New equations of state in simulations of core-collapse supernovae

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Waseda, 8.3.2012

in collaboration with:

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Outline:

1.) introduction
2.) EOS model & constraints
3.) EOS aspects in simulations
    a.) sub-saturation densities
    b.) supra-saturation densities
4.) conclusions

Introduction

- EOS provides the crucial nuclear physics input for astrophysical simulations: thermodynamic quantities and nuclear composition

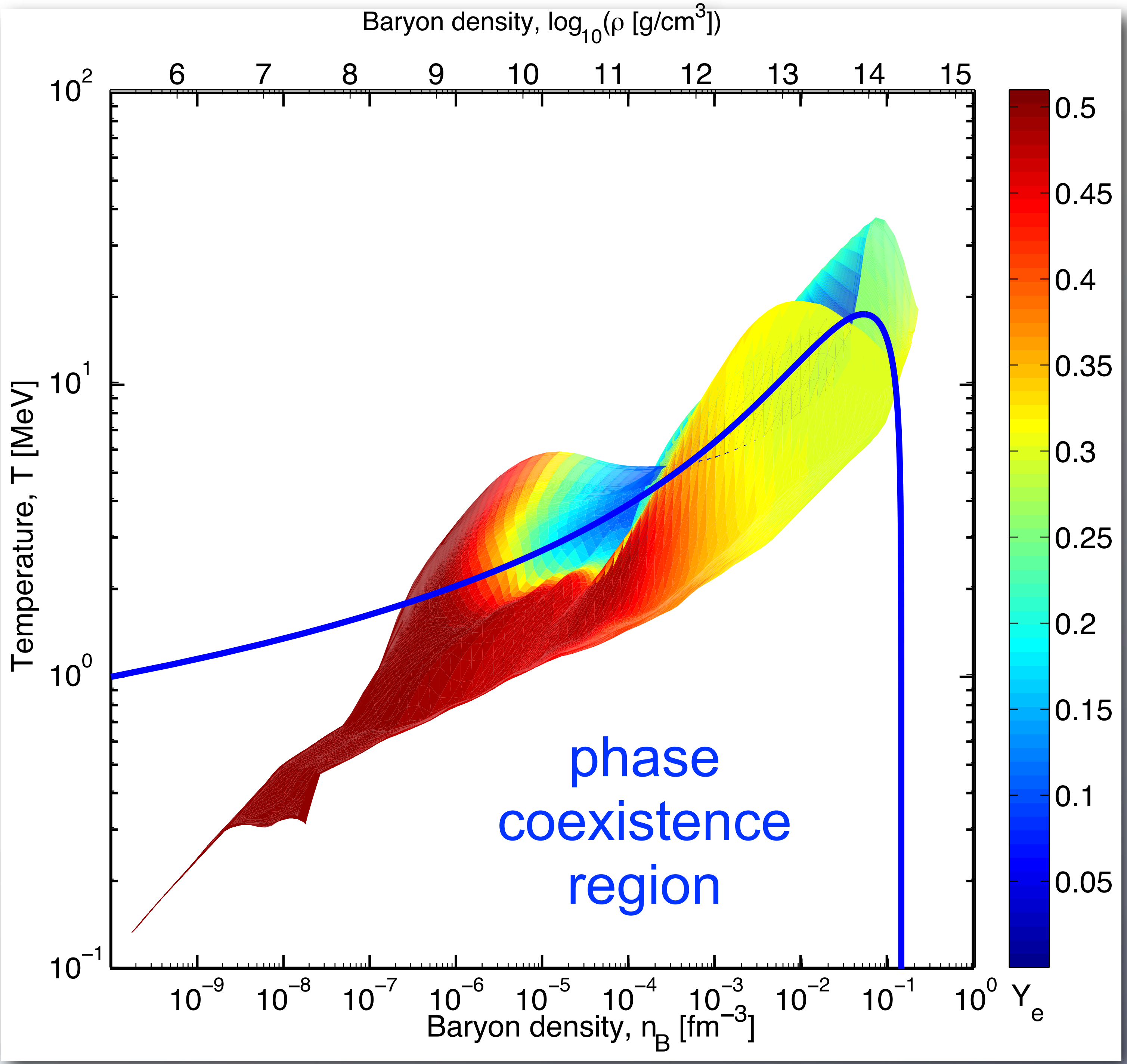
- plenty of EOSs for cold neutron stars

- challenge of the supernova (SN) EOS:
  - finite temperature: $T = 0 – 100$ MeV
  - no weak equilibrium: $Y_p = 0 – 0.6$
  - wide density range: $\rho = 10^4 – 10^{15}$ g/cm^3

- SN EOS: multi-purpose EOS, also applicable to neutron stars and their mergers, proto-neutron stars, heavy-ion collisions, ...

Introduction

state of matter in core-collapse supernovae

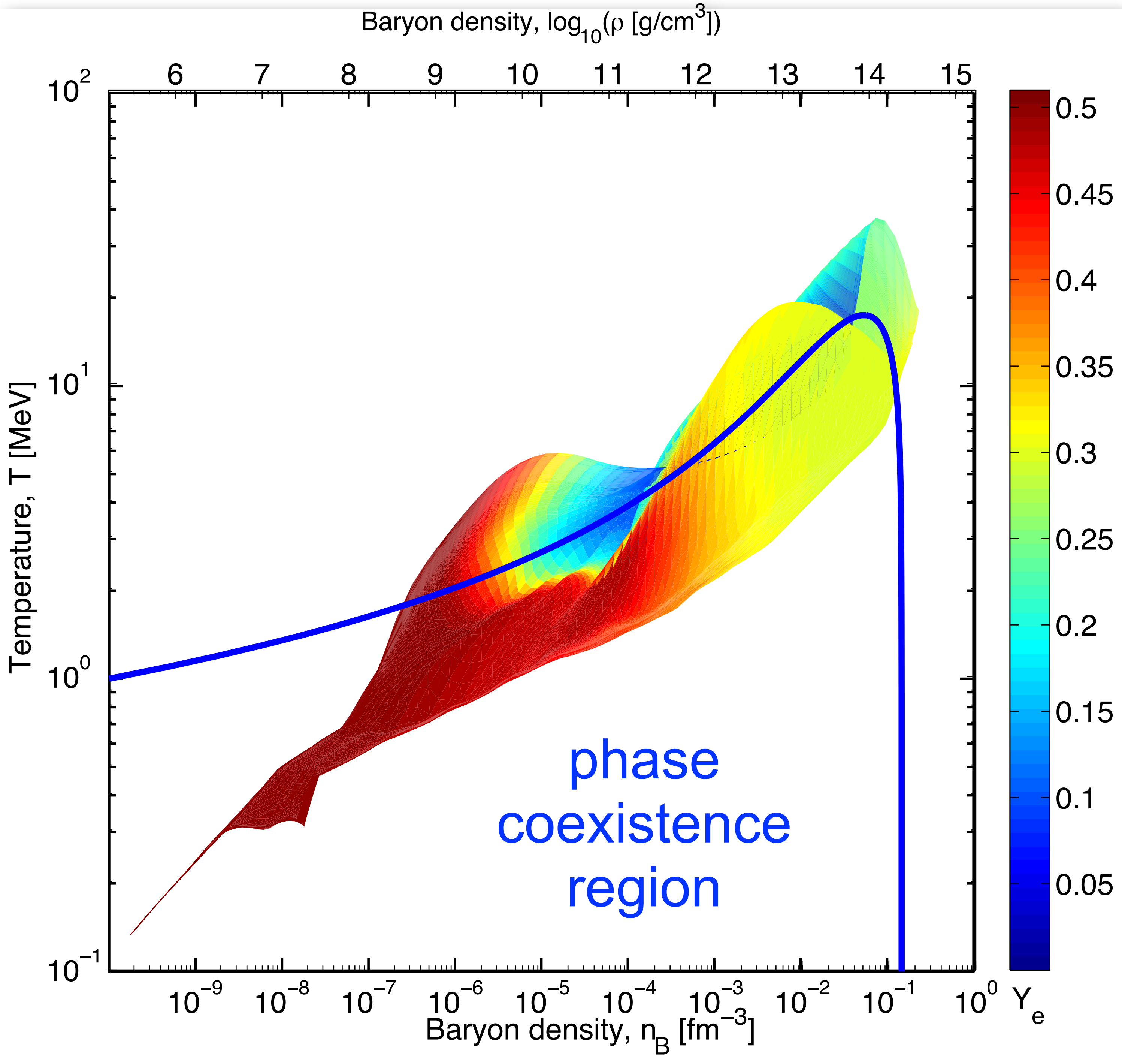


based on: *[Fischer et al., ApJS 2010]*

- without Coulomb, „bulk“: first order liquid-gas phase transition below T_c~15 MeV
- phase coexistence region
- with finite size effects: → non-uniform nuclear matter, mixture of nuclei and nucleons

Introduction

state of matter in core-collapse supernovae



based on: *[Fischer et al., ApJS 2010]*

- 99% of the energy emitted in neutrinos
- detailed knowledge of thermodynamic quantities and nuclear composition crucial
- at ρ ~10^{11} – 10^{13} g/cm³: neutrino spheres
- stall of the shock front

Introduction

commmonly used SN EOSs:

- Lattimer & Swesty 1991 (LS): non-relativistic liquid drop model
- H. Shen, Toki, Omayatsu and Sumiyoshi 1998 (STOS/Shen): relativistic mean-field (RMF), Thomas-Fermi approximation
- both models:
  - one representative nucleus: “single nucleus approximation”
  - no shell effects
  - only α-particles of light clusters

Recent developments

new available SN tables:

Nakazato et al. 2008	quark matter with large n_c added to STOS
Ishizuka et al. 2008	Hyperons added to STOS
Sagert et al. 2009	quark matter with low n_c added to STOS → explosions in 1D
H. Shen et al. 2010	Lambdas added to STOS

strange physics
at large densities

G. Shen, Horowitz, Teige (2010)	tables for NL3 and FSUgold: virial expansion, relativistic mean-field (RMF), Hartree
MH & Schaffner-Bielich (2010)	tables for NL3, TM1, TMA, FSUgold, DD2: NSE, RMF, excluded volume
Typel et al. (2010)	table for DD2 in preparation: gRMF
Furusawa et al. (2011)	table for TM1 in preparation: liquid drop, NSE, RMF

new hadronic EOS

Recent developments

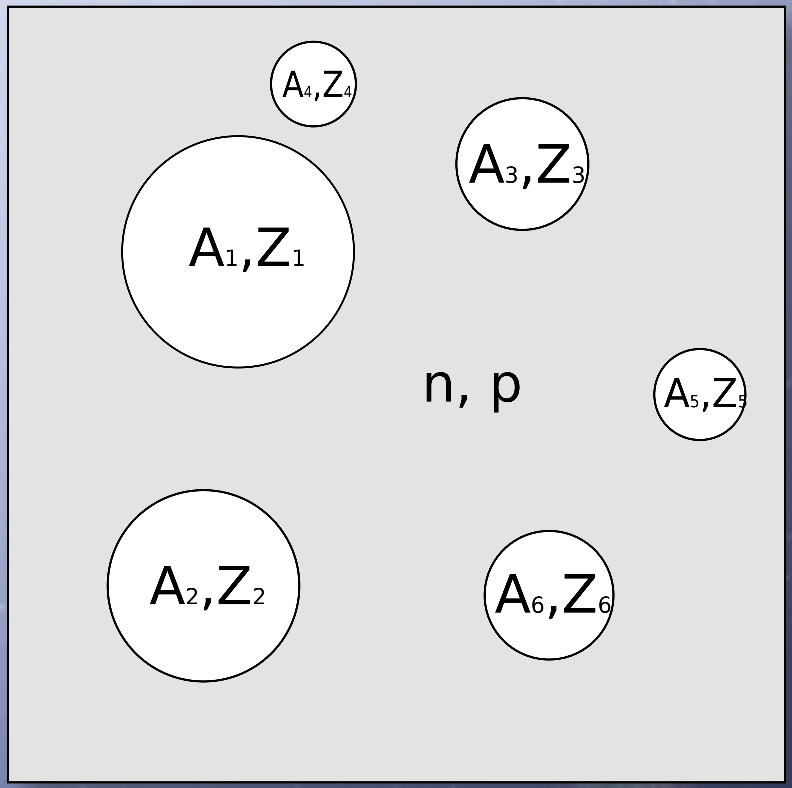
- pasta phases: Horowitz et al. 2004, Newton & Stone 2009
- light nuclei in the (SN) medium:
  Horowitz & Schwenk 2006, Röpke 2009, Typel et al. 2010
- weak reactions:
  Langanke 2003, Arcones et al. 2008
- multifragmentation/NSE models in the supernova context:
  Ishizuka et al. 2003, Botvina & Mishustin 2004, 2010, Blinnikov et al. 2009
- ...

