The Nuclear Equation of State for Core-Collapse Supernovae with the Variational Method

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Contents

Formation of Compact Objects: from the cradle to the grave @ Waseda University, Mar. 8

1. Introduction

The aim of this study is

To construct *a new nuclear Equation of State (EOS)*

for supernova (SN) simulations

based on the realistic nuclear force.

The nuclear EOS plays an important role for astrophysical studies.

1. Lattimer-Swesty EOS : *The compressible liquid drop model* (NPA 535 (1991) 331)
2. Shen EOS : *The relativistic mean field theory* (NPA 637 (1998) 435)

- K. Nakazato EOS : (PRD 77(2008) 103006)
- G. Shen EOS : (PRC 83 (2010) 015806)
- C. Ishizuka EOS : (J. Phys. G 35(2008)085201)
- M. Hempel EOS : (NPA 837 (2010) 210)
- S. Furusawa EOS : (APJ 738 (2011) 178)

These EOSs are based on **phenomenological models** for uniform matter.

*There is **no** nuclear EOS based on **the microscopic many-body theory**.*

We aim at **a new EOS for SN** with the variational method.

Our Plan to Construct the EOS for SN Simulations

Uniform Nuclear Matter

EOS constructed with *the cluster variational method*

CLEAR

Non-uniform Nuclear Matter

EOS constructed with *the Thomas-Fermi (TF) calculation*

1. EOS for non-uniform matter at **zero temperature**

★We are here.★

2. EOS for non-uniform matter at **finite temperature**

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Completion of a Nuclear EOS table for SN simulations

Density ρ : $10^{5.1} \le \rho_m \le 10^{16.0}$ g/cm^3	110 point
Temperature T : $0 \le T \le 400$ MeV	92 point
Proton fraction x : $0 \le x \le 0.65$	66 point

2. EOS for Uniform Nuclear Matter

The Nuclear Hamiltonian

$$H = H_2 + H_3$$

Two-body Hamiltonian

$$H_2 = -\sum_{i=1}^{N} \frac{\hbar^2}{2m}\nabla_i^2 + \sum_{i<j}^{N} V_{ij}$$

the AV18 two-body nuclear potential

Three-body Hamiltonian

$$H_3 = \sum_{i<j<k}^{N} V_{ijk}$$

the UIX three-body nuclear potential

We assume the Jastrow wave function.

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right]\Phi_{\mathrm{F}}$$

Φ_{F}: The Fermi-gas wave function at zero temperature

$P_{ts}{}^{\mu}$: Spin-isospin projection operators

f_{ij} : Correlation function

$$f_{ij} = \sum_{t=0}^{1}\sum_{\mu}\sum_{s=0}^{1}\left[\underbrace{f_{\mathrm{C}ts}^{\mu}(r_{ij})}_{\text{Central}} + \underbrace{s f_{\mathrm{T}t}^{\mu}(r_{ij})}_{\text{Tensor}} S_{\mathrm{T}ij} + \underbrace{s f_{\mathrm{SO}t}^{\mu}(r_{ij})}_{\text{Spin-orbit}}(\boldsymbol{L}_{ij}\cdot\boldsymbol{s})\right] P_{tsij}^{\mu}$$

Two-Body Energy

E_2/N is the expectation value of H_2 with the Jastrow wave function in *the two-body cluster approximation*.

$$\frac{E_2}{N}(\rho, x) = \frac{< H_2 >_2}{N}$$

ρ : Total nucleon number density

ρ_p : Proton number density $\quad x = \rho_\mathrm{p}/\rho$: Proton fraction

E_2/N is minimized with respect to $f_{\mathrm{C}ts}{}^{\mu}(r)$, $f_{\mathrm{T}t}{}^{\mu}(r)$ and $f_{\mathrm{SO}t}{}^{\mu}(r)$ with the following two constraints.

1. Extended Mayer's condition

$$\rho \int \left[F^{\mu}_{ts}(r) - F^{\mu}_{\mathrm{F}ts}(r) \right] d\boldsymbol{r} = 0$$

$F_{ts}{}^{\mu}(r)$: Radial distribution functions

$F_{\mathrm{F}ts}{}^{\mu}(r)$: $F_{ts}{}^{\mu}(r)$ for the degenerate Fermi gas

2. Healing distance condition

Healing distance

$$r_\mathrm{h} = a_\mathrm{h} r_0$$

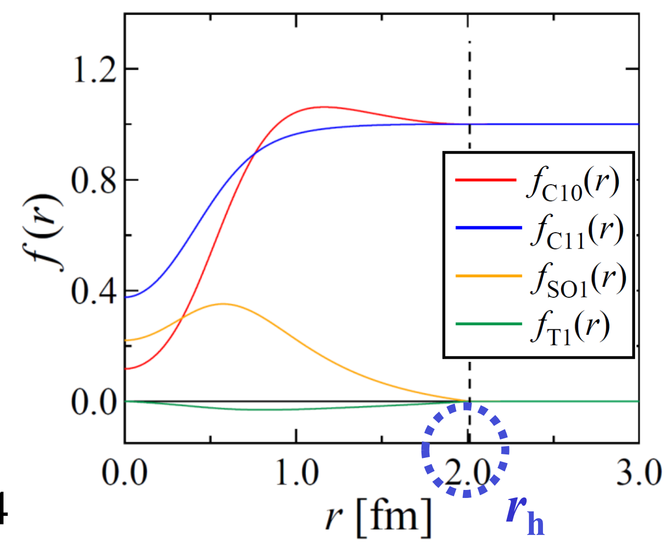
a_h : adjustable parameter

Mean distance between nucleons

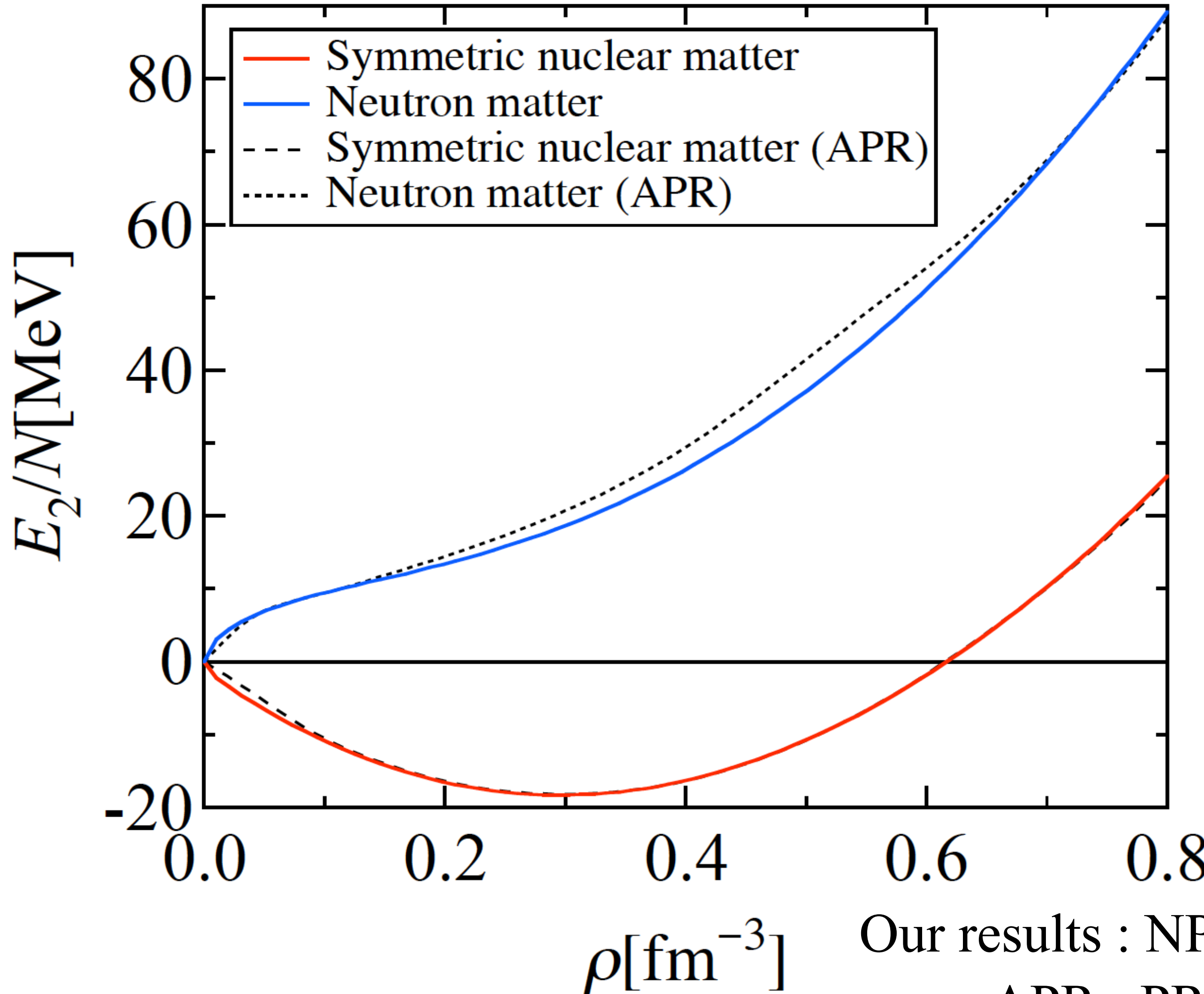
$$r_0 = \left(\frac{3}{4\pi\rho} \right)^{1/3}$$

a_h is determined so that E_2/N reproduces the results by APR(Akmal, Pandharipande and Ravenhall)

APR : PRC58(1998)1804



Two Body Energy



Our results : NPA791(2007)232
APR : PRC58(1998)1804

Our results are in good agreement with the results by APR (FHNC method).

