A New Equation of State with Abundances of All Nuclei at sub-nuclear densities

Shun Furusawa (Waseda Univ.)

Shoichi Yamada (Waseda Univ.)

Kohsuke Sumiyoshi (Numazu college of Tech.)

Hideyuki Suzuki (Tokyo Univ. of Science)

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Outline

1.Introduction

・Astrophysical Background・Nuclear physical Background ・Previous works

2.EOS Model

・NSE method ・Free energy model

3.Results

・Thermodynamics quantities ・Nuclear abundances in (N,Z) plane

4.Summary

1,Introduction

・Astrophysical background

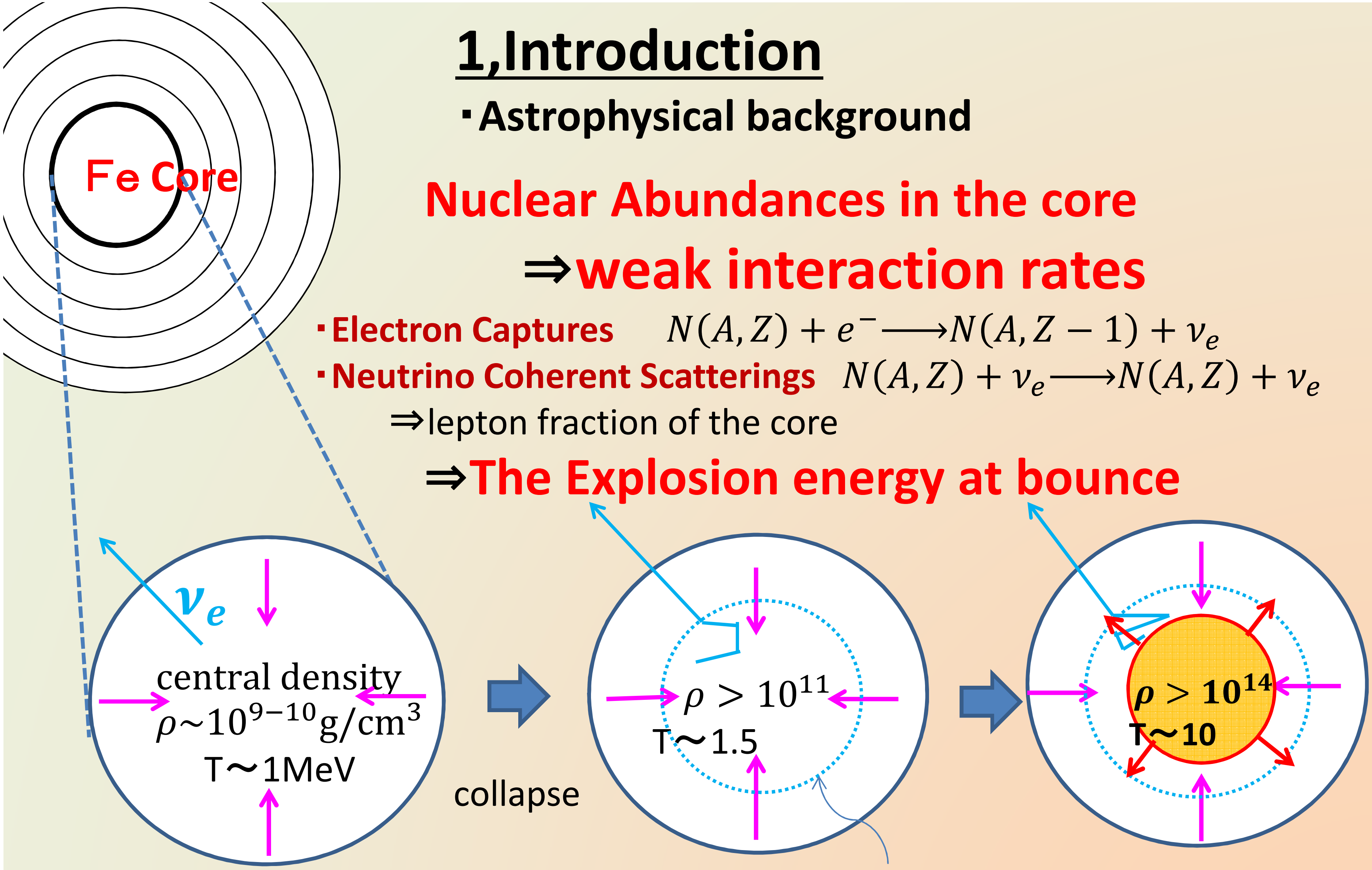
Nuclear Abundances in the core

⇒weak interaction rates

- **Electron Captures** $N(A,Z)+e^{-}\longrightarrow N(A,Z-1)+\nu_e$
- **Neutrino Coherent Scatterings** $N(A,Z)+\nu_e\longrightarrow N(A,Z)+\nu_e$