EOS model & constraints

Excluded volume NSE model with interactions

MH, J. Schaffner-Bieleich; NPA837(2010) (HS)

- chemical mixture of nuclei and interacting nucleons in nuclear statistical equilibrium
- model characteristics:
  - ensemble of heavy nuclei
  - inclusion of all possible light nuclei
  - nuclear shell effects
  - various nucleon interactions
- input: RMF interactions, nuclear mass tables, Coulomb energies, excited states, and excluded volume effects



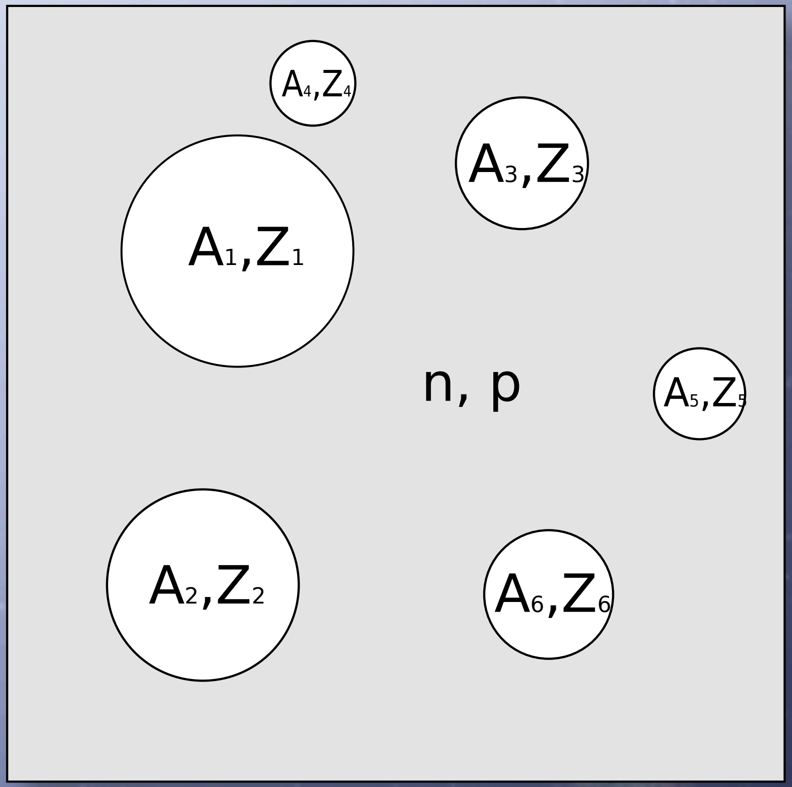
Excluded volume NSE model with interactions

- five tables for different RMF interactions:
  NL3, TM1, TMA, FSUgold, DD2

- T = (0.1 - 200) MeV
- $n_B = (10^{-12} - 10)\ \mathrm{fm}^{-3}$
- $Y_p = 0.01 - 0.6$

- 1.6 million grid-points

- full nuclear composition



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Nucleons – non-linear RMF (TM1, TMA, FSUgold, NL3)

- relativistic mean-field model (RMF)
- interactions mediated via exchange of mesons and meson (self-) interactions

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi \\
& + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - g_\sigma\bar{\psi}\sigma\psi \\
& - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}g_4(\omega_\mu\omega^\mu)^2 - g_\omega\bar{\psi}\gamma^\mu\psi\omega_\mu \\
& - \frac{1}{4}R^a{}_{\mu\nu}R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho^{a\mu} - g_\rho\bar{\psi}\gamma_\mu\tau^a\psi\rho^{\mu a} - \Lambda\omega_\mu\omega^\mu\rho_\nu^a\rho^{a\nu}
\end{aligned}
$$

- alternative: density-dependent coupling constants (DD2)
- coupling constants fitted to experimental data
- well-established description of finite nuclei and nuclear matter

Sugahara & Toki 1994

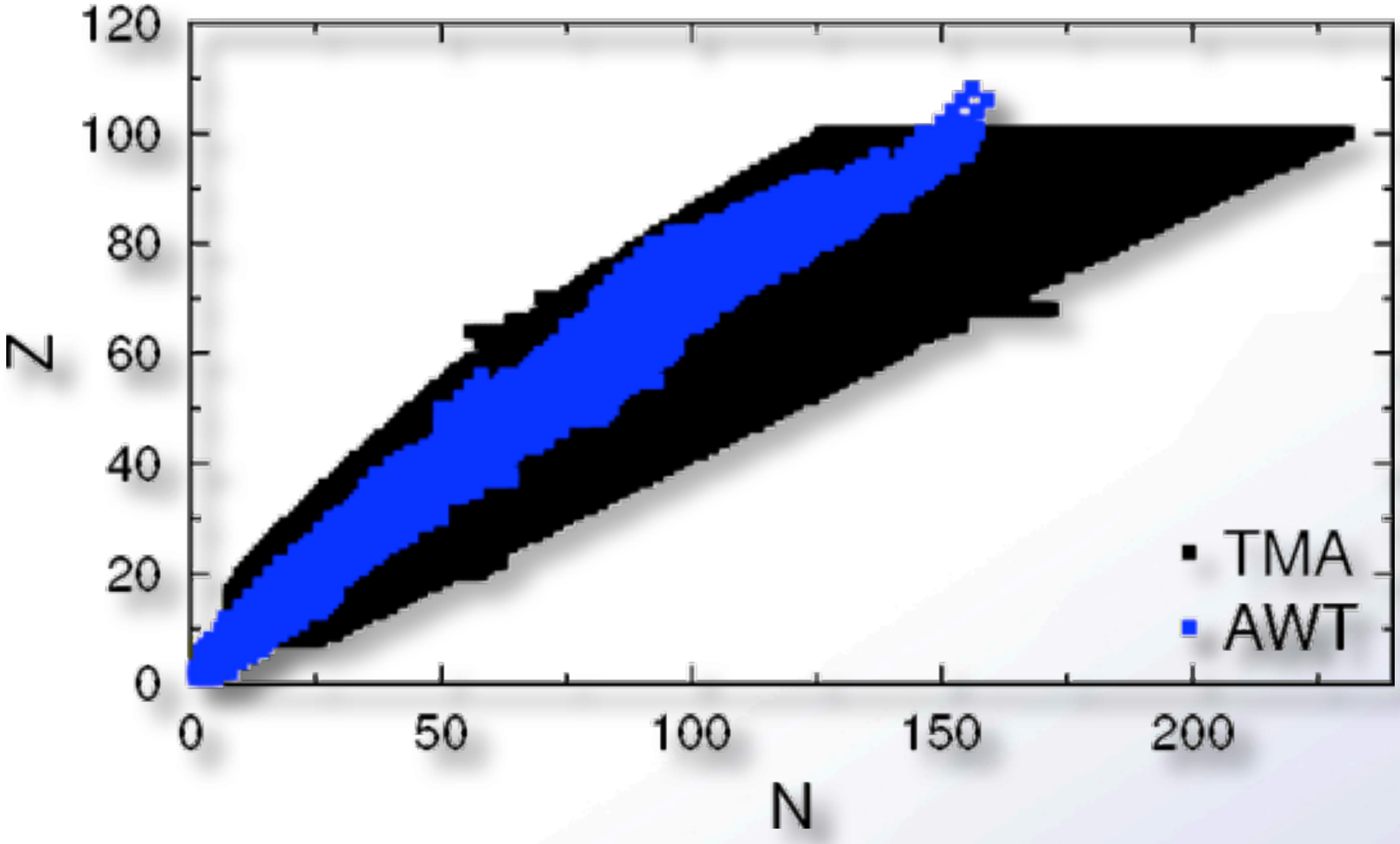
Toki et al. 1995

Todd-Rutel & Piekarewicz 2005

Lalazissis et al. 1997

Typel 2005, Typel et al. 2010

Nuclei - mass tables



- experimental data of AWT (2003): over 2000 precisely measured binding energies

  [Audi et al.; 2003NPA729]

unknown nuclei: mass table generated with theoretical nuclear model

- TMA: BCS δ-force pairing, axial deformations, mass-number dependent coupling constants *[Geng et al.; 2005PTP113]*
- FSUgold: spherical, BCS pairing with constant matrix elements *[Roca-Maza, private communication] [Roca-Maza, Piekarewicz; 2008PRC78]*
- NL3: BCS δ-force pairing, axial deformations *[Lalazissis et al.; 1999ADNDT71]*
- FRDM: finite range droplet model *[Möller et al.; 1995ADNDT59]*

Nuclei - medium effects

- semi-empirical formula for intrinsic partition function of hot nuclei

$$g_i(T) = g_i^0 + \frac{c_1}{A_i^{5/3}} \int_0^{B_i} dE e^{-E/T} \exp(2\sqrt{a_i E})$$

[Fai, Randrup; 1982NPA381]

- Coulomb energies in T=0 Wigner-Seitz approximation

$$E_{Coul} = -\frac{3}{5}\frac{Z_i^2 \alpha}{R_i}\left(\frac{3}{2}x - \frac{1}{2}x^3\right)$$

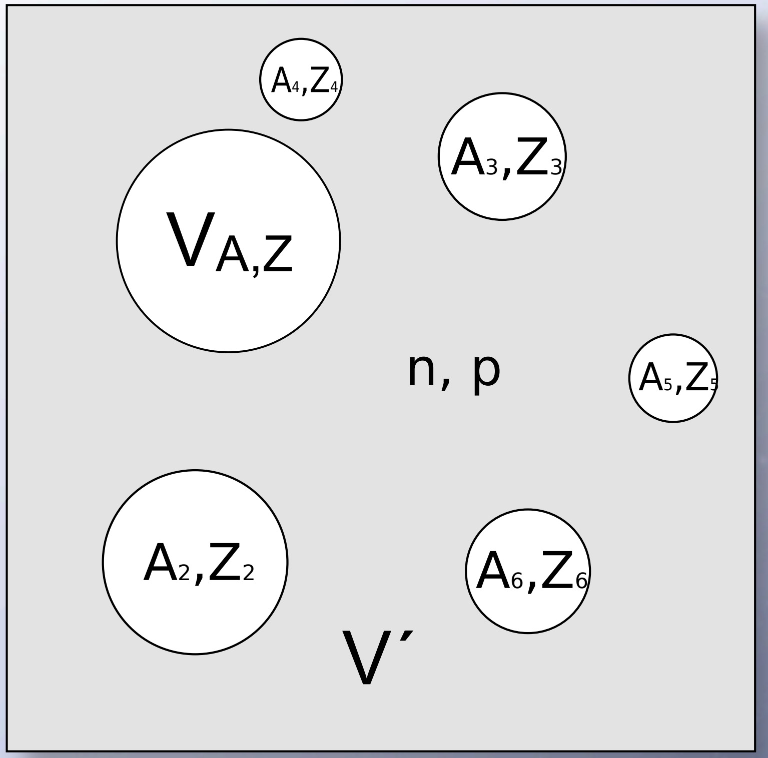
- possible extension: combination with liquid-drop model

  → see talk of S. Furusawa

Excluded volume

model assumptions

- each baryon fills a volume $1/n^0_B$, volume of a nucleus $V_{A,Z} = A/n_B^0$
- nucleons must not be inside of nuclei
- nuclei must not overlap



- filling factor of the nucleons

$$\begin{aligned}\xi &= V'/V \\ &= 1 - \sum_{A,Z} A\, n_{A,Z}/n_B^0\end{aligned}$$

- effect on nuclei given by the free volume fraction

$$\kappa = 1 - n_B/n_B^0$$

Excluded volume - asymptotics

- thermodynamic potential: free energy density

$$\begin{aligned} f &= f_e^0(T,n_e) + \sum_{A,Z} f_{A,Z}^0(T,n_{A,Z}) + f_{Coul}(n_e,\{n_{A,Z}\}) \\ &\quad + \xi f_{RMF}^0(T,n_n',n_p') - T \sum_{A,Z} n_{A,Z} \ln(\kappa) \ , \end{aligned}$$

ideal gas expressions, Coulomb contribution, filling factor for nucleons, direct excluded volume contribution

- low fraction of nuclei, ξ ~ 1: unmodified RMF model ✓
- $\kappa \rightarrow 0$ for $n_B \rightarrow n_B^0 \Rightarrow f \rightarrow \infty$ if nuclei are present
→ dissociation of nuclei above saturation density ✓