Three-Body Energy

UIX potential

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{R}$$

$V_{ijk}^{2\pi}$:2 pion exchange part

V_{ijk}^{R} :Repulsive part

Expectation values with the Fermi-gas wave function

$$\frac{E_3^R}{N} = \frac{1}{N}\sum_{i<j<k}^{N} \langle V_{ijk}^{R} \rangle_{\mathrm{F}} \qquad \frac{E_3^{2\pi}}{N} = \frac{1}{N}\sum_{i<j<k}^{N} \langle V_{ijk}^{2\pi} \rangle_{\mathrm{F}}$$

Three Body Energy

$$\frac{E_3}{N}(\rho, x) = \alpha \frac{E_3^R}{N}(\rho, x) + \beta \frac{E_3^{2\pi}}{N}(\rho, x) + \gamma \rho^2 e^{-\delta\rho}\left[1 - (1-2x)^2\right]$$

$\alpha, \beta, \gamma, \delta$: adjustable parameters

Correction term

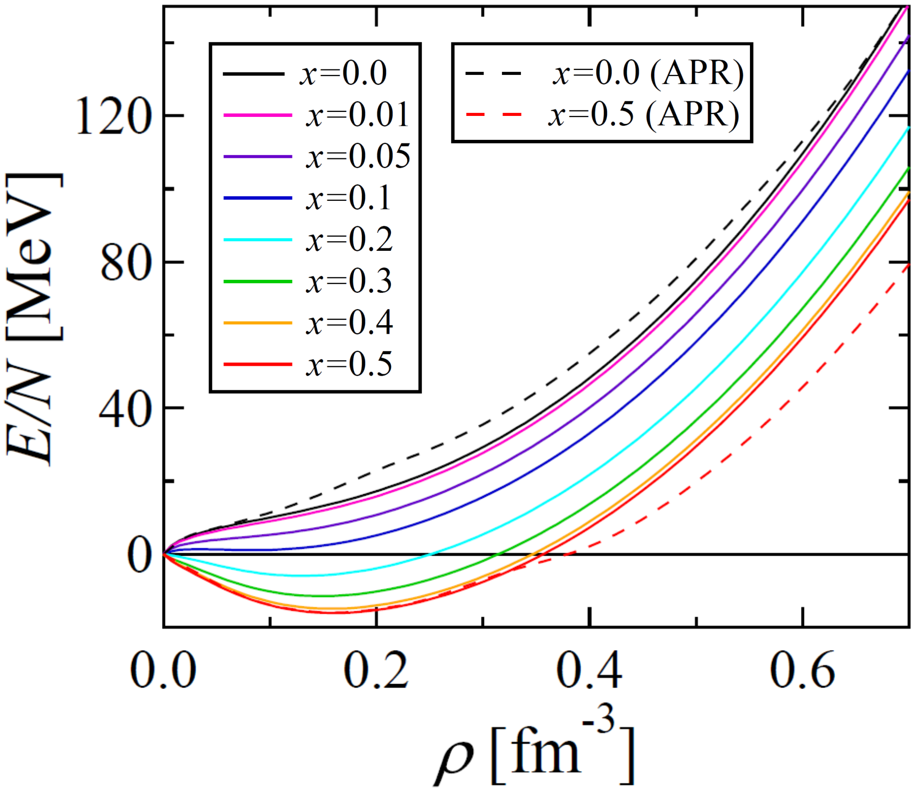
Parameters of E_3/N are determined so that

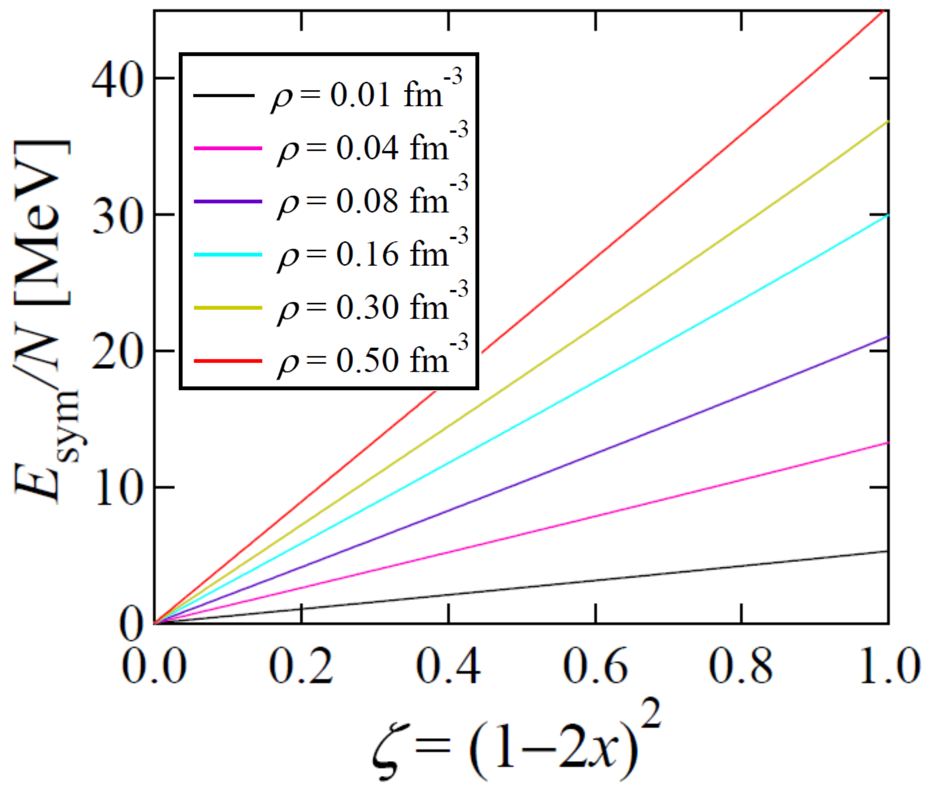
TF calculation for atomic nuclei reproduces the gross feature of the experimental data.

Total Energy per Nucleon at Zero Temperature

Total energy per nucleon

$$\frac{E}{N} = \frac{E_2}{N} + \frac{E_3}{N}$$





ρ_0[fm^{-3}]	E_0[MeV]	K[MeV]	E_{sym}[MeV]
0.16	−16.1	240	30.0

Symmetry Energy

$$\frac{E_{sym}(x)}{N} = \frac{E(x)}{N} - \frac{E(x=1/2)}{N}$$

Free Energy at Finite Temperatures I

We follow the prescription proposed by *Schmidt and Pandharipande.*

(Phys. Lett. 87B(1979) 11) (A. Mukherjee et al., PRC 75(2007) 035802)

Free Energy

$$\frac{F}{N} = \frac{E_0}{N} - T\frac{S_0}{N}$$

$\frac{E_0}{N}$: Approximate Internal Energy

$\frac{S_0}{N}$: Approximate Entropy

S_0/N is expressed with the averaged occupation probabilities $n_i(k)$

Approximate Internal Energy

$$\frac{E_0}{N} = \frac{E_2}{N} + \frac{E_3}{N}$$

chosen to be the same as at 0 MeV

E_2/N : Expectation value of H_2 with the Jastrow-type wave function at finite temperature.

$$\Psi(T) = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right] \Phi_\mathrm{F}(n_\mathrm{p}(k), n_\mathrm{n}(k))$$

Φ_F: The Fermi-gas wave function expressed with $n_i(k)$

Free Energy at Finite Temperatures II

The averaged occupation probability

$$n_i(k) = \left\{1 + \exp\left[\frac{\varepsilon_i(k) - \mu_i}{k_\mathrm{B} T}\right]\right\}^{-1} \qquad (i = \mathrm{p, n})$$

μ_i is determined with the normalization condition.

