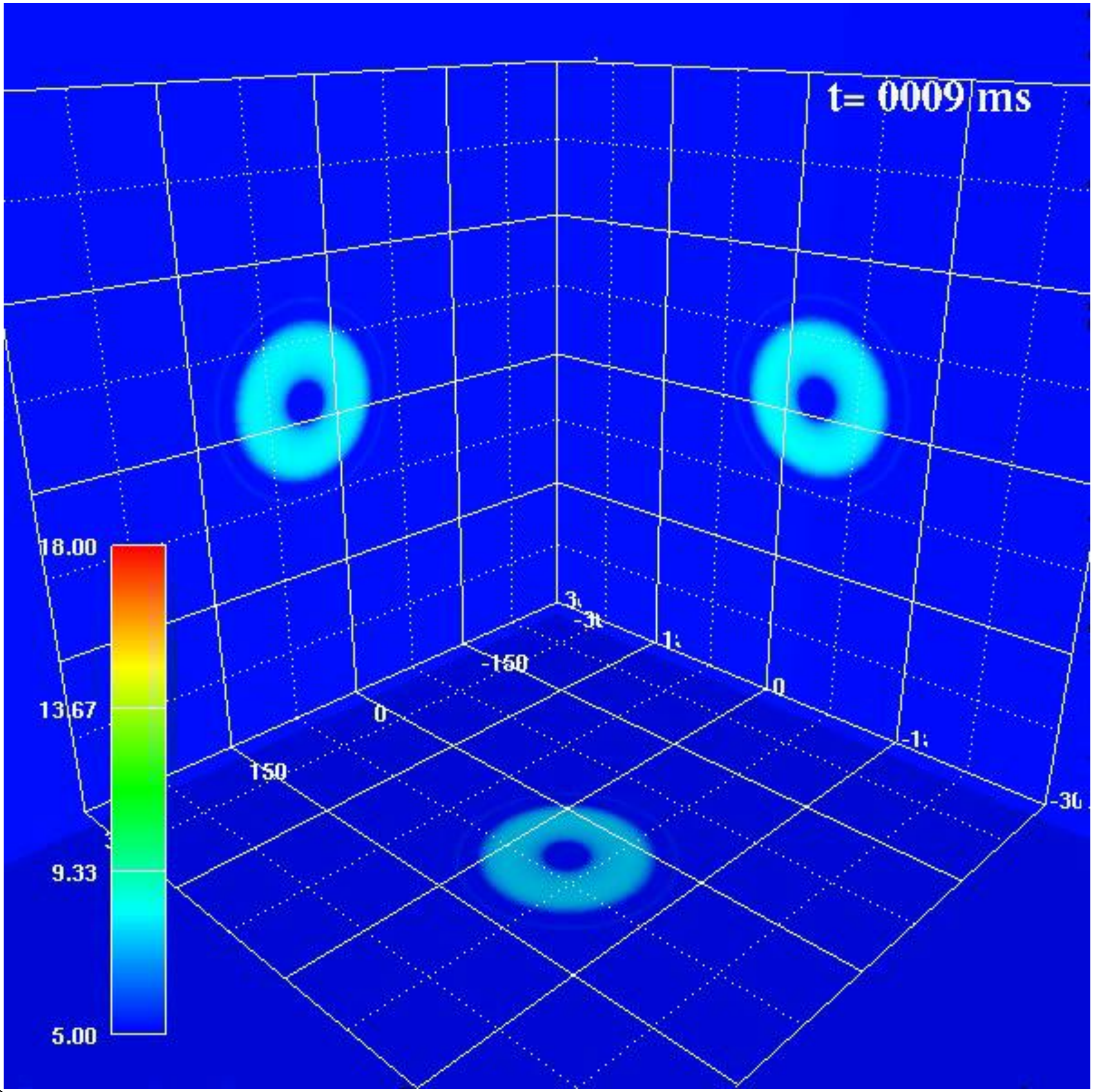
That is mainly because the neutrino luminosity of that models is larger than 2D.

Dwell time of 3D high res. model is longer than the other model.

The difference might gives critical difference in the case of a heavy progenitor.

Anyway to conclude robustly, more high-resolution study will be necessary.

Test Computation with K computer



Using K computer, we can perform a study with longer duration.