- thermodynamic fully consistent

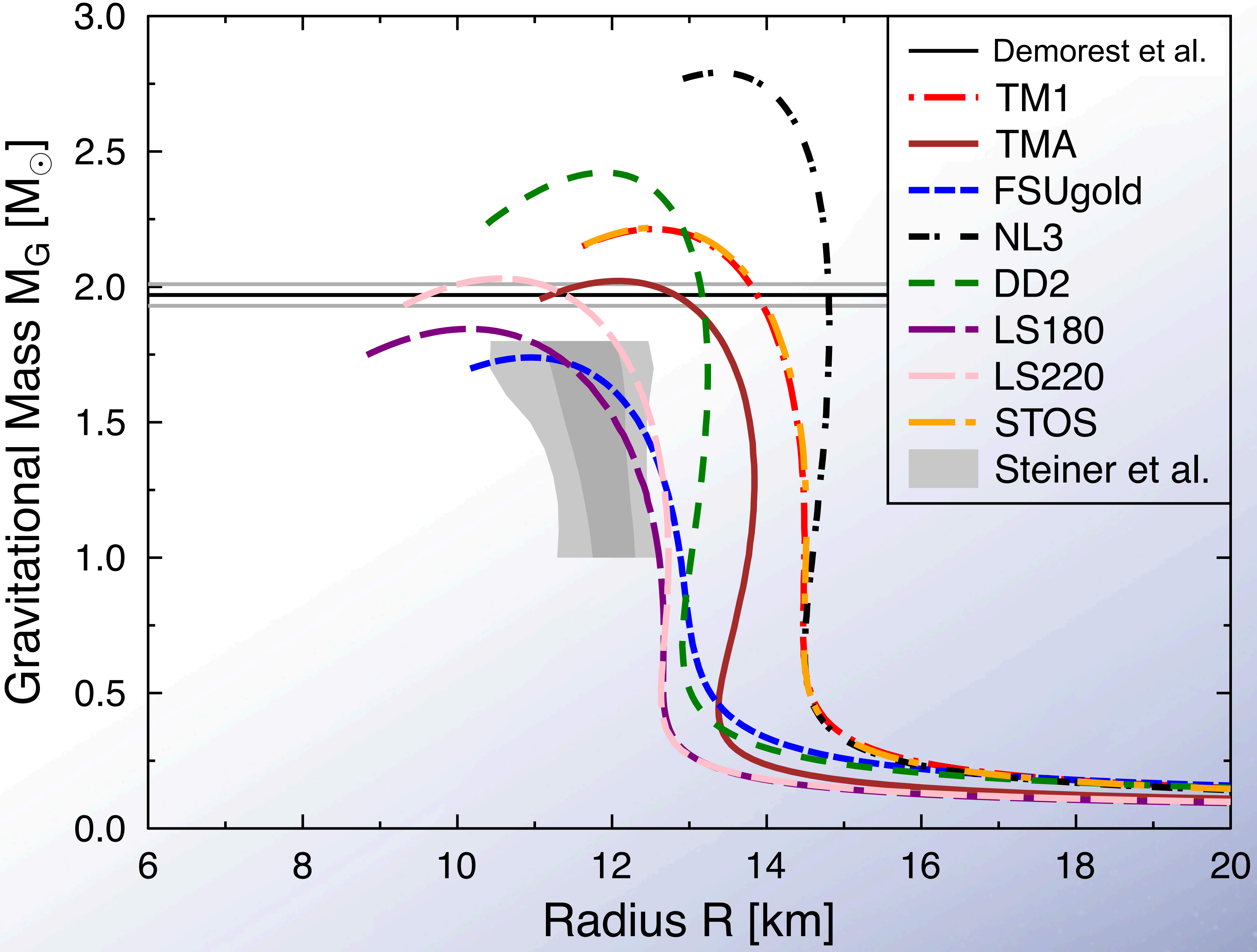
EOS constraints - saturation properties & maximum mass

	n_B^0 [fm^{-3}]	E_0 [MeV]	K [MeV]	J [MeV]	L [MeV]	$M_{\max}$ [M$_\odot$]
TM1	0.146	-16.31	282	36.95	110.99	2.213
TMA	0.147	-16.03	318	30.66	90.14	2.022
FSUgold	0.148	-16.27	230	32.56	60.44	1.739
NL3	0.148	-16.24	271	37.39	118.50	2.791
DD2	0.149	-16.02	243	31.67	55.04	2.422
LS180	0.155	-16.00	180	28.61	73.82	1.828
LS220	0.155	-16.00	220	28.61	73.82	2.031
Exp.	~ 0.15	~ -16	240 ± 10 [1]	30 - 34 [2]	40 - 110 [2]	$> 1.97 \pm 0.04$ [3]

- span a broad range of possible RMF models

	references	type of constraint
[1]	Piekarewicz JPG 2010	compilation of measurements of isoscalar giant monopole resonances
[3]	Tsang et al. PRL 2009 Carbone et al. PRC 2010	compilation of experiments: isospin diffusion,pygmy dipole resonance, nuclear masses, GDR, isoscaling, antiprotonic atoms
[4]	Demorest et al. Nature 2010	measurement of Shapiro delay

EOS constraints - mass and radius measurements



- bayesian analysis of observations of six NS (X-ray burster, low -mass X-ray binaries), Steiner et al. ApJ 2010

→ see talk of A. Steiner for further discussion

Results - low density

- including simulations for 15 M_{sun} progenitor of Woosley & Weaver ApJS 101 (1995)
- HS TM1 in comparison with STOS TM1 and/or LS 180

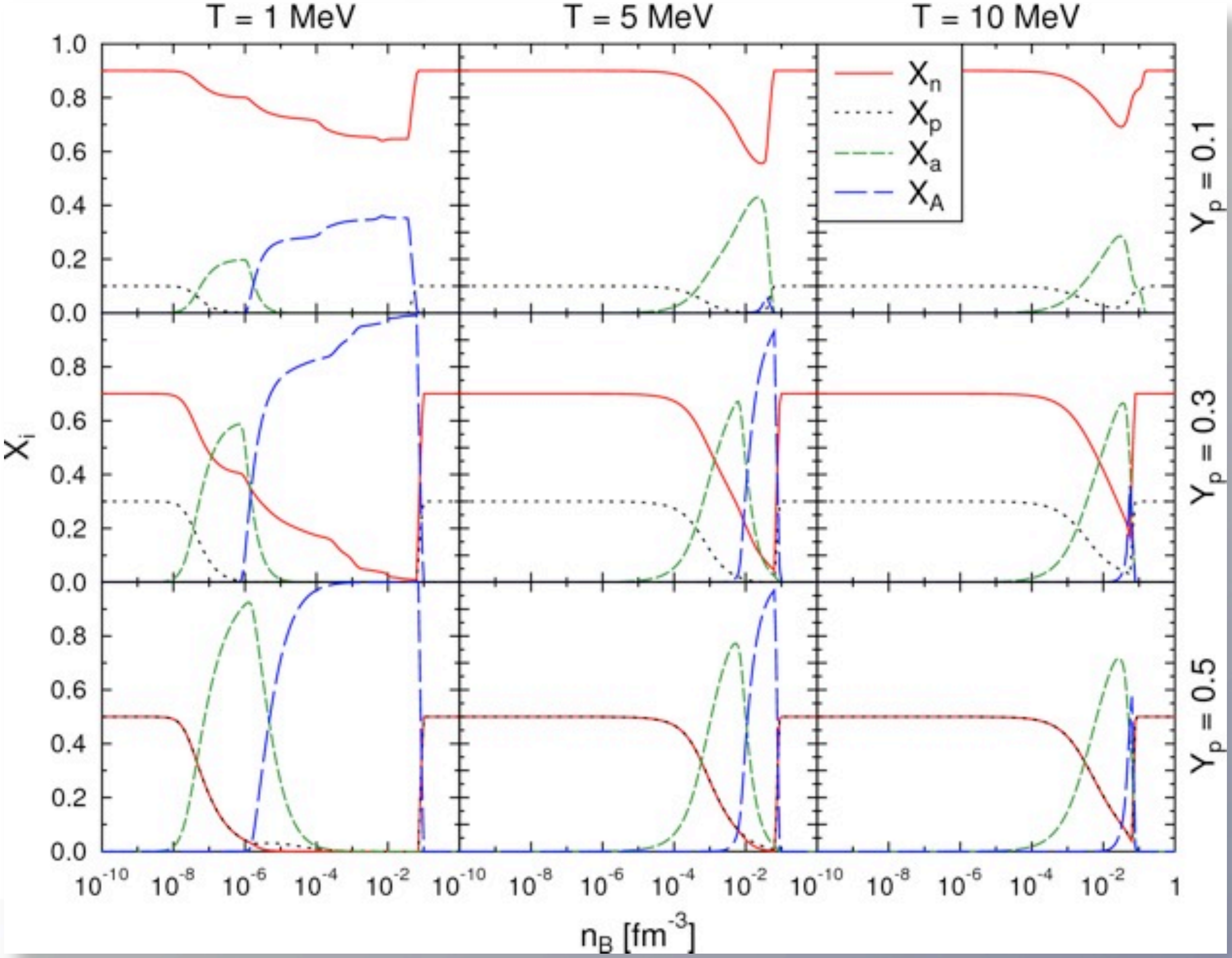
MH, T. Fischer, J. Schaffner-Bielich, M. Liebendörfer, ApJ 748, 70 (2012)

Supernova simulations

- core-collapse SN simulations by Tobias Fischer, GSI Darmstadt
  - spherical symmetry
  - general relativistic radiation hydrodynamics
  - three flavor neutrino transport
- weak reactions:
  - all light clusters treated as alpha-particles
  - only average heavy nucleus

$$\begin{aligned}
e^- + < A,Z > &\longleftrightarrow < A, Z-1 > + \nu_e \\
e^- + p &\longleftrightarrow n + \nu_e \\
e^+ + n &\longleftrightarrow p + \bar{\nu}_e \\
\nu + e^\pm &\longleftrightarrow \nu + e^\pm \\
\nu + N &\longleftrightarrow \nu + N \\
N + N &\longleftrightarrow N + N + \nu + \bar{\nu} \\
\nu_e + \bar{\nu}_e &\longleftrightarrow \nu_{\mu/\tau} + \bar{\nu}_{\mu/\tau}
\end{aligned}$$

Nuclear composition



- general isothermal density dependence:

  nucleons → light → (heavies) → uniform nucleon matter
- low temperature:

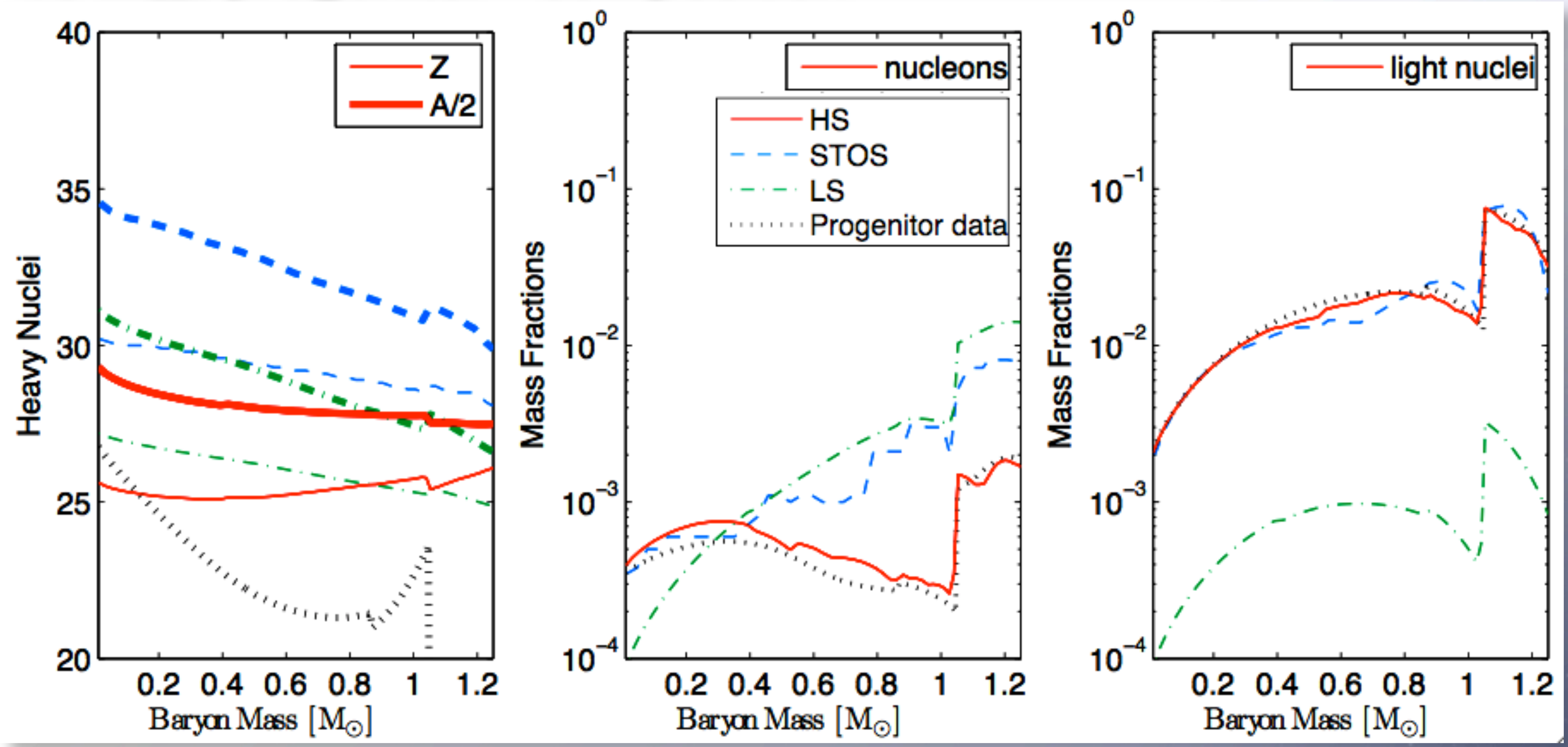
  shell effects

- distinction of light and heavy nuclei by charge Z=6:

$$X_a = \sum_{A,Z\leq 5} A n_{A,Z}/n_B$$

$$X_A = \sum_{A,Z\geq 6} A n_{A,Z}/n_B$$

15 M_{sun} progenitor - composition

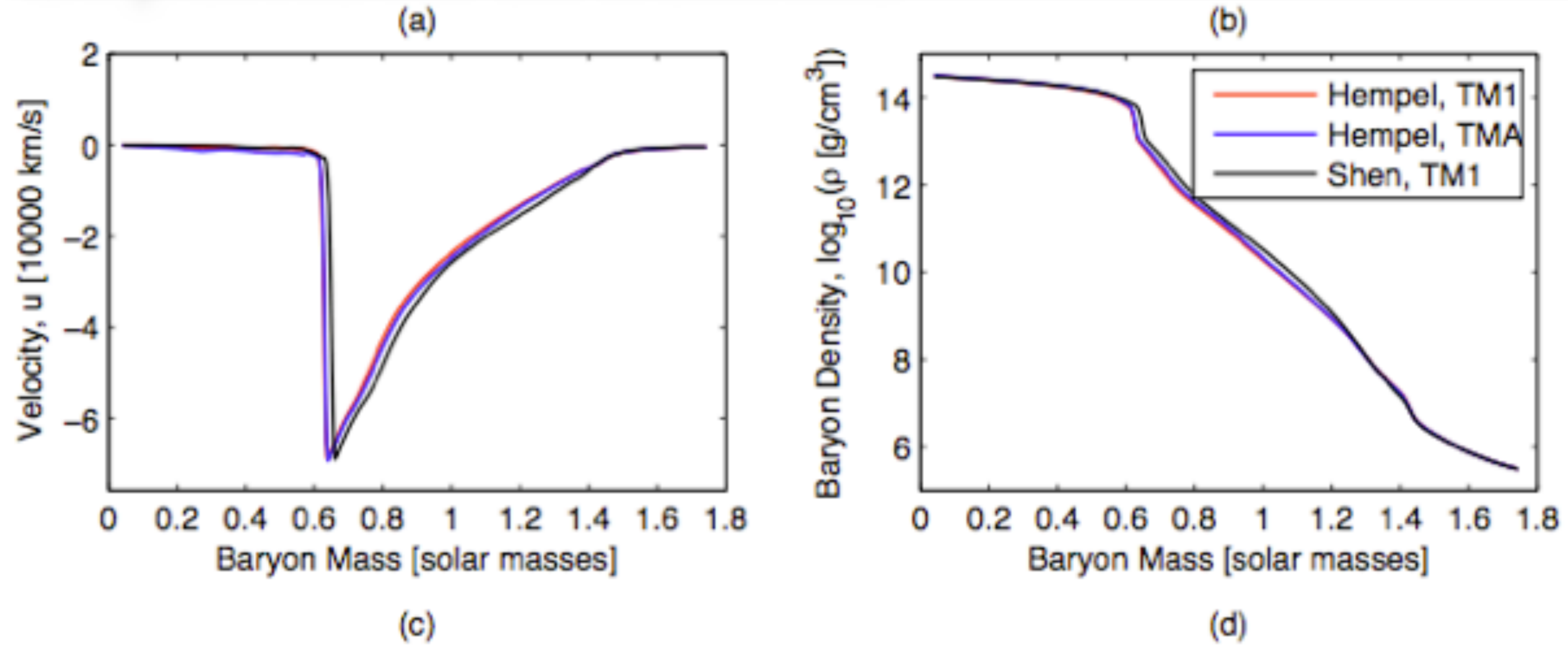


- no iron group nuclei in STOS and LS
- progenitor not in full NSE
- still good agreement of composition in HS

- HS: Hempel, Schaffner-Bielich (TM1)
- STOS: Shen, et al. (TM1)
- LS: Lattimer, Swesty (K=180 MeV)

Supernova at bounce

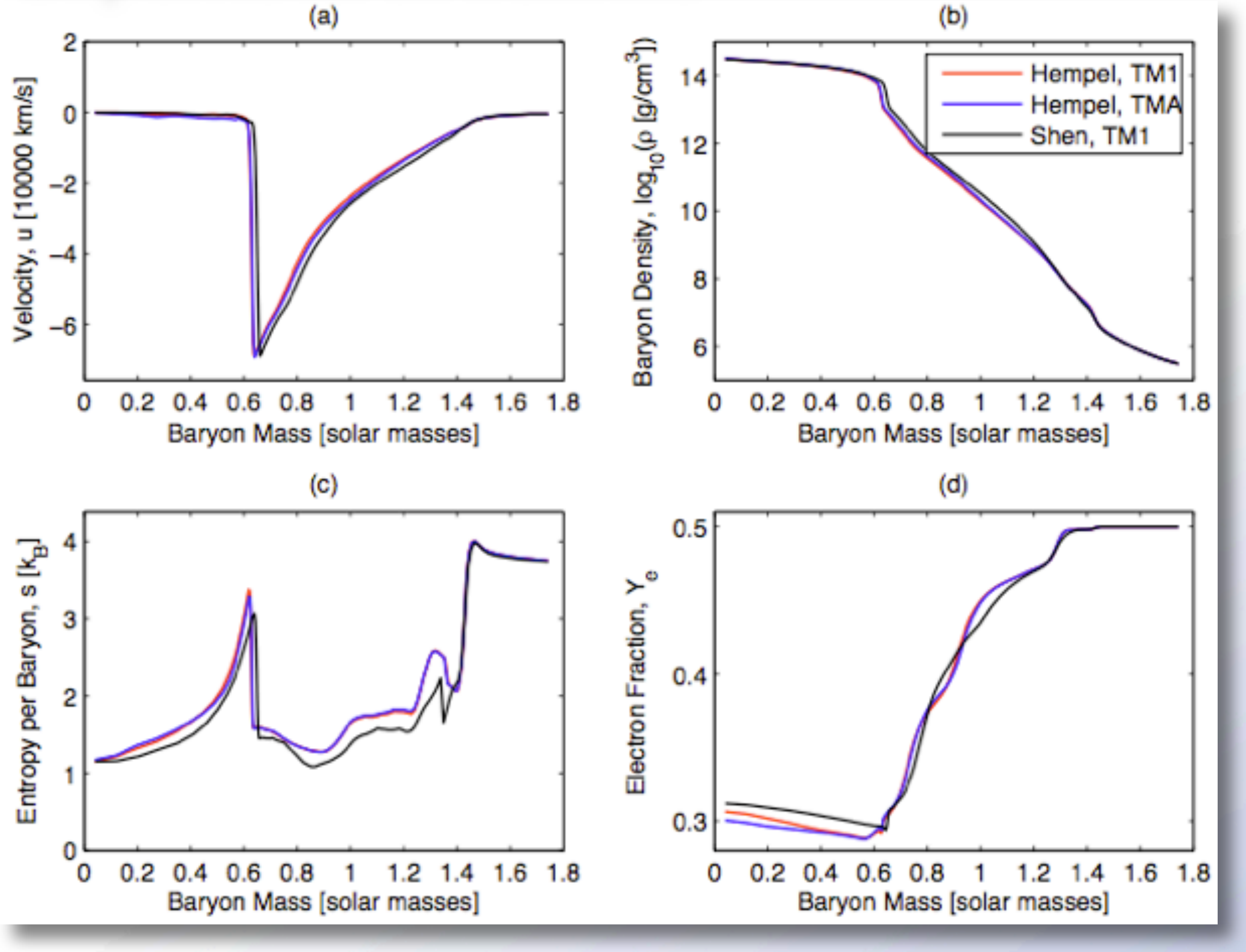
15 M_{sun} progenitor (Woosley & Weaver 1995)



- similar evolution
- slightly more compact PNS core

Supernova at bounce

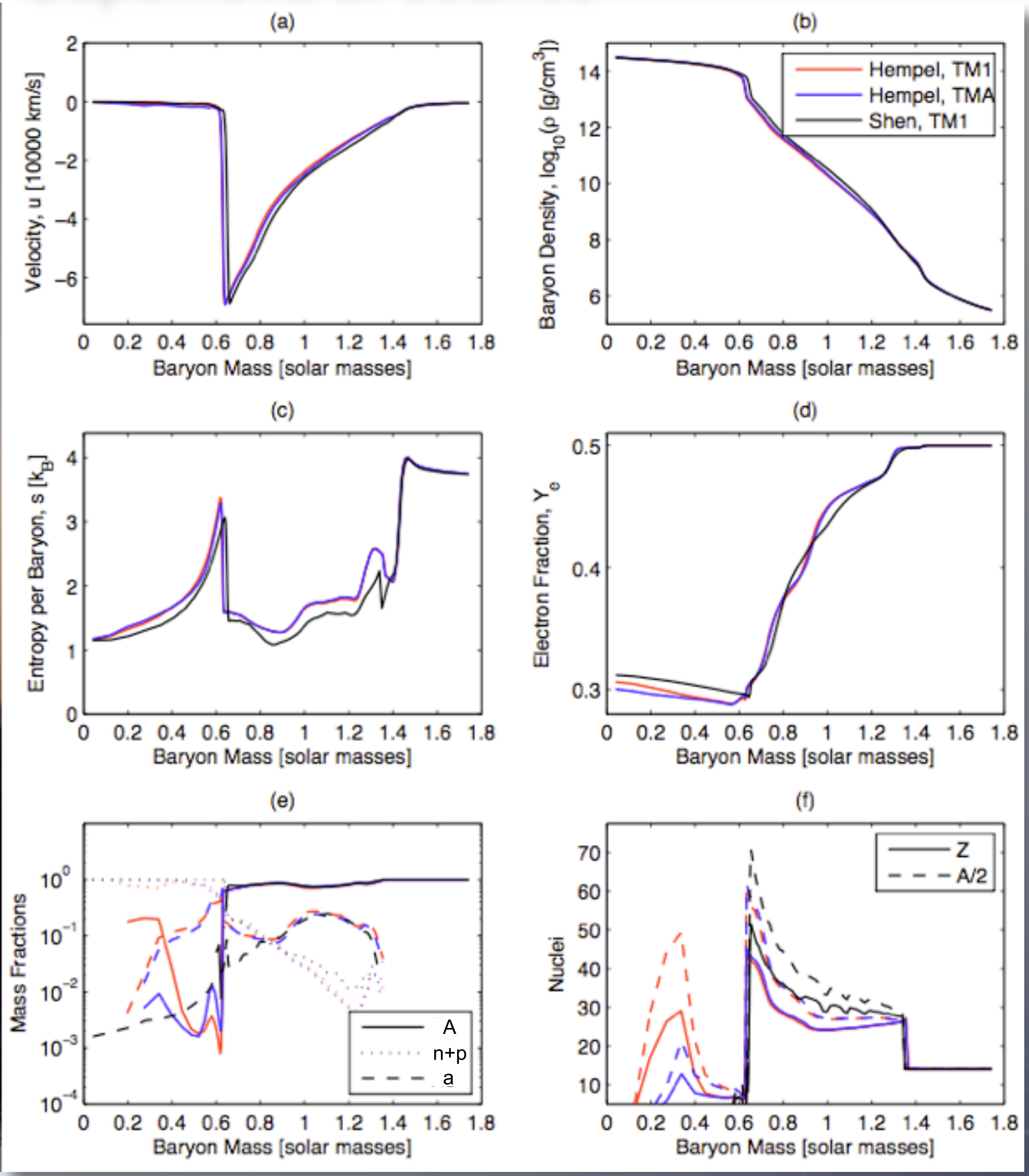
15 M$_{sun}$ progenitor (Woosley & Weaver 1995)



- similar evolution
- slightly more compact PNS core
- entropy difference due to missing kinetic entropy of heavy nuclei in Shen et al.
- slightly more e-captures in HS compared to Shen et al.