$\varepsilon_i(k)$: Single particle energy

$$\varepsilon_i(k) = \frac{\hbar^2 k^2}{2m_i^*}$$

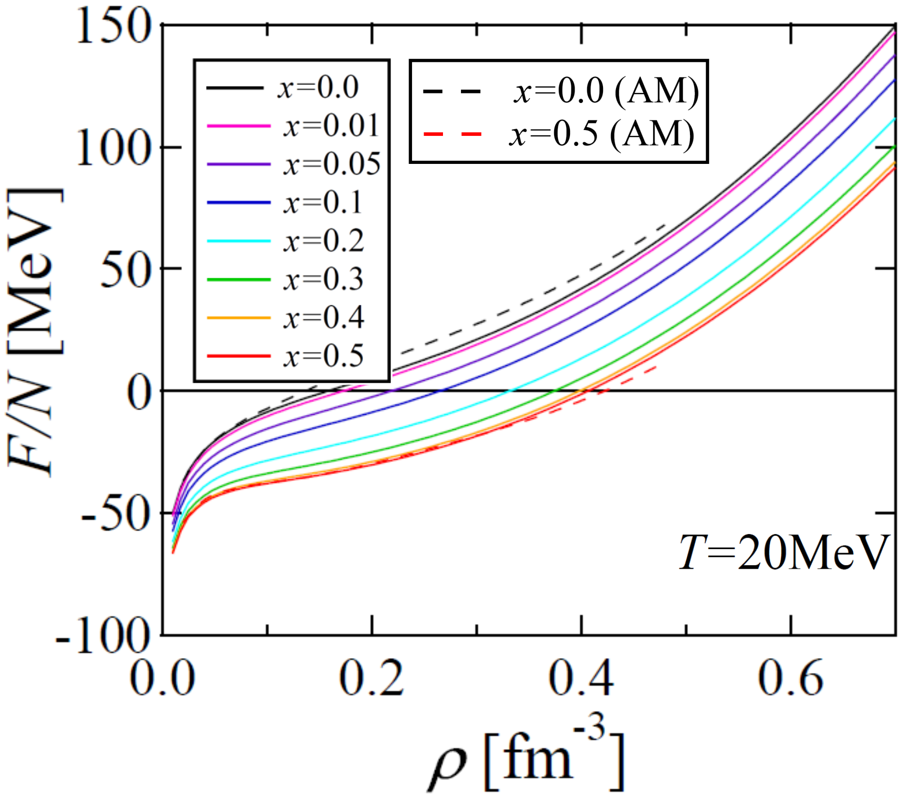
m_i*: Effective mass of nucleons

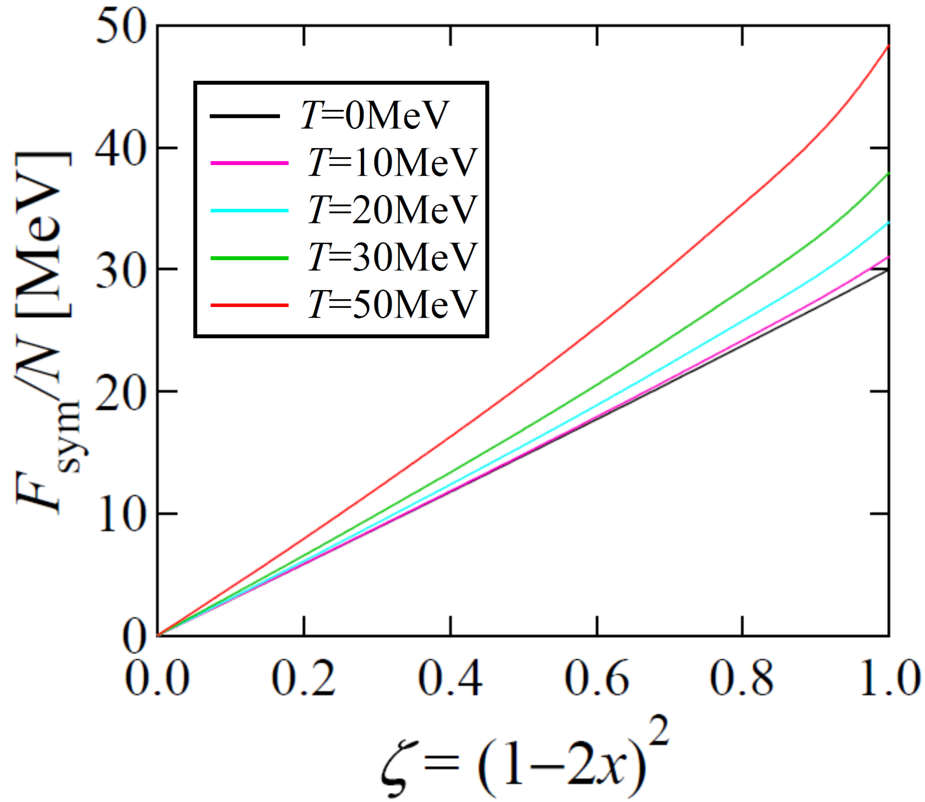
Approximate Entropy

$$\frac{S_0}{N} = -\frac{k_\mathrm{B}}{N} \sum_{i=\mathrm{p,n}} \sum_{\mathrm{spin}} \sum_{k} \left\{[1 - n_i(k)] \ln [1 - n_i(k)] + n_i(k) \ln n_i(k)\right\}$$