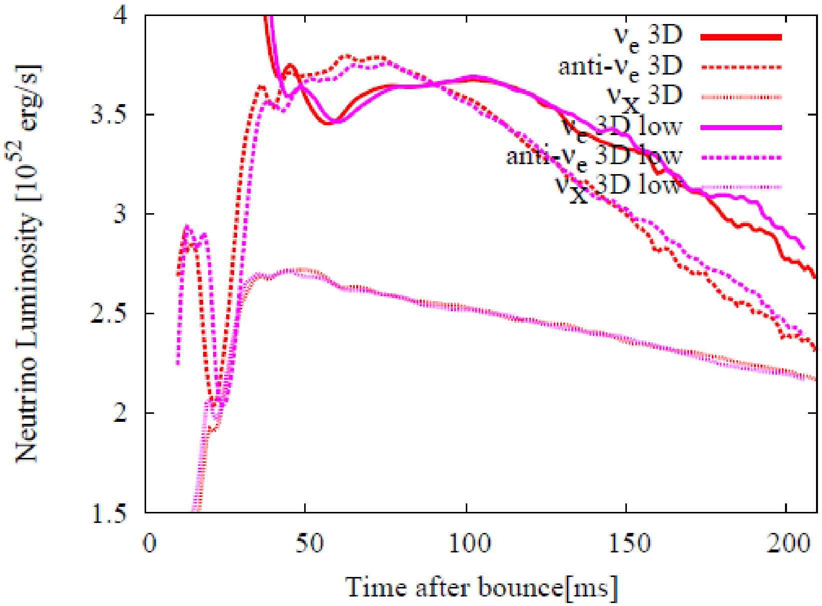
320x64x128

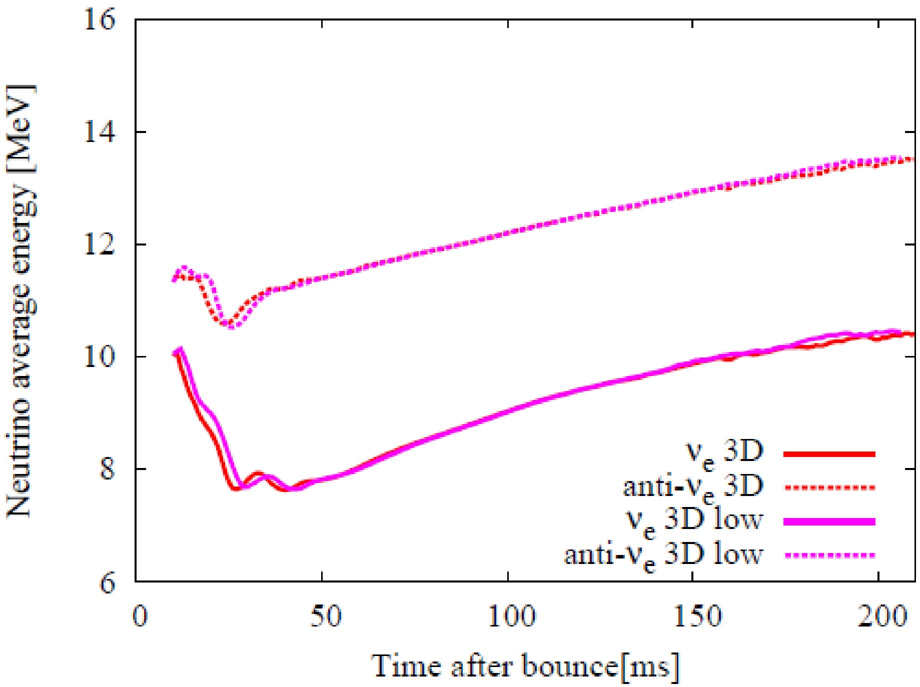
4096 parallel

This is just a test, we aim studies with higher resolution.