⇒lepton fraction of the core

⇒The Explosion energy at bounce



Fe Core

ν_e

central density $\rho \sim 10^{9-10}\,\mathrm{g/cm^3}$

T～1MeV

collapse

$\rho > 10^{11}$

T～1.5

Neutrino Sphere

$\boldsymbol{\rho > 10^{14}}$

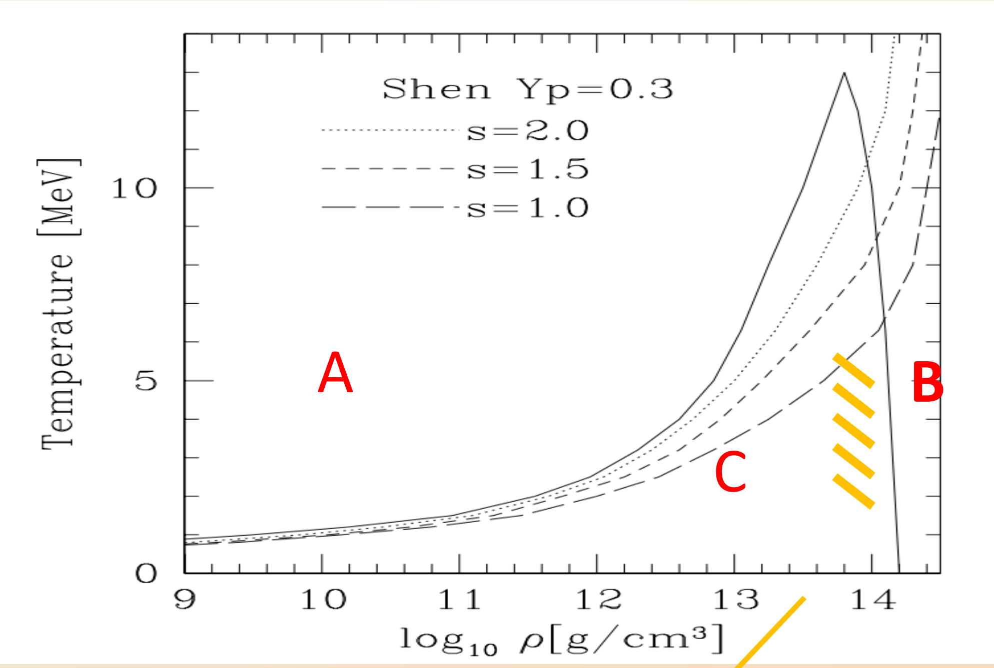
T～10

Bounce ($\rho=\rho_0$)
⇒Shock Wave

The core of massive stars (M>10Msolar)