Free energies are minimized with respect to m_p* and m_n*

Free Energy per Nucleon at Finite Temperatures



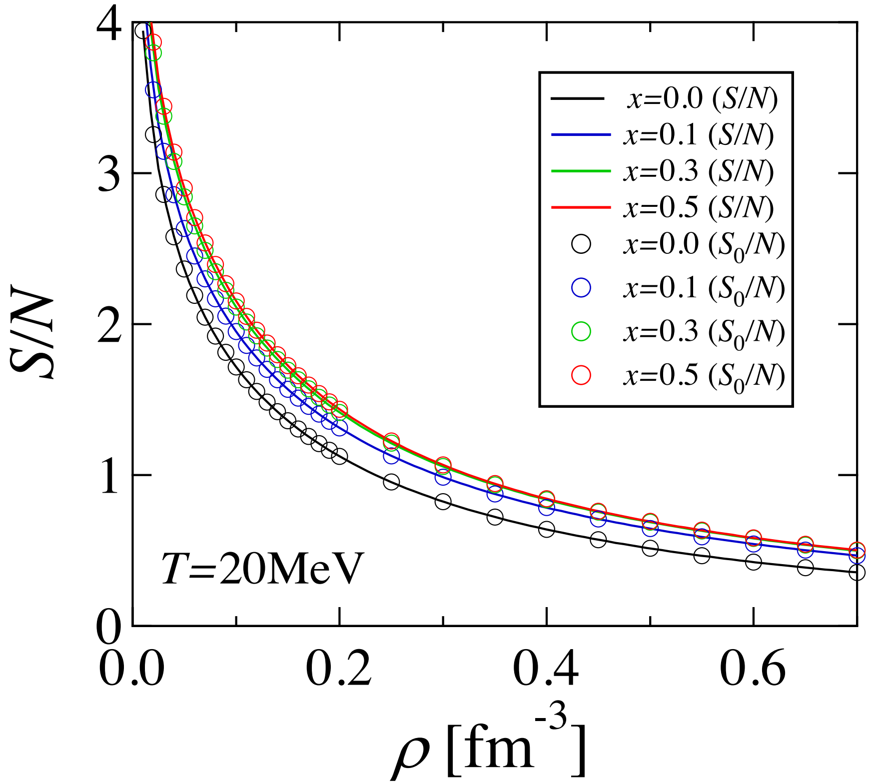


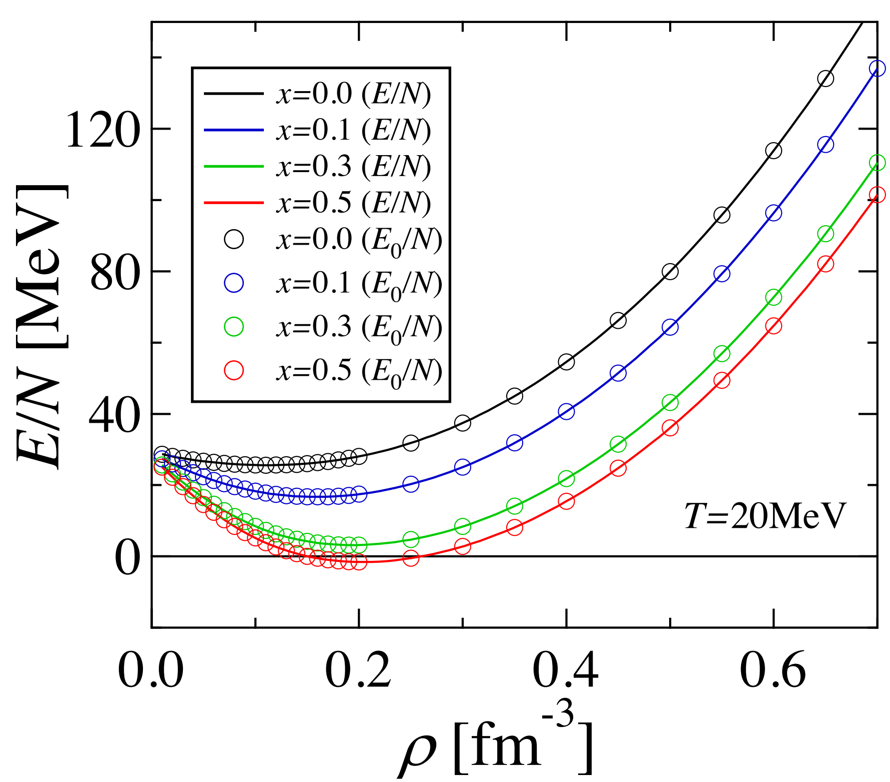
Symmetry Free Energy

$$\frac{F_{\mathrm{sym}}(x)}{N} = \frac{F(x)}{N} - \frac{F(x = 1/2)}{N}$$

AM : A. Mukherjee, PRC 79(2009) 045811

Entropy and Internal Energy





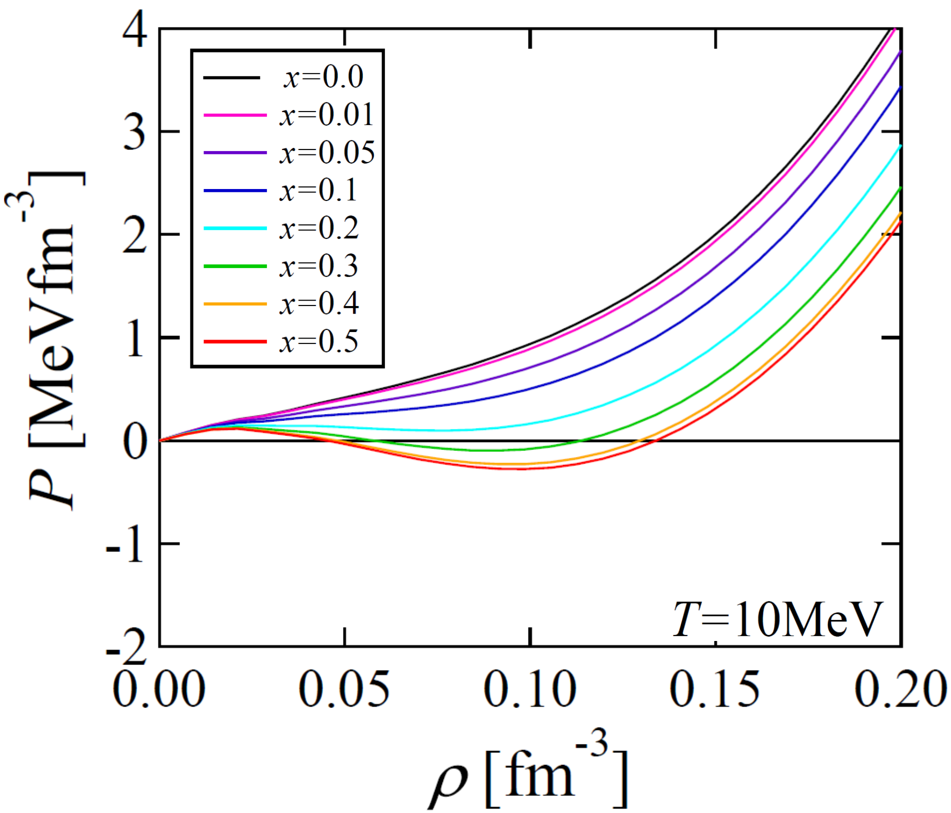
Entropy at T=20MeV

Internal energy at T=20MeV

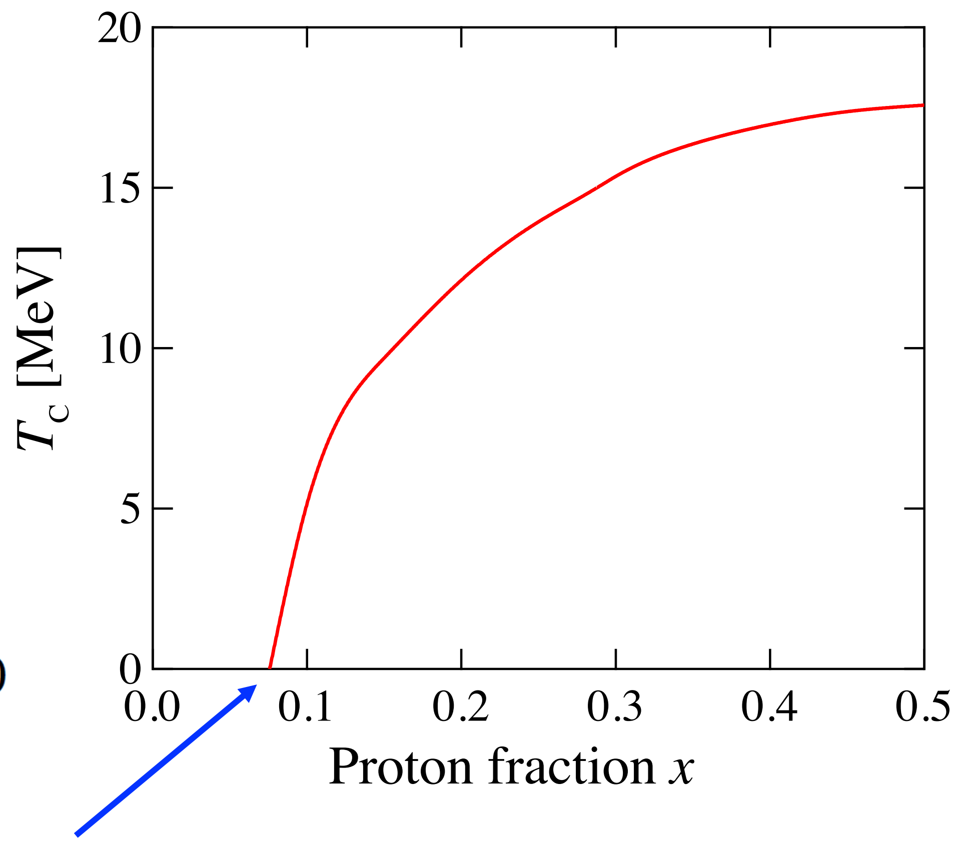
Entropies are in good agreement with the approximate entropies.

This variational calculation is Self Consistent.

Pressure and Critical Temperature



Pressure at T=10MeV



$x = 0.08$

Critical temperature

Critical Temperature T_C is defined by

$$\left.\frac{\partial P}{\partial \rho}\right|_{x,T=T_C} = \left.\frac{\partial^2 P}{\partial \rho^2}\right|_{x,T=T_C} = 0$$

3. EOS for Non-uniform Nuclear Matter

We follow the TF method by Shen et. al. (NPA637(1998)435)

Energy in the Wigner-Seitz (WS) cell

Bulk energy **Gradient energy**

$$E = \int d\boldsymbol{r}\varepsilon(n_{\mathrm{p}}(r), n_{\mathrm{n}}(r)) + F_0 \int d\boldsymbol{r}|\nabla n(r)|^2 + \frac{e^2}{2}\int d\boldsymbol{r}\int d\boldsymbol{r}' \frac{[n_{\mathrm{p}}(r) - n_{\mathrm{e}}][n_{\mathrm{p}}(r') - n_{\mathrm{e}}]}{|\boldsymbol{r} - \boldsymbol{r}'|} + c_{\mathrm{bcc}}\frac{(Ze)^2}{a}$$

Coulomb energy

$F_0 = 68.00$ MeV fm^5

ε : Energy density of uniform nuclear matter

Parameter	Minimum	Maximum	Mesh	Number	
$\log_{10}(T)$ [MeV]	−1.24	1.40	0.12	23 + 1	(0MeV)
$\zeta = (1-2Y_{\mathrm{p}})^2$	0.0	1.0	0.1	11+2	($\zeta = 0.85$, 0.95)
n_{B} [fm^{-3}]	0.0001	0.1600	0.0001	1600	