Supernova at bounce

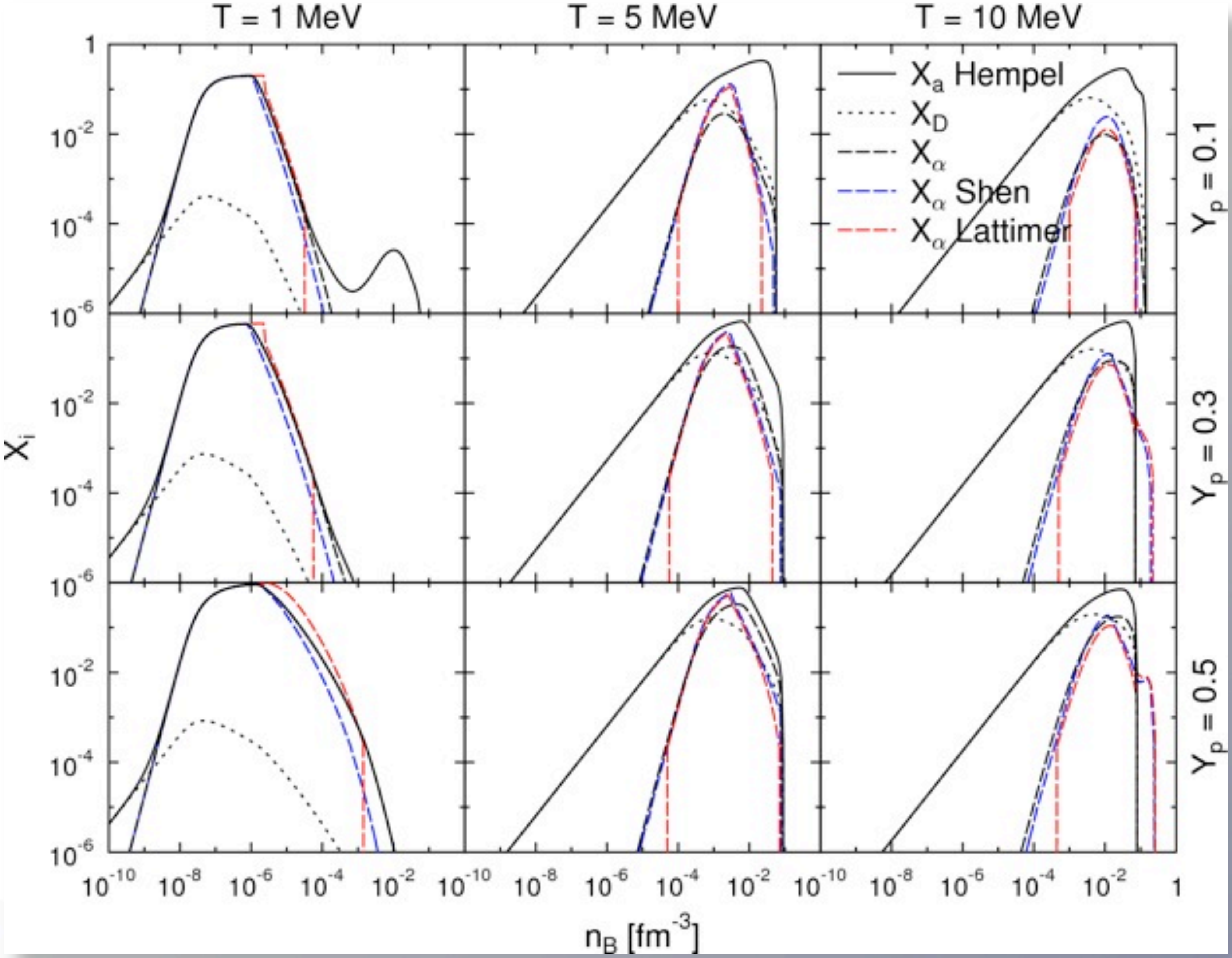
15 M_{sun} progenitor (Woosley & Weaver 1995)



- similar evolution
- slightly more compact PNS core
- entropy difference remains
- slightly more e-captures in HS compared to Shen et al. due to smaller nuclei
- appearance of light clusters behind the shock: deuterons, ^{3}H, Li

→ change of the low-density model more important than change TM1↔TMA

Light nuclei - comparison with other EOS



- T = 1 MeV:

  light clusters well represented by alphas

- T ≥ 5 MeV:

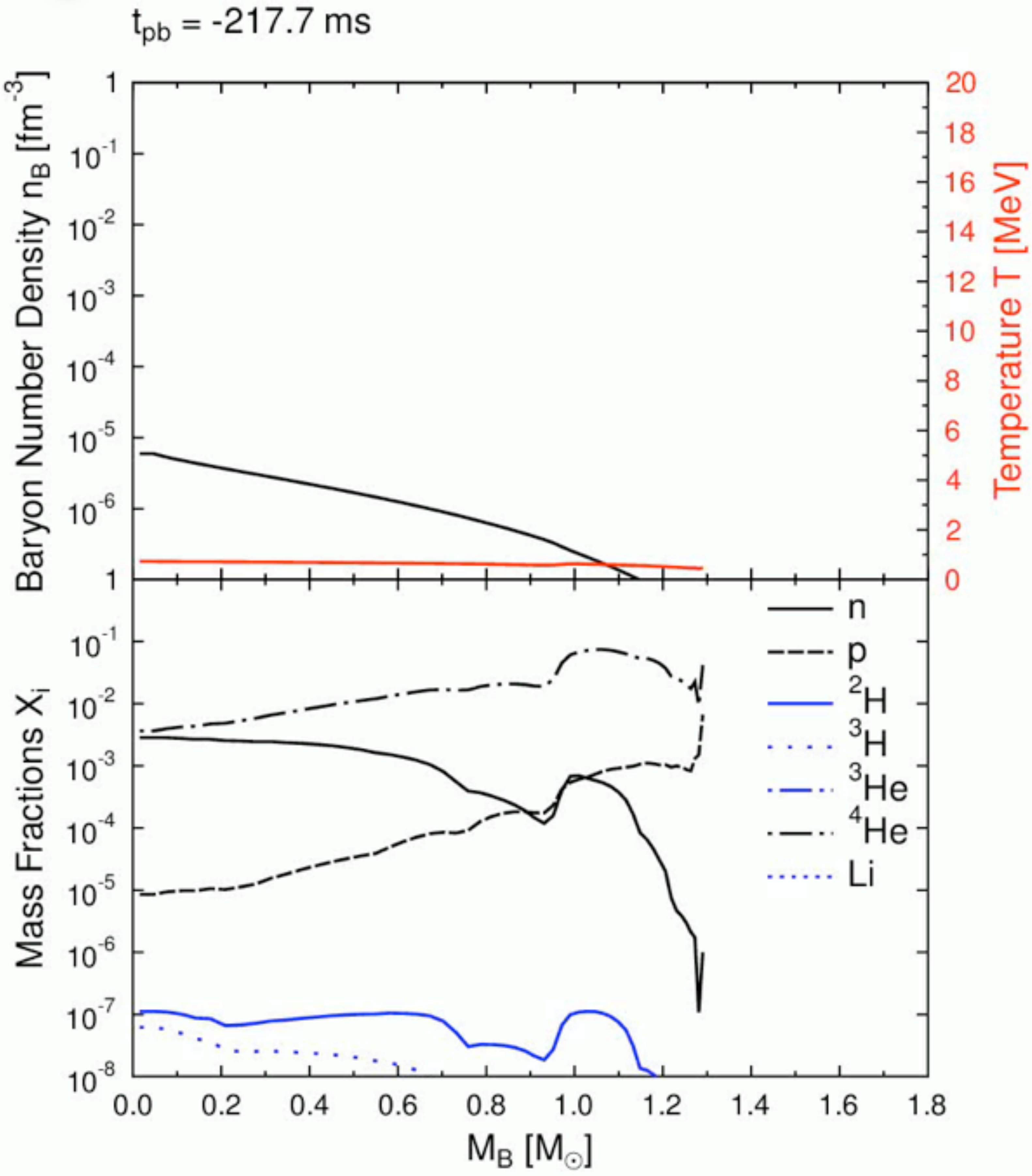
  large Deuteron contribution at low n_B,

  differences for alphas at large n_B

- n_B~ 10^{-2} fm^{-3}:

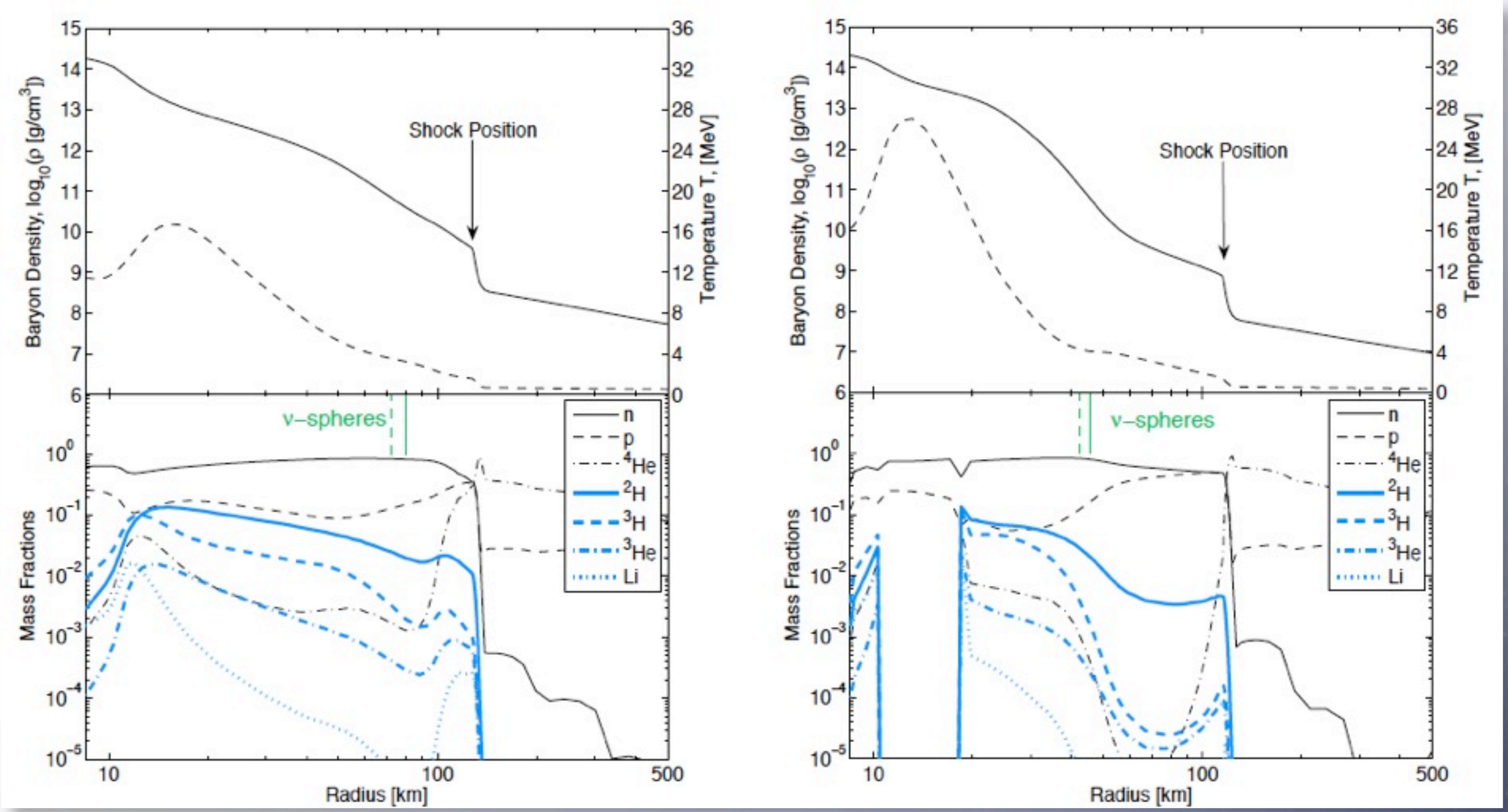
  whole distribution of light clusters important, not representable by alphas alone

Light nuclei - animation from simulation



Light nuclei - postbounce evolution

t_{pb} = 50 ms t_{pb} = 200 ms

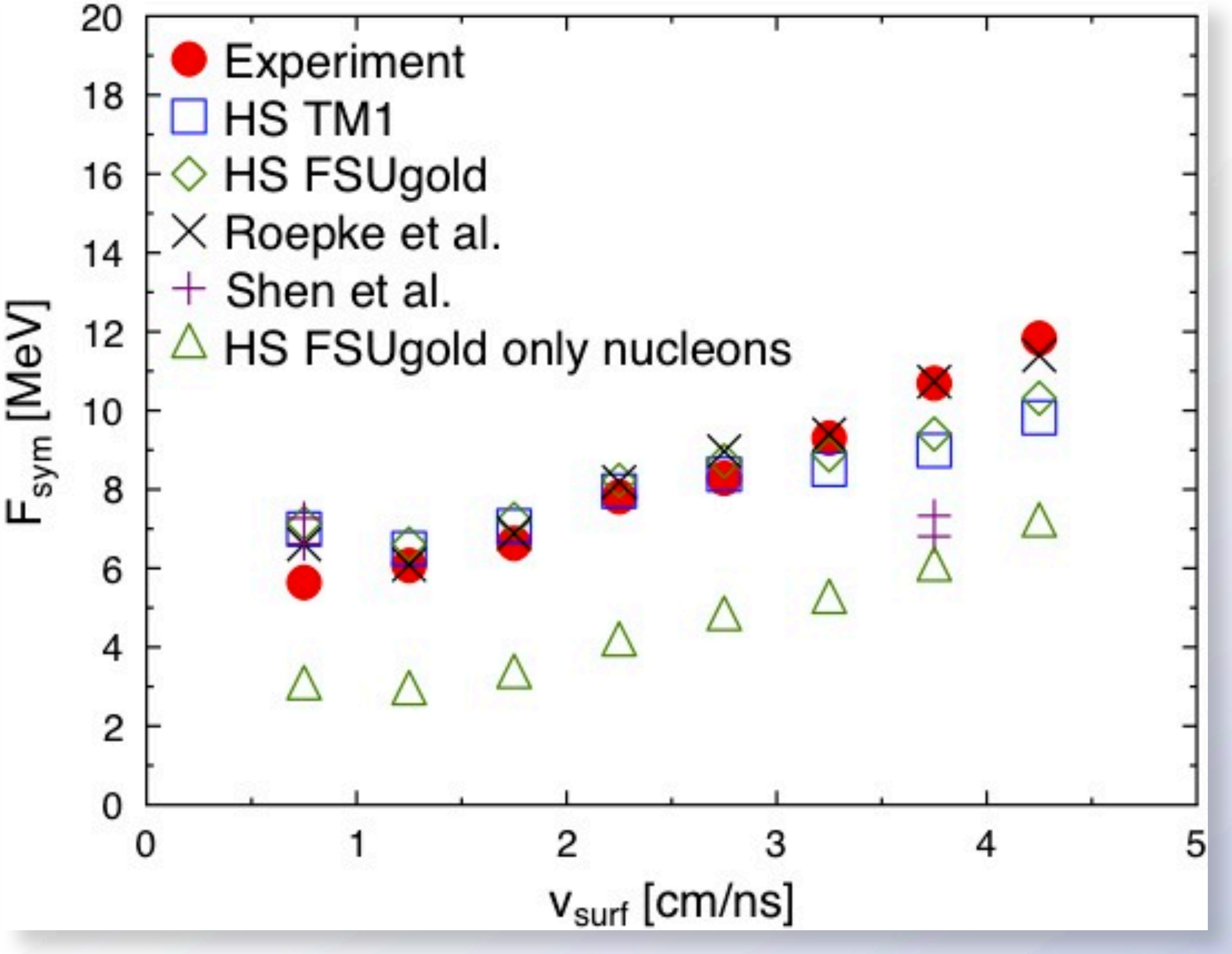


- PNS core: nucleons
- PNS envelope & shock heated matter: light nuclei
- infalling matter: heavies, alphas, nucleons