Appendix

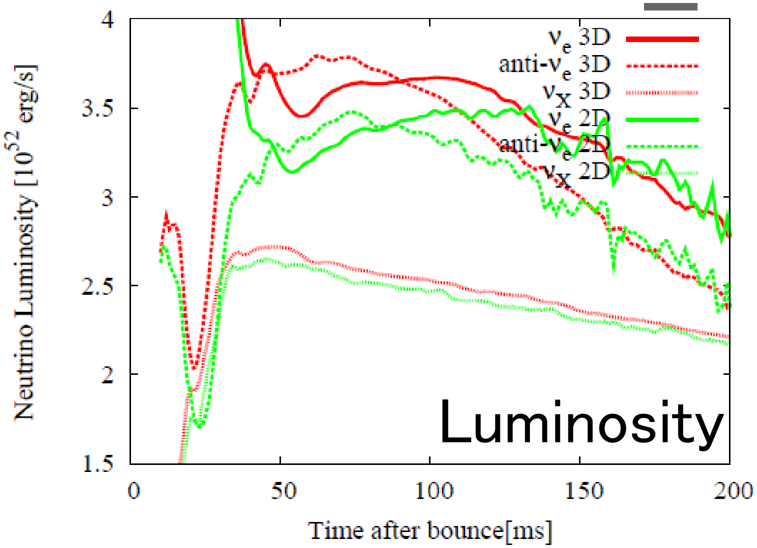
Neutrino Heating 3D vs 3D low

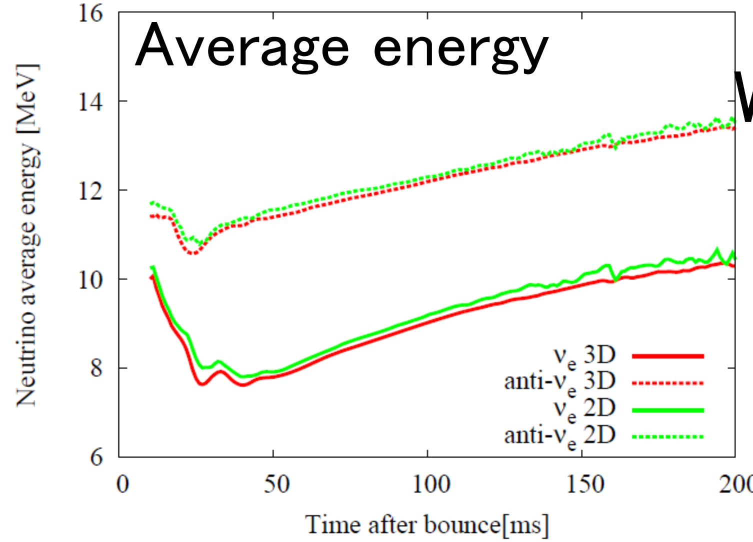




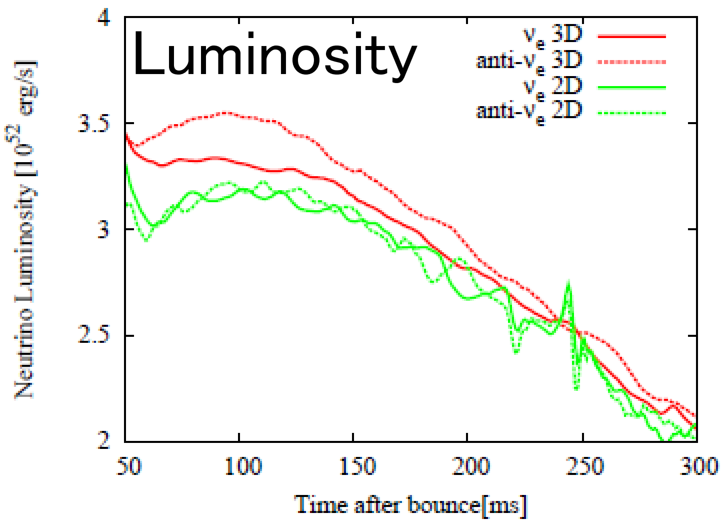
Luminosity is not so different between models.

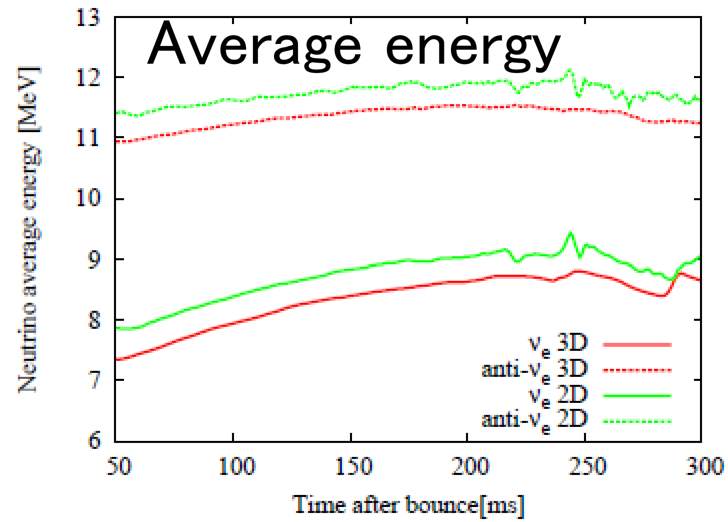
The effect of ν_X





Withν _X





Without ν _X

Takiwaki +12

In Takiwaki+ 2012, the shock of 2D is more energetic than 3D.

Because average energy of neutrino is bigger than that of 3D.