24×13×1600
≃ 500000 point

Nucleon density distribution

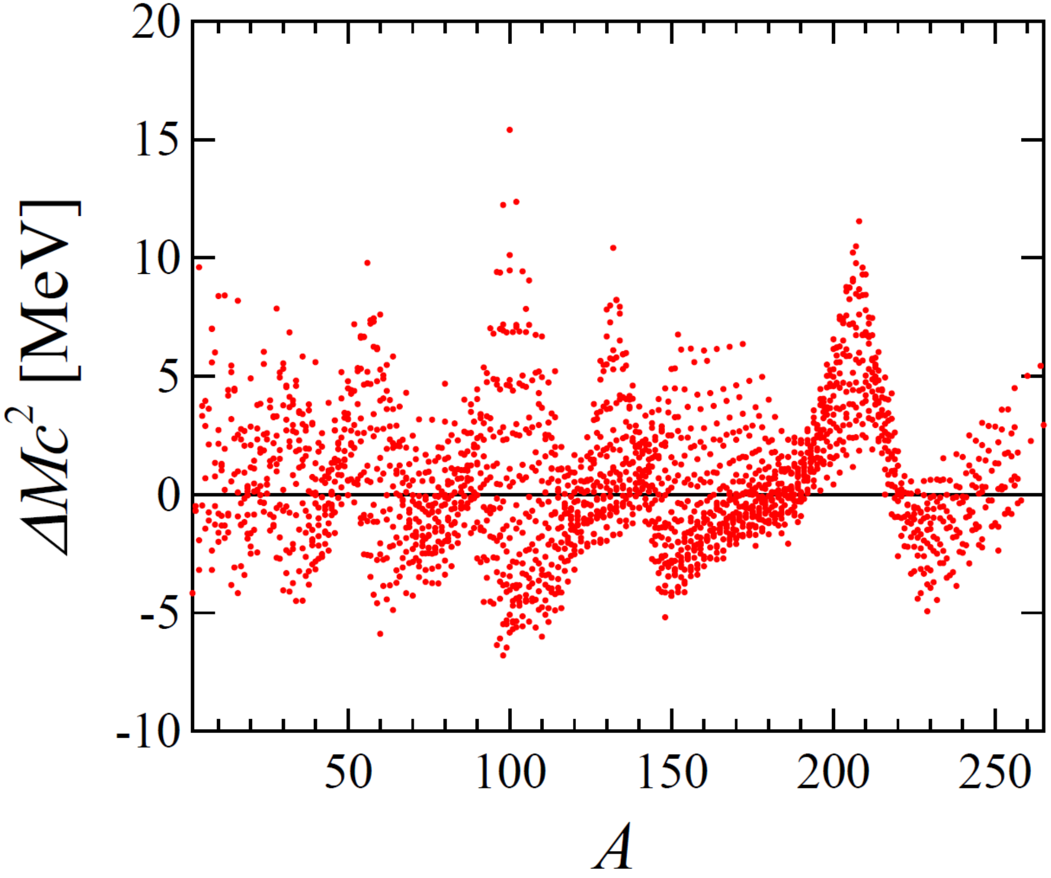
$$n_i(r) = \begin{cases} n_i^{\mathrm{in}}[1 - (r/R_i)^{t_i}]^3 + n_i^{\mathrm{out}} & (0 \le r \le R_i) \\ n_i^{\mathrm{out}} & (R_i \le r \le R_{\mathrm{cell}}) \end{cases} \quad (i = \mathrm{p, n})$$

a : Lattice constant

$$V_{\mathrm{cell}} = \frac{4\pi R_{\mathrm{cell}}^3}{3} = a^3$$

E/V_{cell} is minimized with respect to n_i^{out}, n_i^{in} R_i, t_i, a at given density and proton fraction.

TF Calculation for Atomic Nuclei



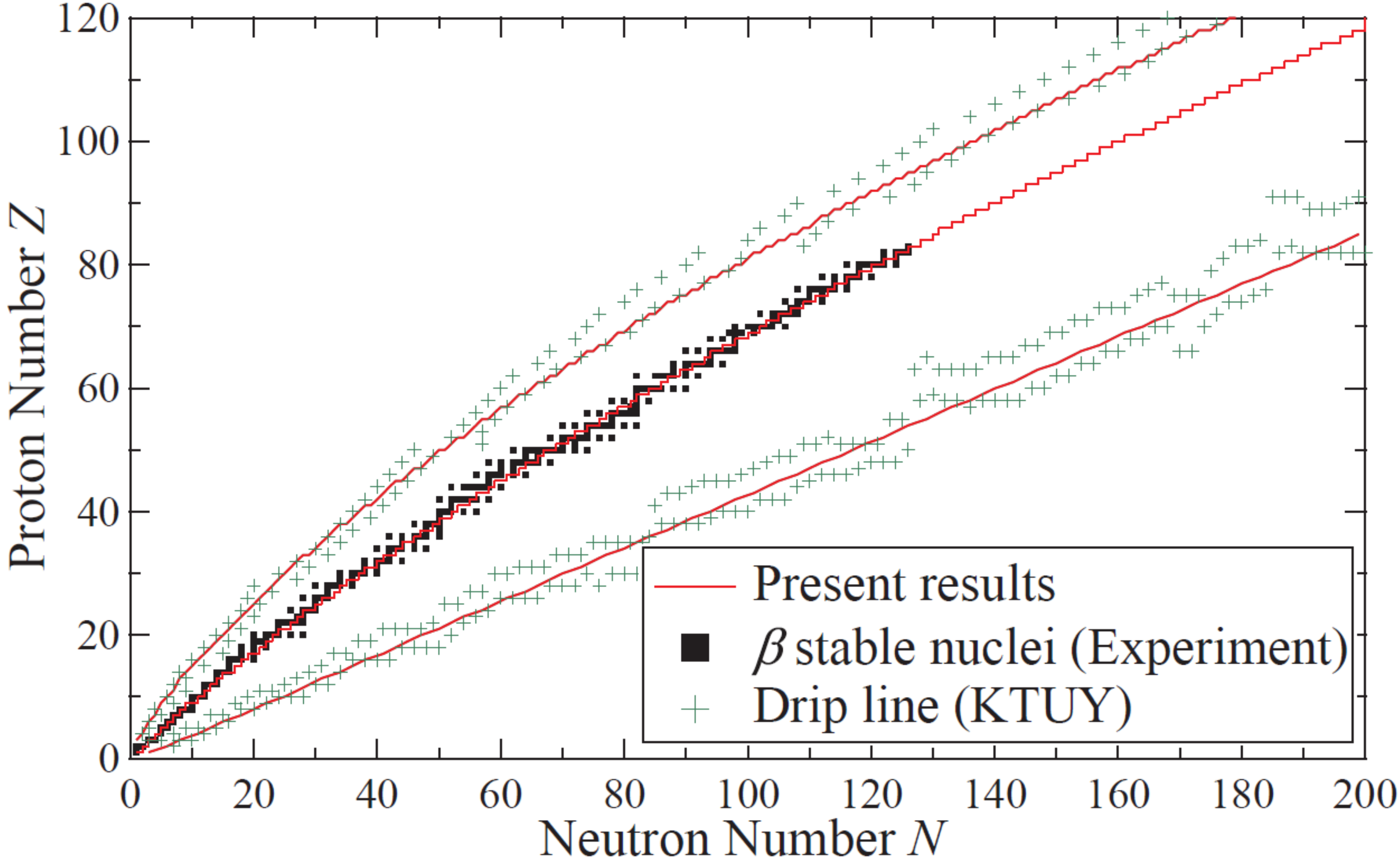
$\Delta M = M_{TF} - M_{exp}$

M_{TF} : Mass by the Thomas-Fermi calculation

M_{exp} : Experimental data

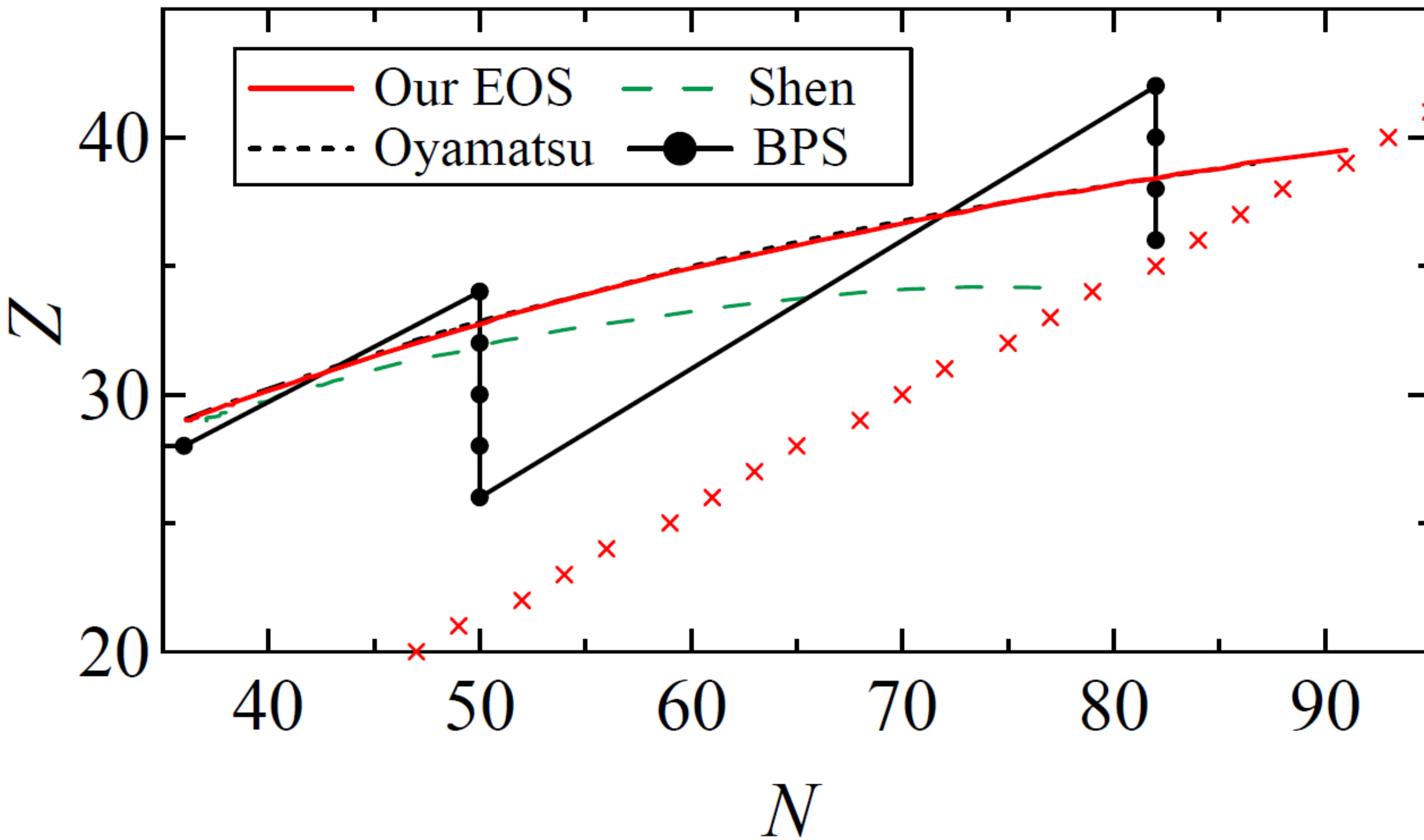
RMS deviation (for 2226 nuclei) 2.99 MeV