Supernovae

・Nuclear physical background



Nuclear Phase diagram

A. dilute free nucleons

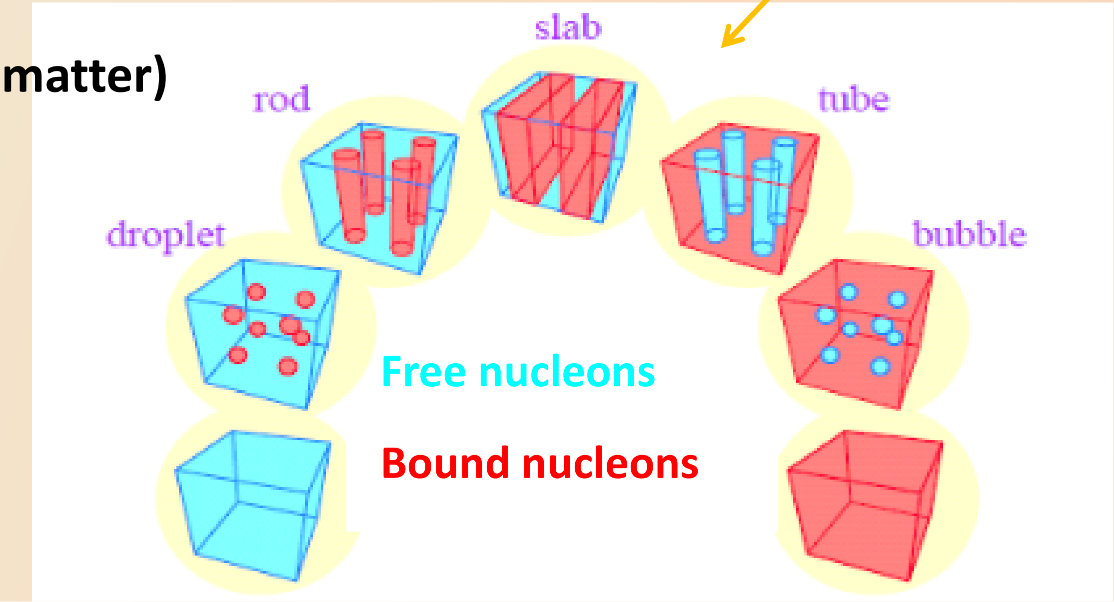
B. strongly interacting nucleons

C. the ensemble of various nuclei

Nuclear pasta

(from droplet nuclei to uniform matter)

⇒Droplet
Rod,
Slab,
Tube(Rod-hole),
Bubble(Droplet-hole)



Previous works and Motivation

●Standard EOSs

1,Lattimer et al.(1991)

- Skyrme Type Interaction & Compressible Liquid Drop model

2,Shen et al.(1998)

- Relativistic Mean Field(RMF) & Thomas Fermi approximation

Only one heavy nucleus (Single Nucleus Approximation(SNA))
For actual calculations of weak interaction rates,
we need the EOS with multi species of nuclei.（Motivation）

●Multi-Nuclei EOS

3,Hempel et al. (2009)

- Nuclear Statistical Equilibrium(NSE) & RMF
- Mass data + coulomb correction (proton number Z < 100)

4,Botvina et al. (2010)

- NSE & Mass Formula (mass number A<1000)

2, EOS model

Baryonic EOS with Multi Species of Nuclei (Z<1000)

・Formulations

NSE (Nuclear Statistical equilibrium)

$T \; > \sim 5 \times 10^9$ [K]

$$(A, Z) \leftrightarrow (A-1, Z) + n$$
$$(A, Z) \leftrightarrow (A-1, Z-1) + p$$

[time scale of all nuclear interactions] < [dynamical time scale]

We solved **the minimized Free Energy *F* (*X*p, *X*n, {*X*i})**

for **Abundances *X*p, *X*n, {*X*i}** on given *ρ*, *T* & *Y*p .

⇒ Thermodynamics Quantities & Abundances ***X*p, *X*n, {*X*i}**

The Free Energy density

$$f = f_{p,n} + \sum_i n_i \{ E_i^{trans} + \underline{E_i^{bulk} + E_i^{surf} + E_i^{coul}} \}$$

free nucleons

nuclei

Translational Energy

Nuclear Mass

i : index of a nucleus (1<Z<1000,1<N<1000)

Free Nucleons: RMF theory (TM1 parameter set)
and excluded volume effect

Nuclei:
- approximate excluded volume effect for translational energy
- A Mass Formula based on Liquid Drop Model
(including Nuclear Shell term & approximate nuclear pasta phase)

The points of the free energy
@Low densities : Boltzmann gasses
with experimental mass data (ordinary NSE)
@ Saturation density : a continuous transition
to the EOS for supra-nuclear density (RMF)