compare also:
Sumiyoshi & Röpke PRC77 2008
Typel et al. PRC81 2010

Light nuclei - measurement of symmetry energy

based on: *[Natowitz et al.; 2010PRL104]*



- symmetry energy extracted from low-energy heavy ion collisions
- thermodynamic conditions: $T = 3 – 7$ MeV and $n_B = 1/100 – 1/20\ n_B^0$
- cluster formation leads to increased symmetry energy

→ experimental evidence for appearance of light clusters in SN

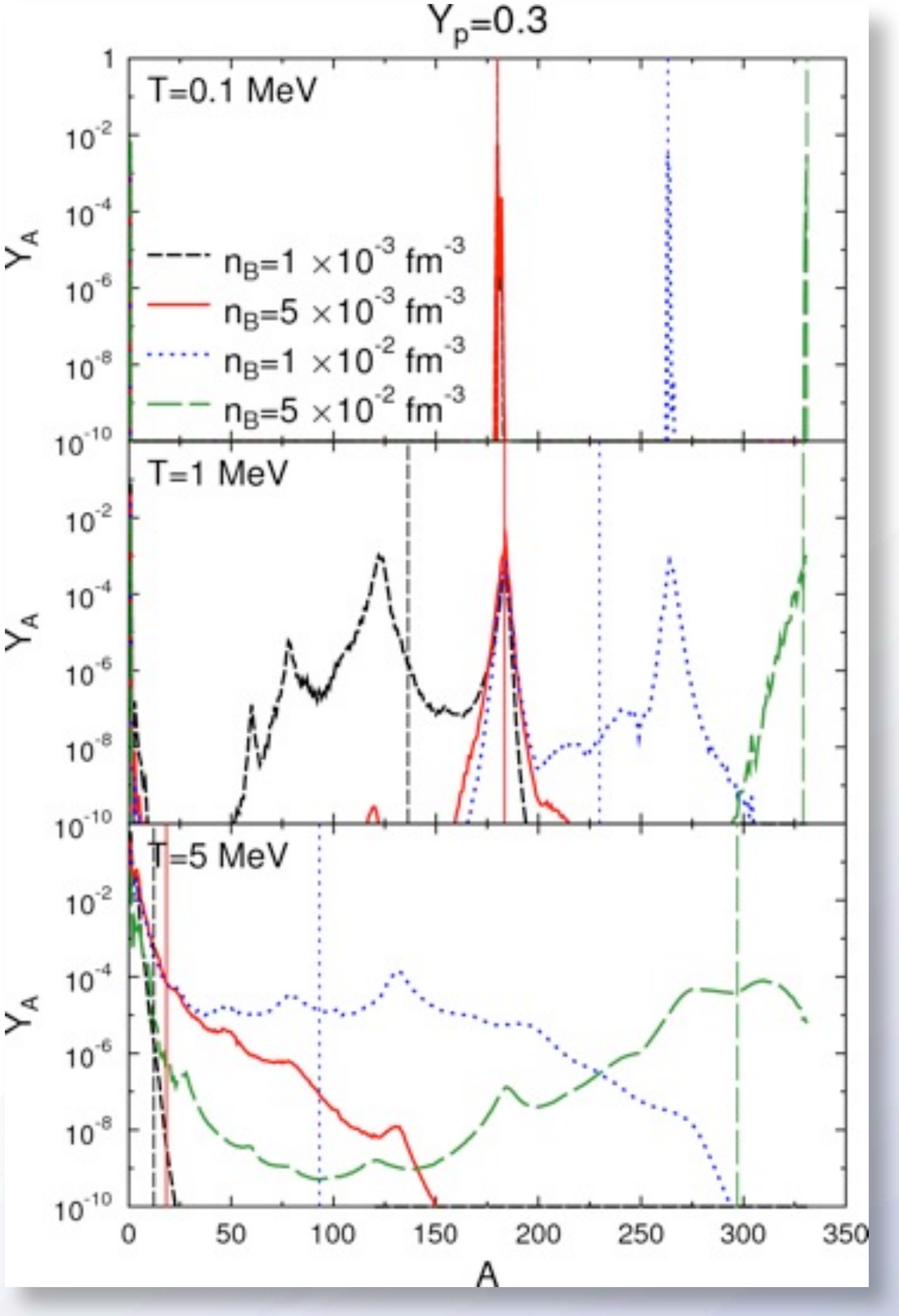
T [MeV]	3.3	3.3	3.6	4.2	4.7	5.3	6.2	7.5
n_B [10^{-3} fm^{-3}]	2.1	1.7	2.3	3.8	4.7	4.9	5.5	6.4

$$E_{sym}(n_B, T) = 1/2\ (E_{sym}(n_B, T, Y_p = 1) + E_{sym}(n_B, T, Y_p = 0)) - E_{sym}(n_B, T, Y_p = 0.5)$$

[Typel et al.; 2010PRC81]

[Kowalski et al.; 2007PRC75]

Nuclear distributions



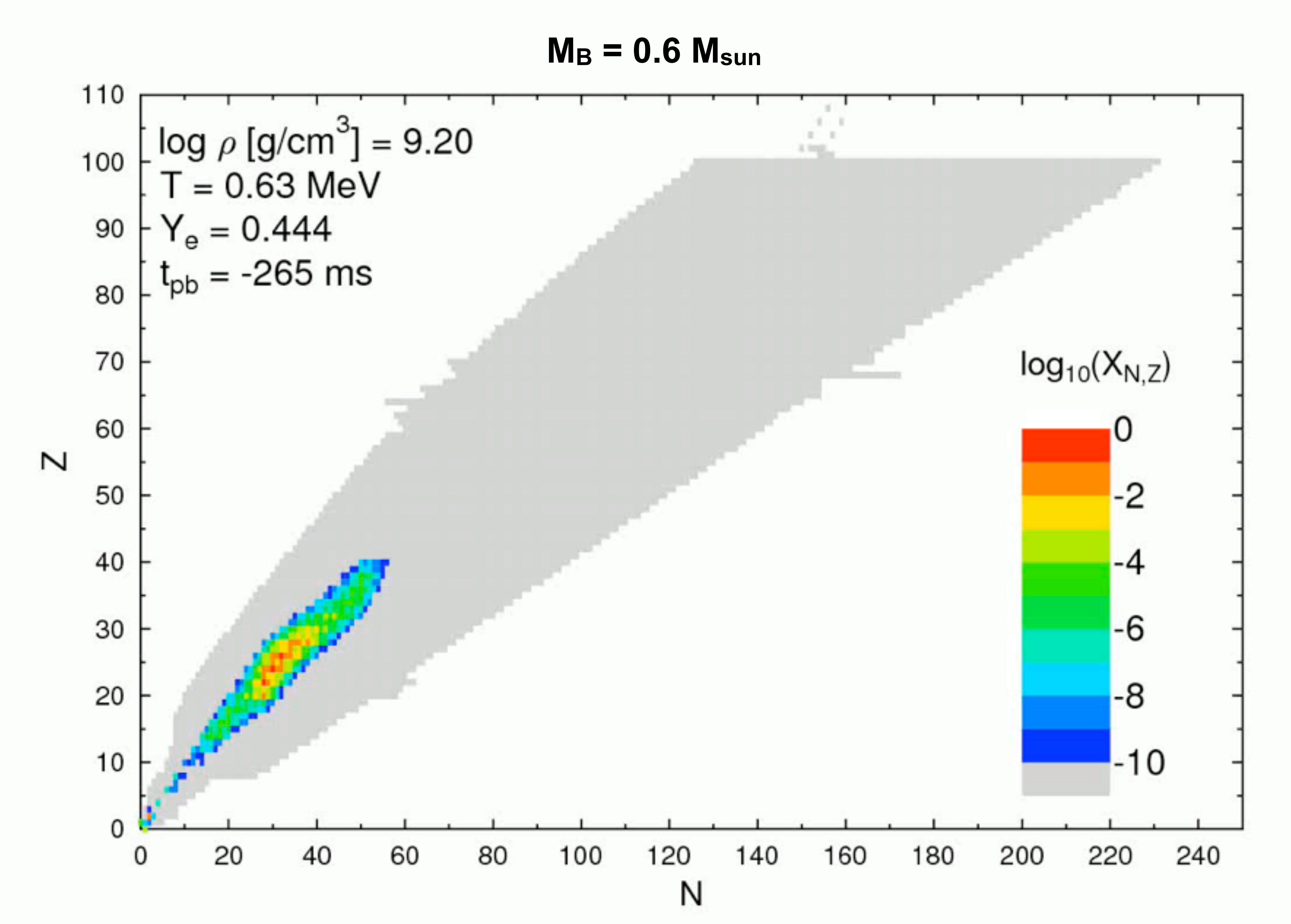
- T=0.1 MeV: only one nucleus
- peaks caused by neutron shell effects
- neutron magic numbers (40), 50, 82, 126, 184

- T=1 MeV: shell effects leading to narrow distributions with multiple peaks

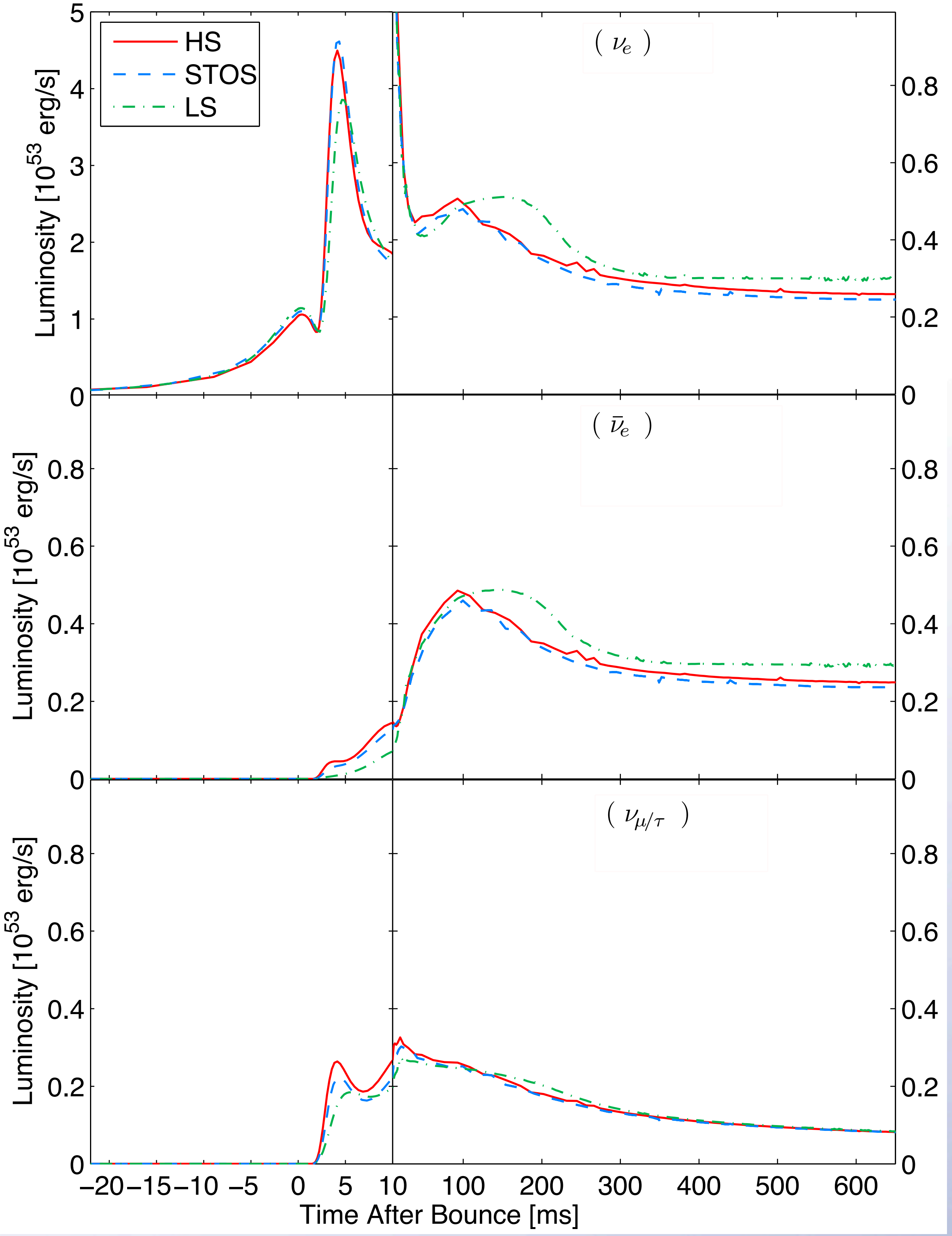
- T ≥ 5 MeV:

  broad distributions, weakening of shell effects

Nuclear distributions in the SN



Neutrino signal - 15 M_{sun} progenitor



- STOS TM1 and HS TM1: negligible differences
- softer LS EOS: hotter PNS, larger luminosities