TF Calculation for Atomic Nuclei



Our results are in good agreement with
the experimental data and the sophisticated atomic mass formula.

TF Calculation for Outer Crust of Neutron Star

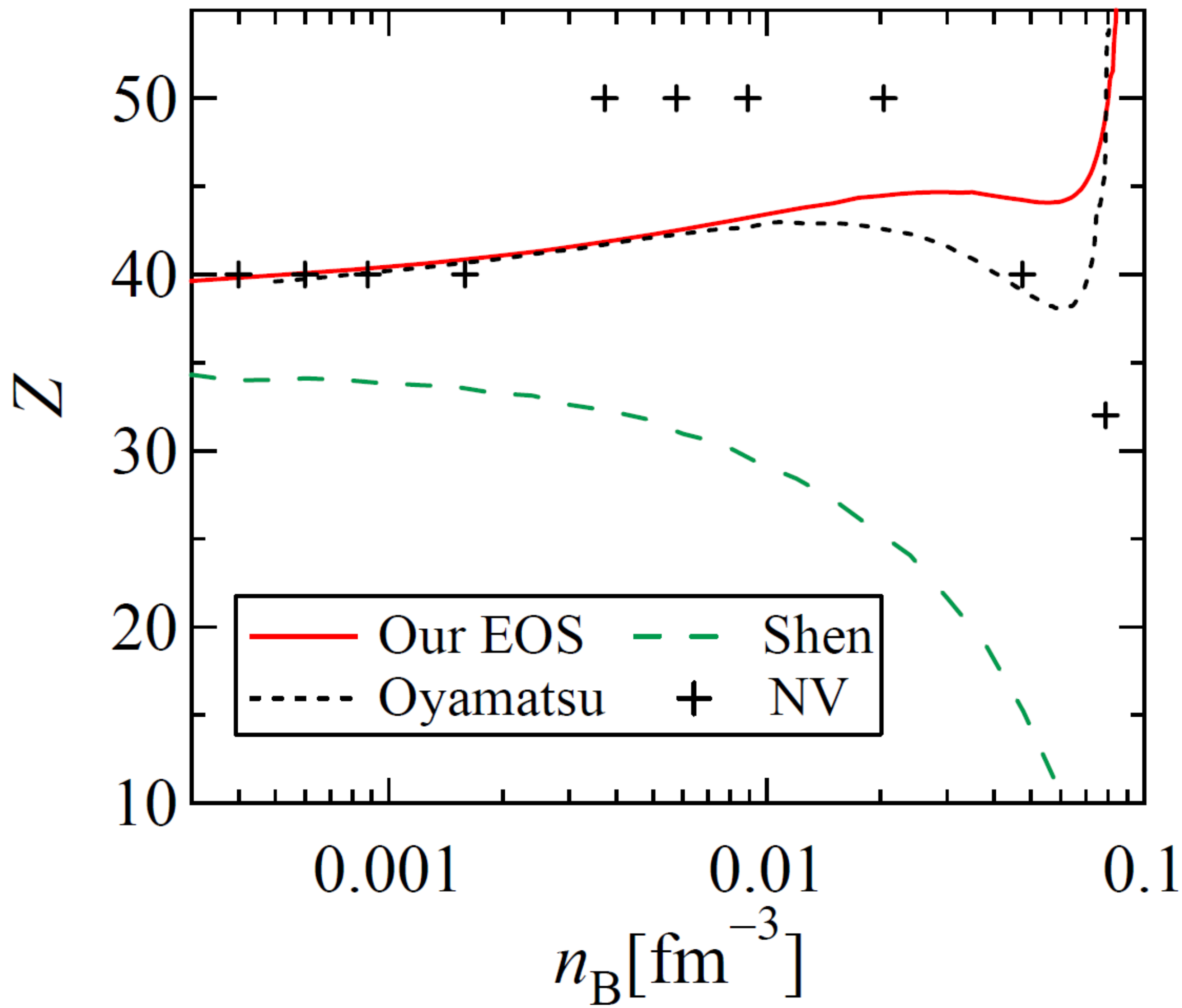


Nuclei in the outer crust of Neutron Star

BPS : G. **B**aym, C. J. **P**ethick and P. **S**utherland, *Astro*. *Phys*. *J* **170** (1971) 299.

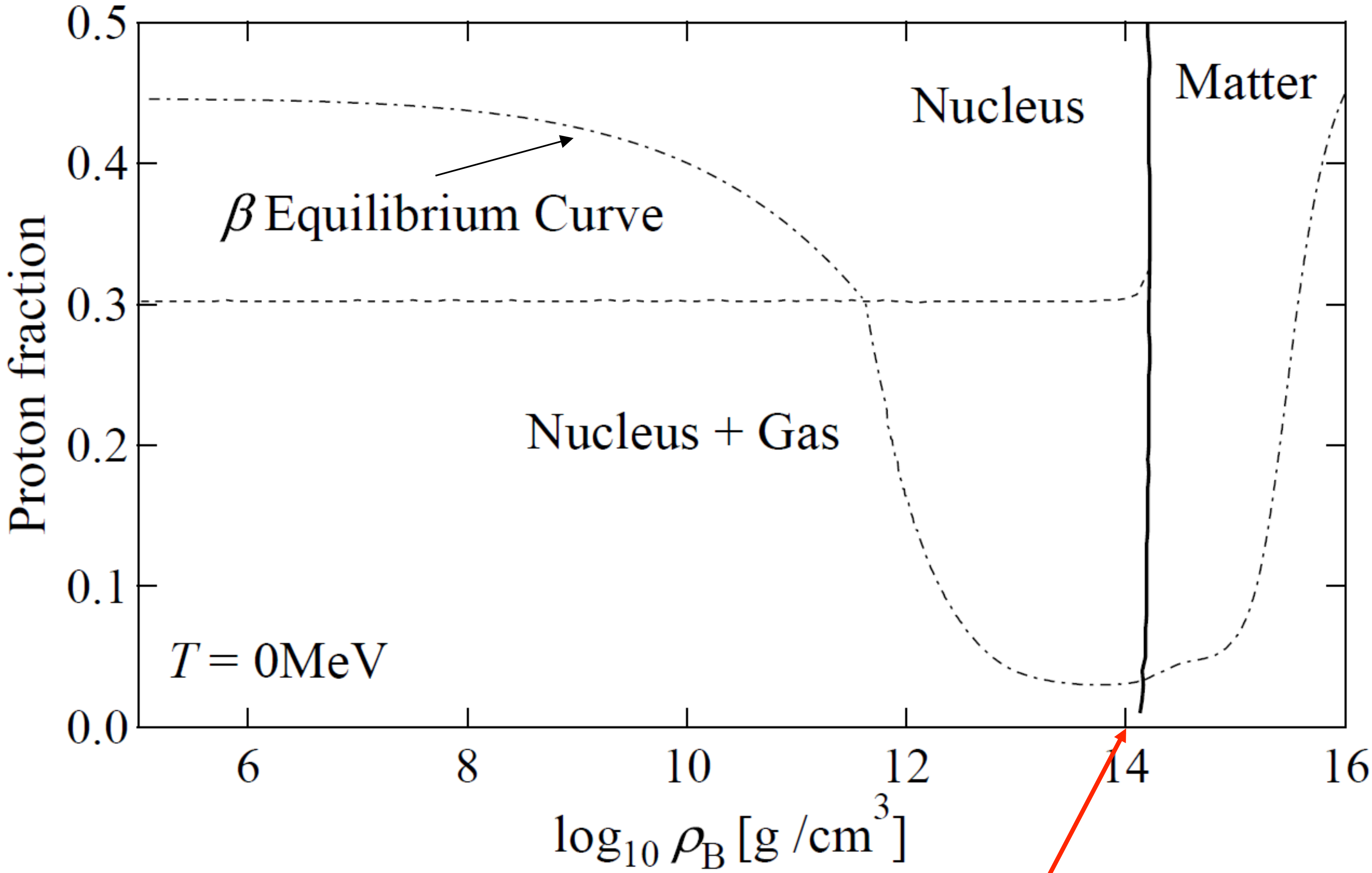
Oyamatsu : K. Oyamatsu, *Nucl*. *Phys*. *A* **561** (1993) 431.