Coulomb and Surface Energies and pasta phases

$$M_i = E_i^{bulk} + E_i^{surf} + E_i^{coul}$$

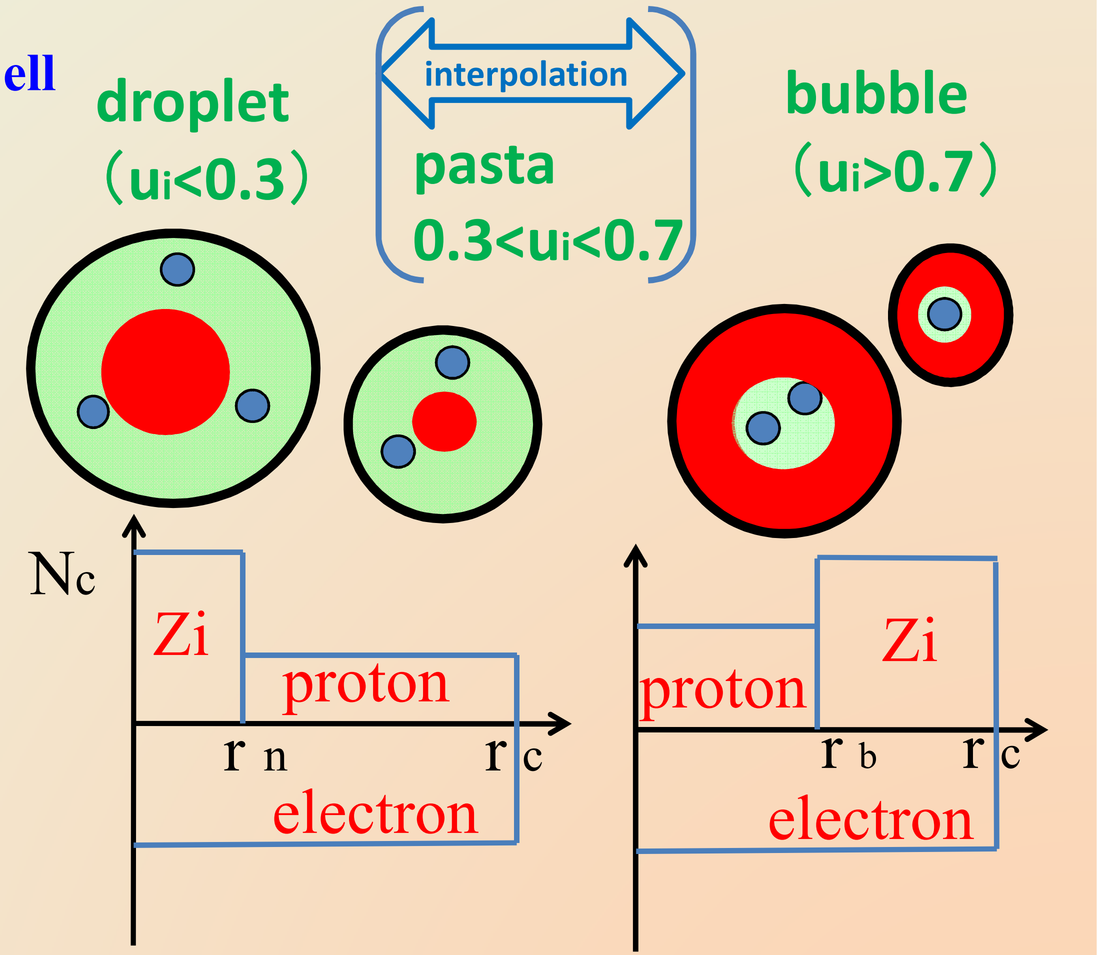
Charge neutrality in Wigner-Seiz Cell

$$\boxed{V_i n_e = Z_i + (V_i - V_i^N) n'_p}$$

$$\boxed{u_i = V_i^N / V_i}$$

Volume of a cell V_i ○

Volume of a nucleus V_i^N ●



droplet (ui<0.3)

interpolation

pasta 0.3<ui<0.7

bubble (ui>0.7)

- **Surface: Surface tensions σ_i (Lattimer 1991) +high density correction**

$$\boxed{E_i^s = \sigma_i \times (surface\ area) \times (1 - n_{nucleon}/n_{si})^2}$$

- **Coulomb :Integration of Coulomb force in WS cell**

Bulk Energy(including Symmetric energy)

$$M_i = \boxed{E_i^{bulk}} + E_i^{surf} + E_i^{coul}$$

RMF calculation at $n_B = n_{si}$ (saturation density) $Y_p = Z_i/A_i$,T

Ⓐ $$\boxed{E_i^{bulk}(T) = A_i \underline{F^{RMF}(n_{si}, T, Z_i/A_i)}}$$

Free Energy per baryon

Experimental mass data should be used whenever available **at low densities**
(shell energies(magic numbers) are included) $(\boldsymbol{\rho < 10^{12}}\mathrm{g/cm^3})$

Ⓑ $$\boxed{E_i^{bulk} = M_i^{\mathrm{data}} - [E_i^{coul}]_{\rho, n'_p, n'_n = 0} - [E_i^{surf}]_{\rho, n'_p, n'_n = 0}}$$