→ only slight impact of the low density EOS on the neutrino signal

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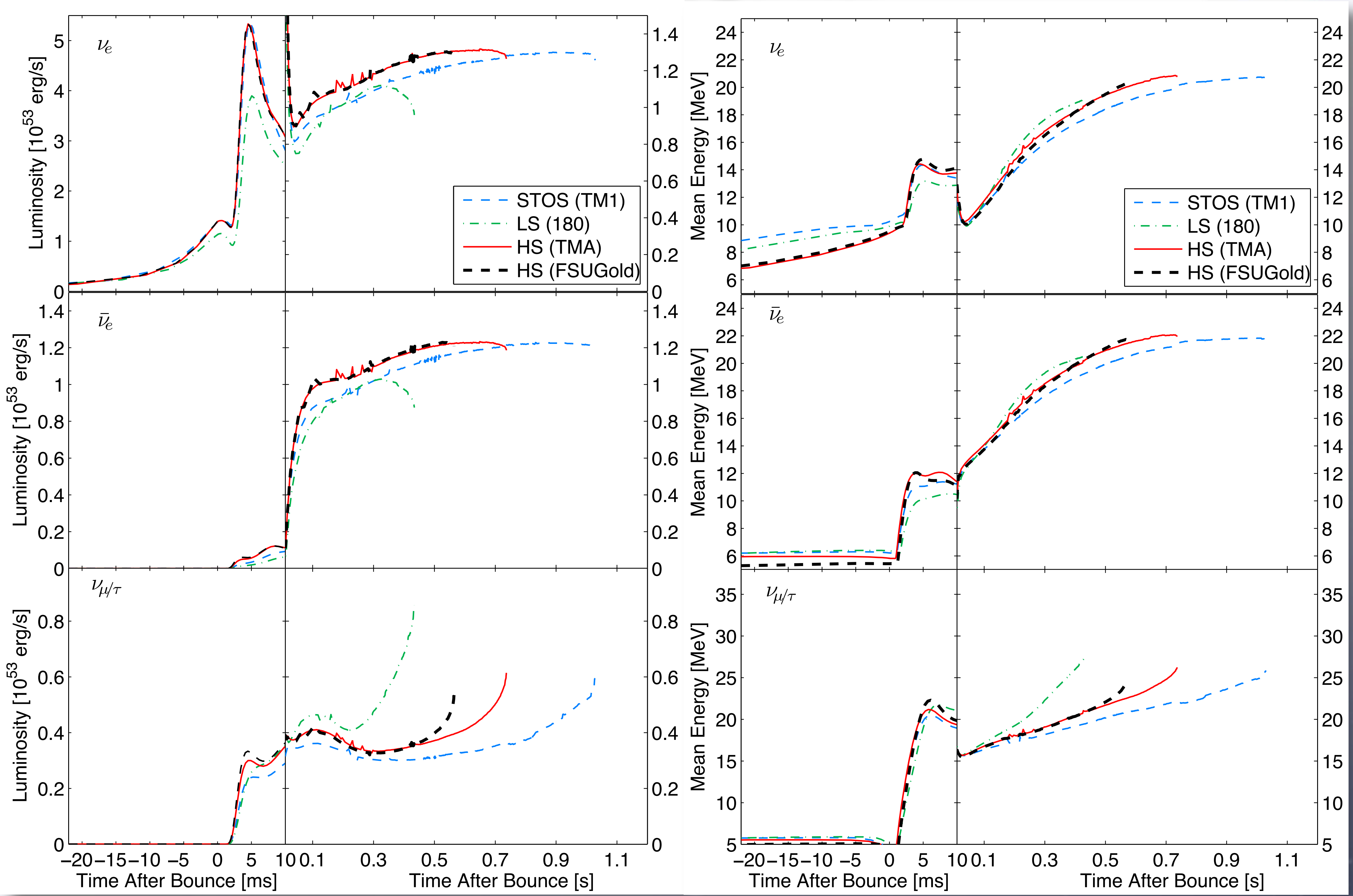
Results - high density

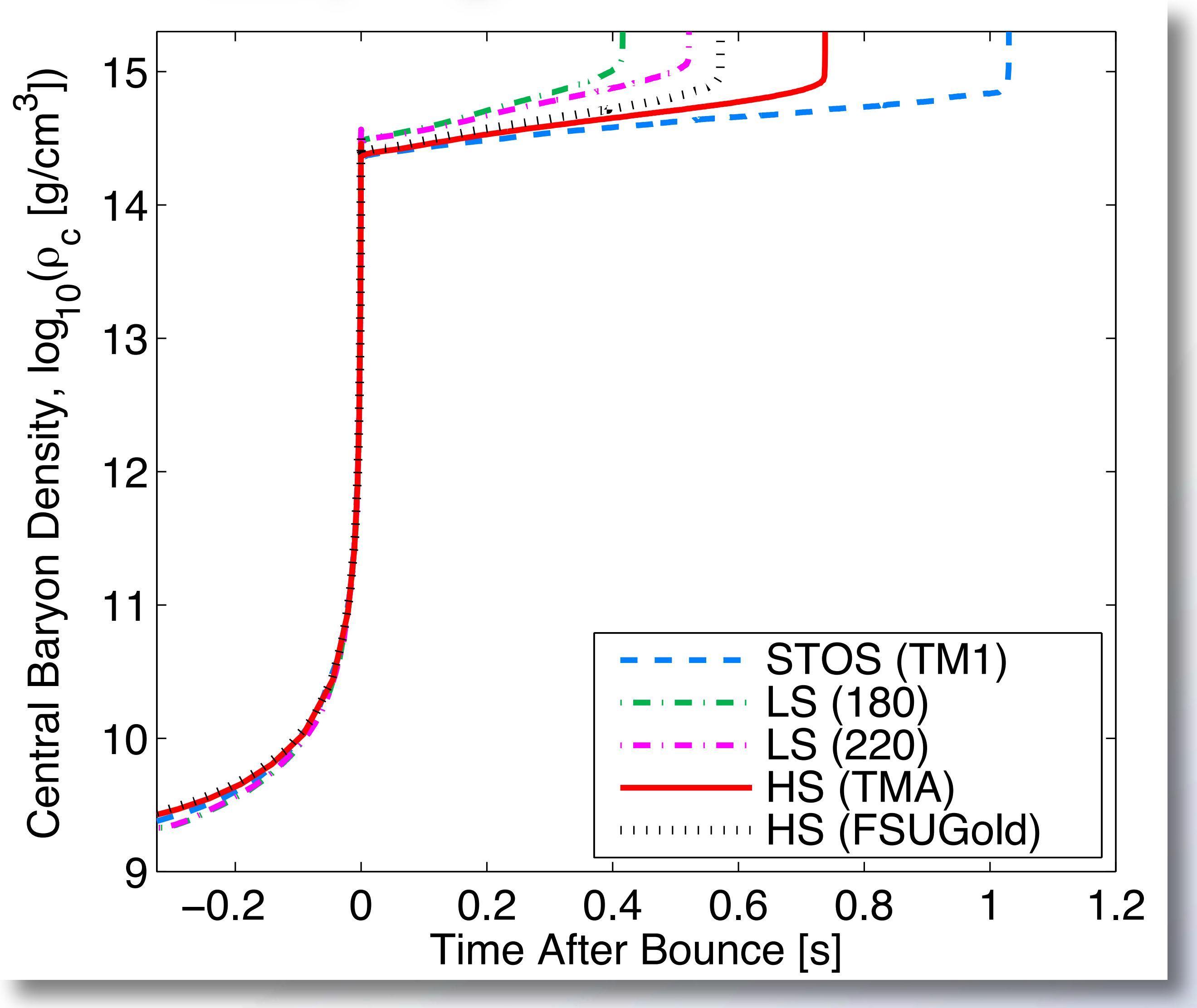
- simulations for 40 M_{sun} progenitor of Woosley & Weaver ApJS 101 (1995)
- set of EOS tables: STOS, LS 180, LS 220, HS (TMA), HS(FSUgold)

Neutrino signal - 40 M_{sun} progenitor



- different times until black hole formation
- μ/τ-neutrinos most sensitive to EOS because emitted from deeper layers

40 M_{sun} progenitor - time until black hole formation



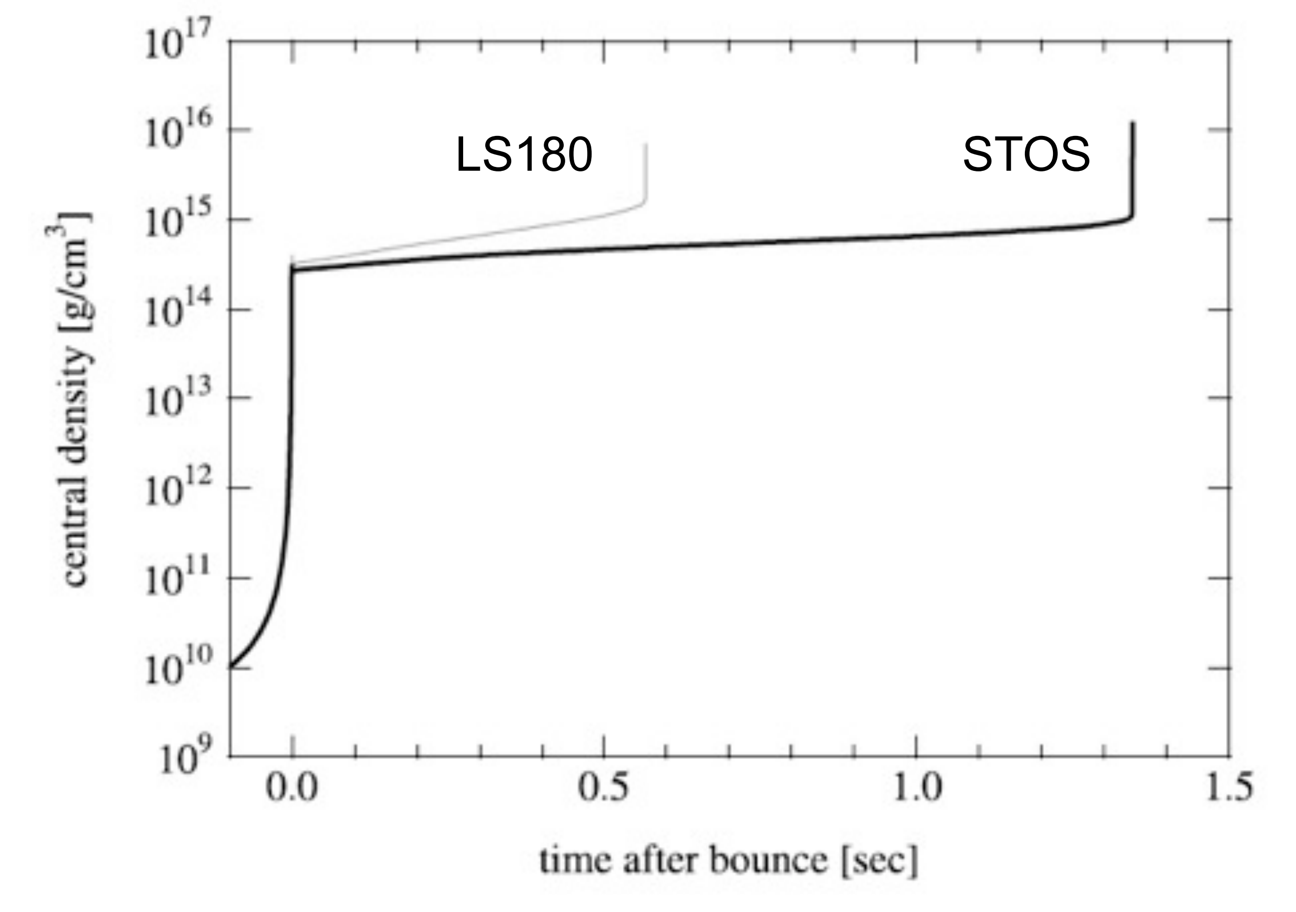
- surprising result: maximum mass of cold neutron stars in not directly correlated with t_{BH}
- non-relativistic LS collapse earlier than expected
- note: accretion rate not much affected by EOS