TF Calculation for Inner Crust of Neutron Star



NV : J. W. Negele and D. Vautherin, *Nucl*. *Phys*. *A* **207** (1973) 298.

TF Calculation for Non-uniform Nuclear Matter

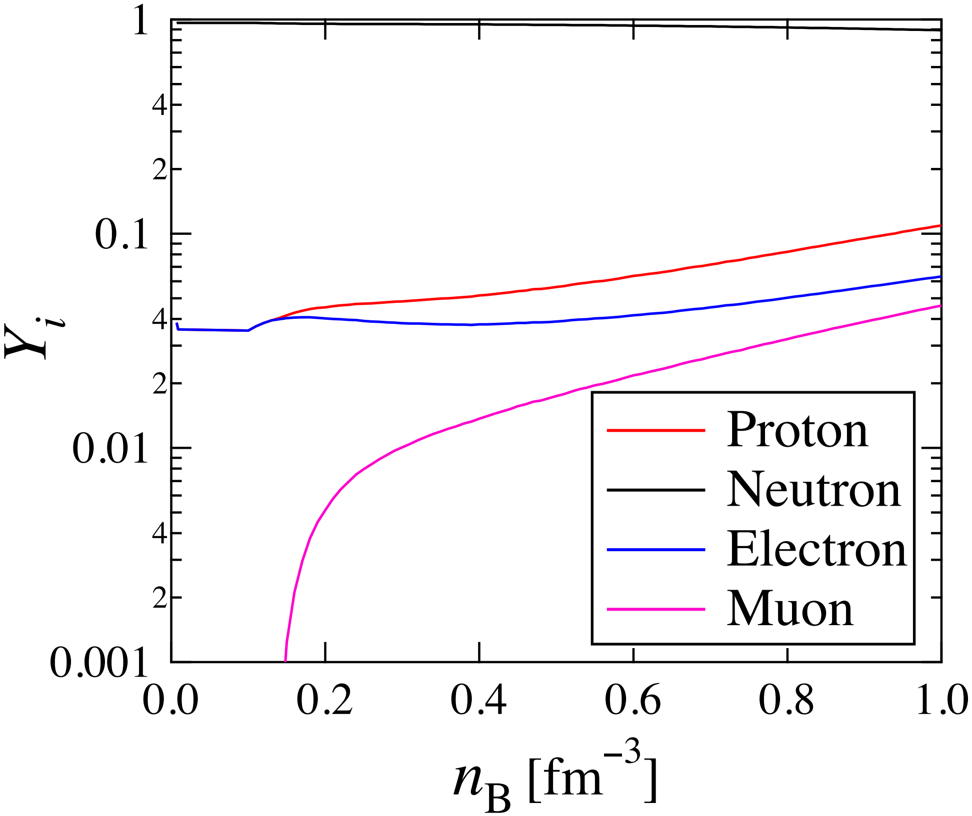


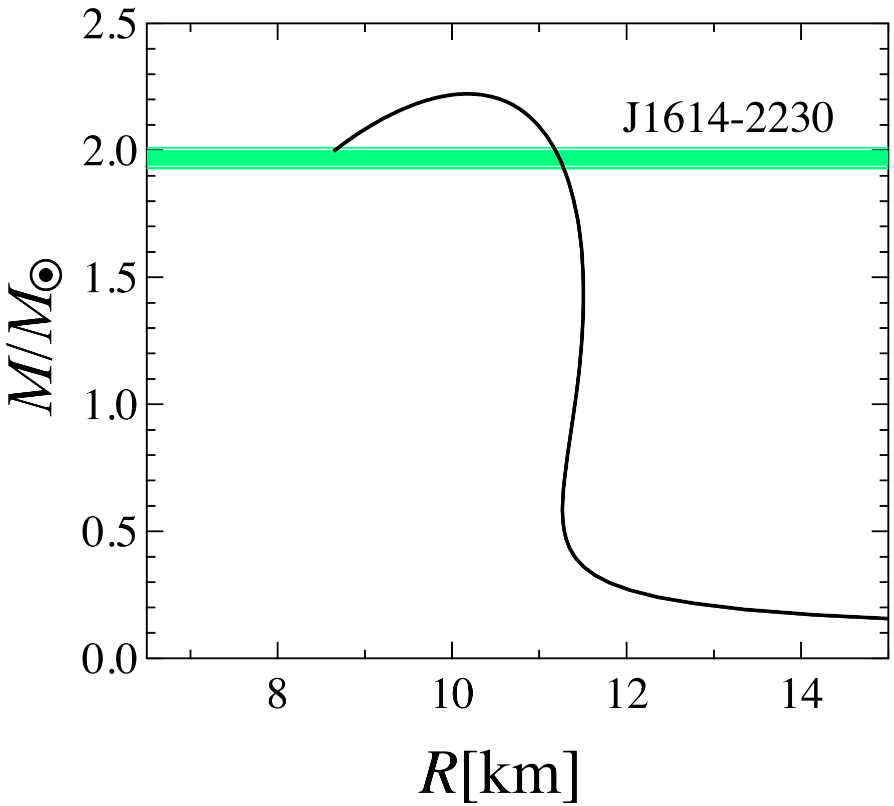
Phase diagram of nuclear matter at $T = 0\text{MeV}$

$\rho_B = 10^{14.23}\ \text{g/cm}^3$

4. Application to Neutron Star

Our EOS at 0MeV is applied to cold neutron star (NS) .





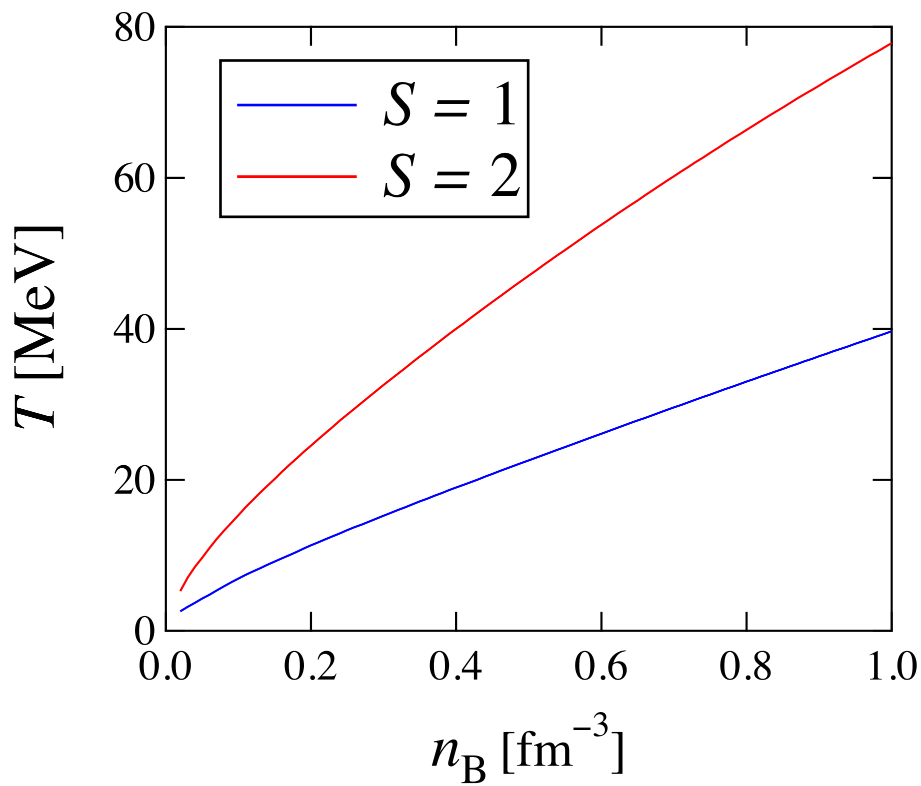
The maximum mass of NS is 2.20 $M_\odot$

Our EOS is consistent with the observational data.