Interpolation of density

experimental data ($10^{12}\mathrm{g/cm^3}$) Ⓑ ⟷ Ⓐ RMF ($\sim 10^{14.2}\mathrm{g/cm^3}$)

The two points of the Free Energy Density

@Low densities

$$f = \underbrace{f_{p,n}}_{\Rightarrow\text{Boltzmann}} + \sum_i n_i \{ \underbrace{E_i^{trans}}_{\Rightarrow\text{Boltzmann}} + \overbrace{\underbrace{E_i^{bulk} + E_i^{surf} + E_i^{coul}}_{\Rightarrow\sim\text{Mass data}}\}}^{\text{nuclear mass}}$$

The ordinary NSE EOS (e.g. Frank Timmes EOS)

@nuclear saturation density

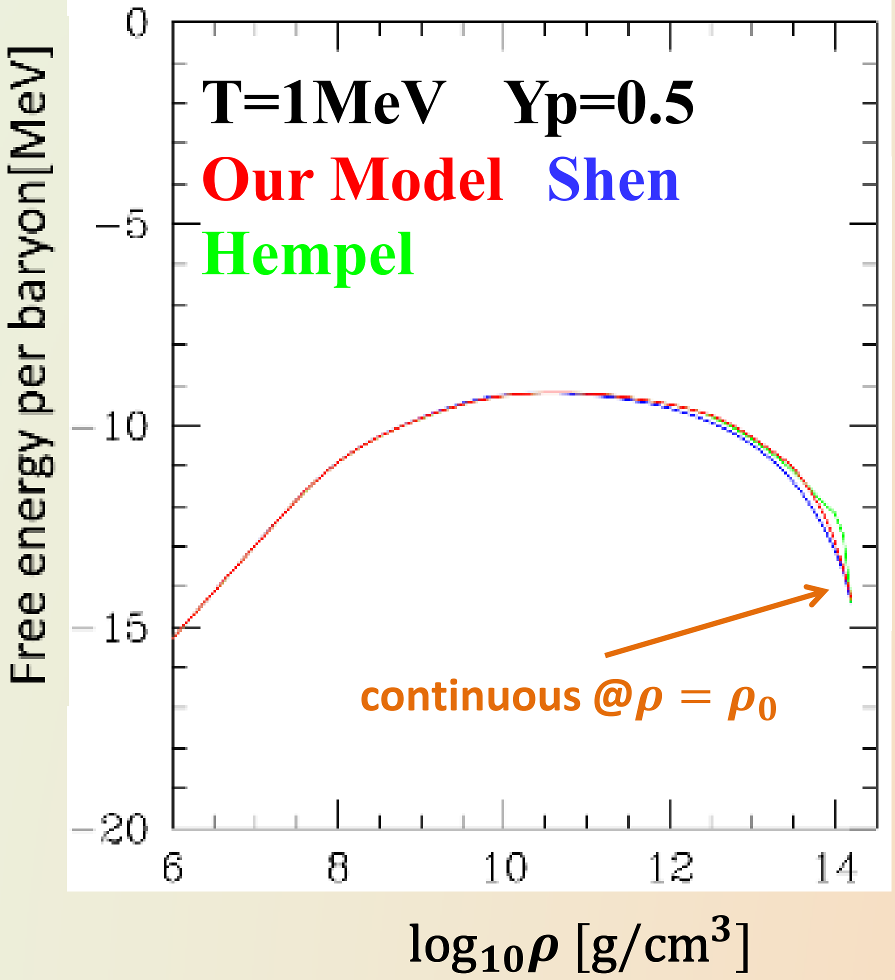
$$f = \underbrace{f_{p,n}}_{\Rightarrow\text{RMF}} + \sum_i n_i \{ \underbrace{E_i^{trans}}_{\substack{\Rightarrow 0 \\ \text{(excluded volume)}}} + \underbrace{E_i^{bulk}}_{\Rightarrow\text{RMF}} + \underbrace{E_i^{surf} + E_i^{coul}}_{\substack{\Rightarrow 0 \\ \text{(bubble volumes} \Rightarrow 0)}} \}$$

Continuous Connection to RMF EOS for uniform nuclear matter (same as Shen EOS at supra-nuclear densities)