	M_{max}(T=0) [M_{sun}]	t_{BH} [ms]
TM1	2.22	1028
TMA	2.02	737
FSUgold	1.74	571
LS220	2.06	521
LS180	1.83	415

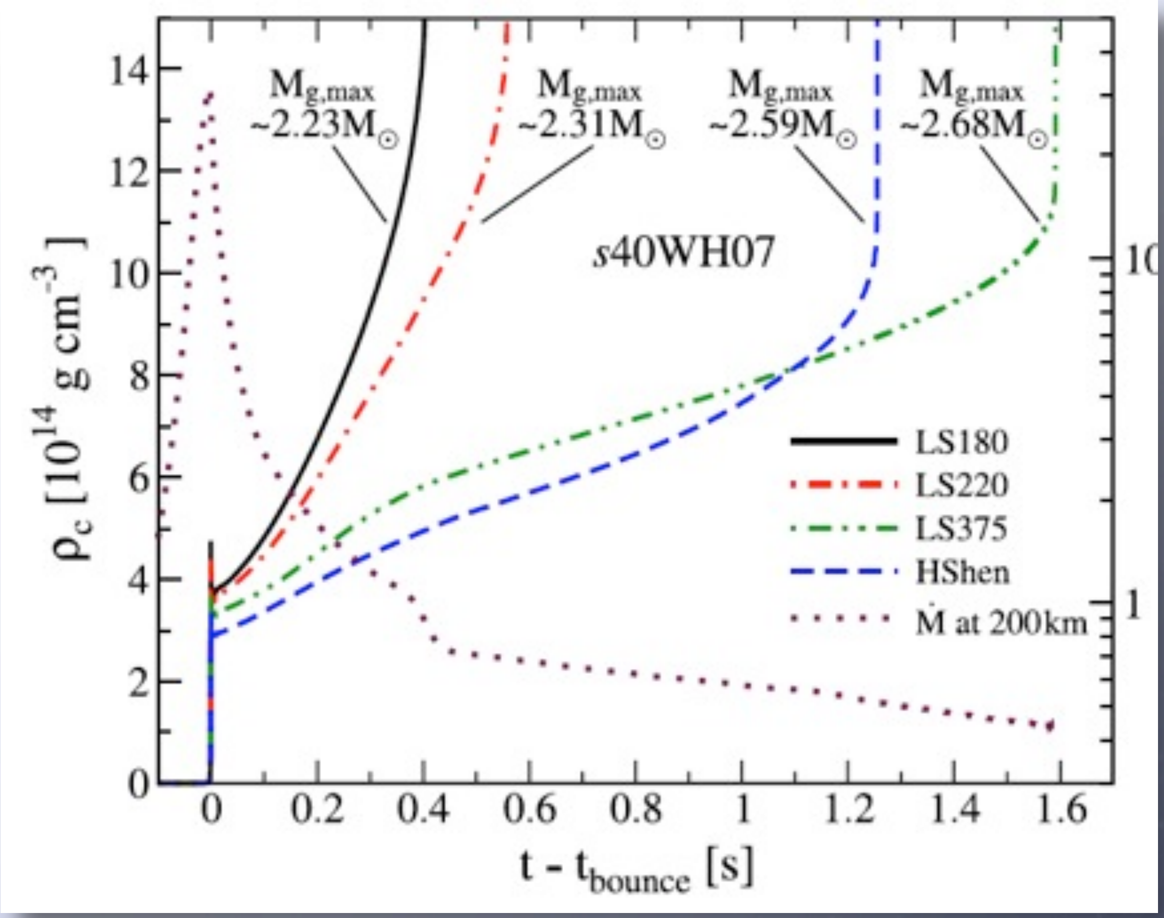
40 M_{sun} progenitor - time until black hole formation

- this was not found in previous studies, because only STOS, the stiffest RMF EOS with TM1 was available:

[Sumiyoshi et al.; 2007ApJ667]

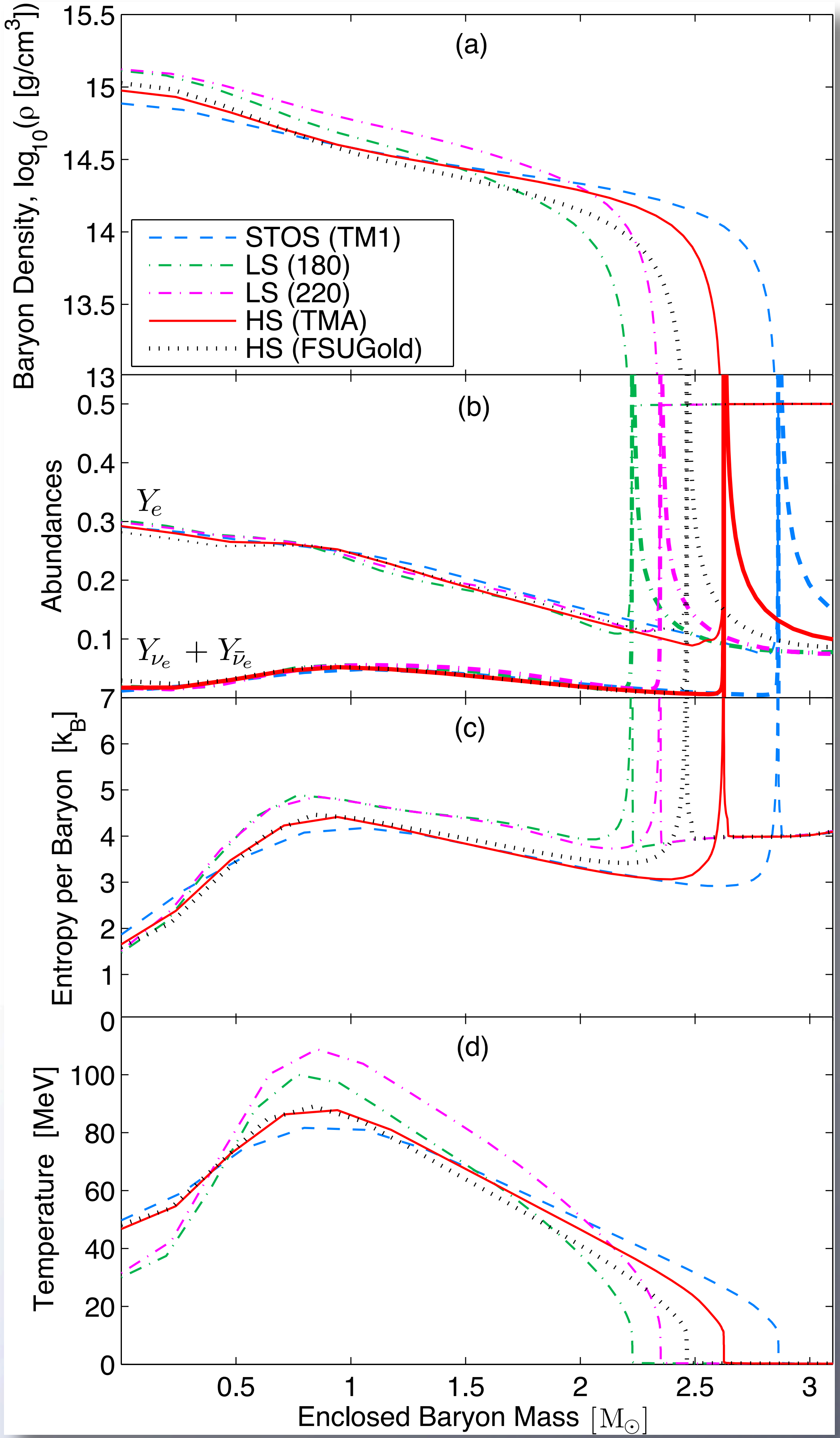


[O‘Connor & Ott.; 2011ApJ730]



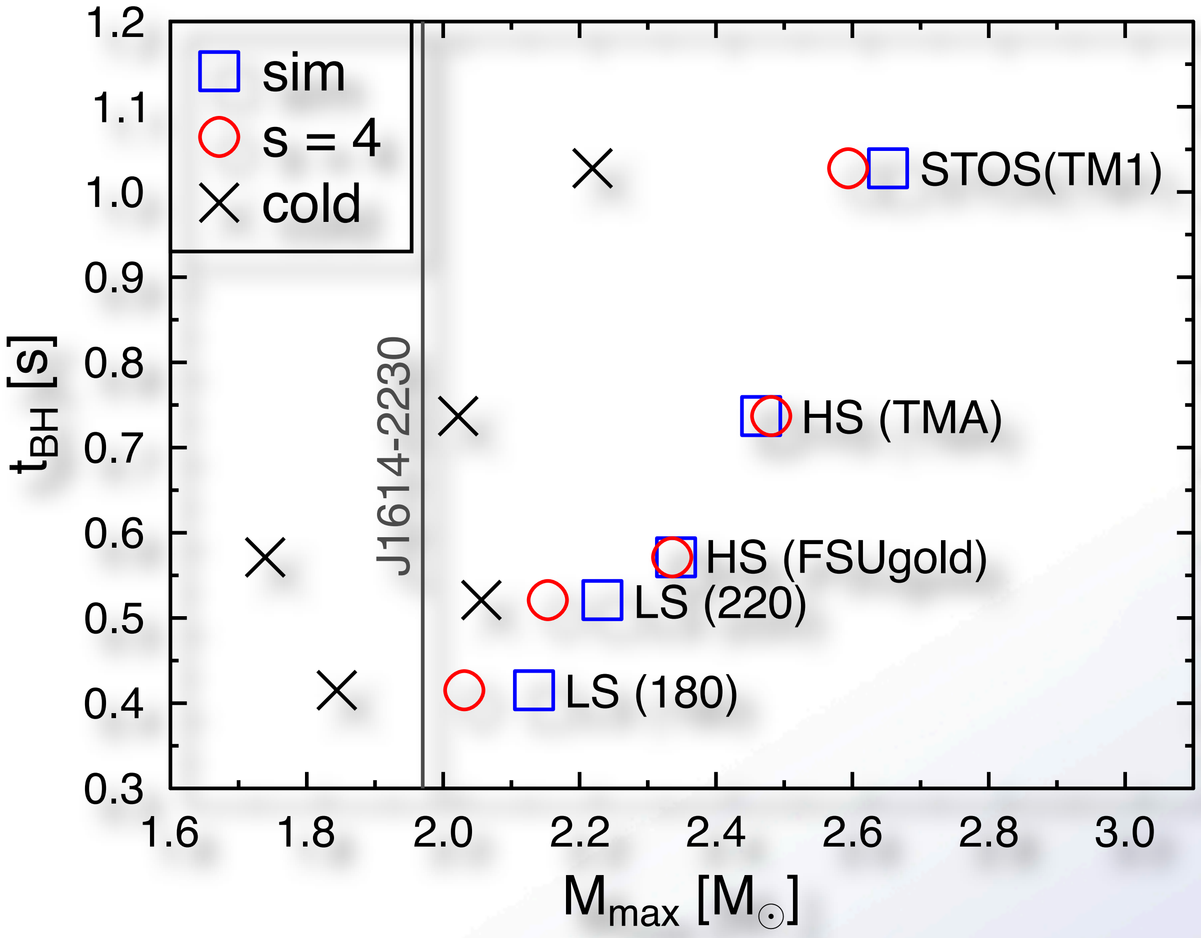
- why do the non-relativistic LS EOS collapse so early, respectively why do they behave so soft in the CCSN?

40 M_{sun} progenitor - state before black hole formation



- similar configurations of the five models at the state before the collapse to black hole
- main differences in the temperature profiles
- idea: use s=4 k_B, beta-equilibrium and the TOV equations to analyze the maximum mass

40 M_{sun} progenitor - correlation of t_{BH} with maximum mass



- the maximum gravtitational masses found in the simulation are significantly enhanced (up to 0.6 M_{sun}) compared to T=0
- the s=4 configuration reproduces the results of the simultions

- the LS EOS show a significantly weaker stiffening by temperature than the RMF EOS
- due to: effective mass, symmetry energy, non-relativistic description
- new correlation: the time until black hole formation gives information about the finite entropy EOS and the maximum mass of proto-neutron stars!

Conclusions

- phenomenological, thermodynamic consistent model for an ensemble of nuclei in equilibrium with an interacting nucleon gas

- EOS tables and routines for composition available for NL3, TM1, TMA, FSUgold, DD2:

  http://phys-merger.physik.unibas.ch/~hempel/eos.html

- results from SN simulations (low densities):
  - – broad distributions of heavy nuclei during collapse
  - – large yields of light clusters in the shock heated matter
  - – notable impact of the low-density EOS model

  → role of light nuclei and distributions on neutrino transport?

- high densities:
  - – SN of massive progenitors give information about the finite entropy EOS
  - – temperature effects are EOS dependent

  → extract information about nucleon effective mass or symmetry energy?

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