Application to Proto-Neutron Star

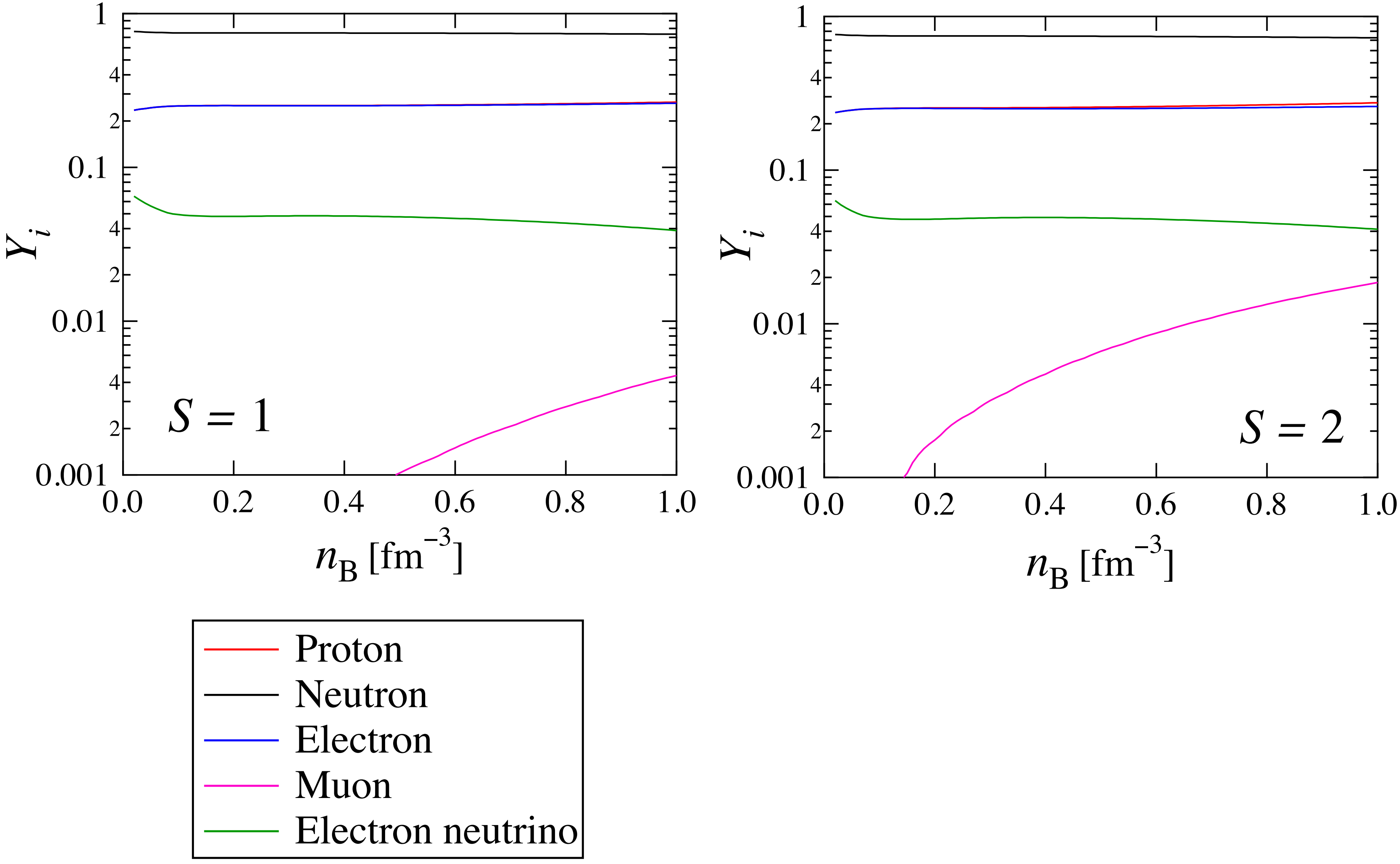
We construct *the EOS for proto-neutron star (PNS)* with the following conditions.

1. Isentropic β-stable matter with trapped neutrinos
2. Electron lepton number fraction : $Y_{\mathrm{L}e} = 0.3$
3. Muon lepton number fraction : $Y_{\mathrm{L}\mu} = 0.0$

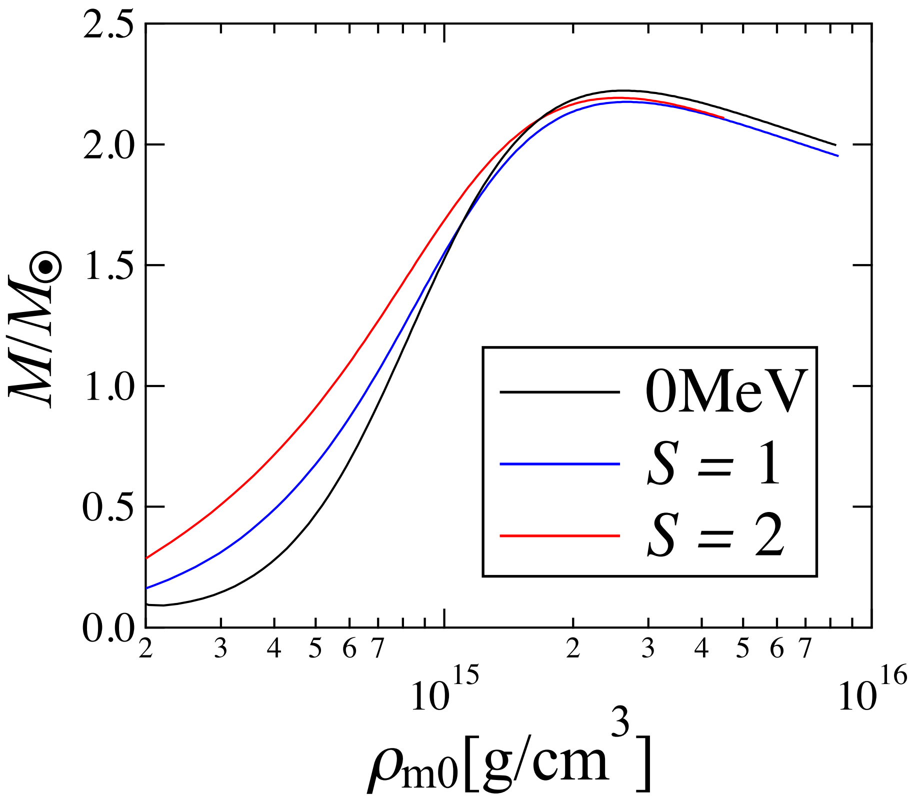


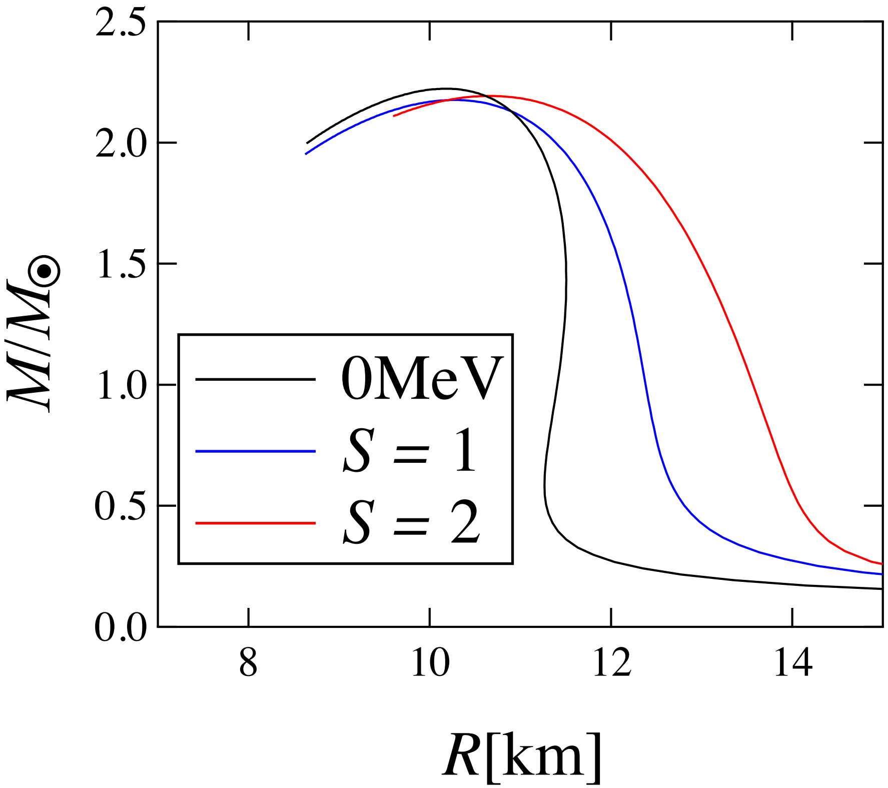
Temperature of PNS matter as a function of nucleon number density

Composition of Proto Neutron Star



Gravitational Mass of Proto Neutron Star





Maximum mass of PNS is smaller than that of cold NS.

Crust EOS is same as at 0 MeV.

5. Summary

- The EOS for uniform nuclear matter is constructed with the cluster variational method. (zero and finite temperatures)
- The EOS for non-uniform nuclear matter at zero temperature is calculated in the Thomas-Fermi approximation.

Uniform nuclear matter

The obtained thermodynamic quantities are reasonable.

Non-uniform nuclear matter

Phase diagram at zero temperature is reasonable.

Application to Neutron Star

Maximum mass of PNS is smaller than that of cold NS.

Future Plans

- Construction of the EOS table for non-uniform matter
- Contribution of the α-particle mixing

⇩

Construction of the EOS for supernova simulations