3,Results

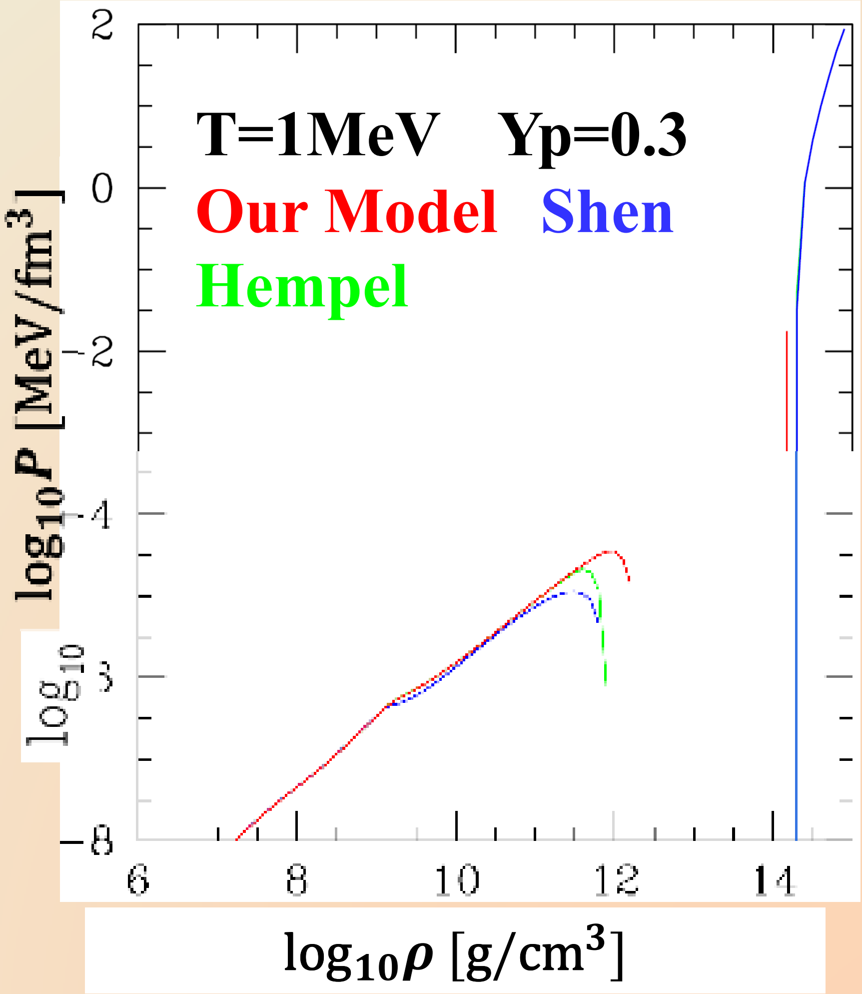
Thermodynamical quantities are not very different from other EOSs.

①Free Energy/baryon



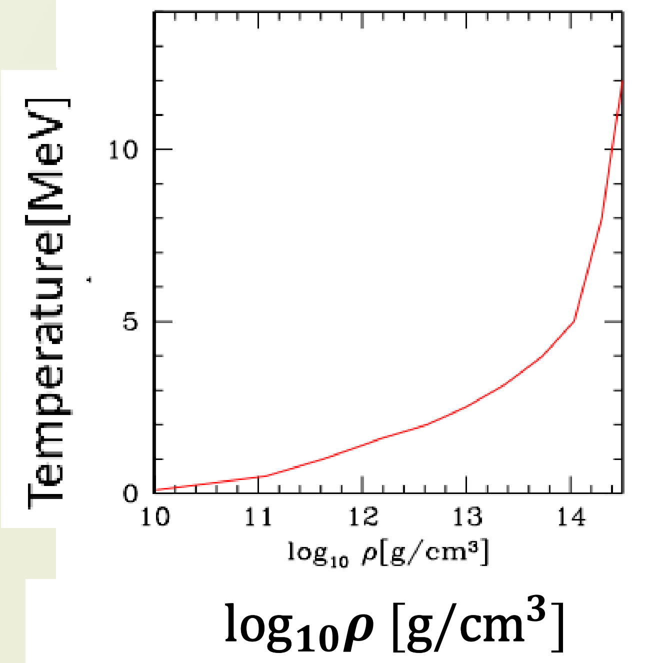
continuous @$\rho = \rho_0$

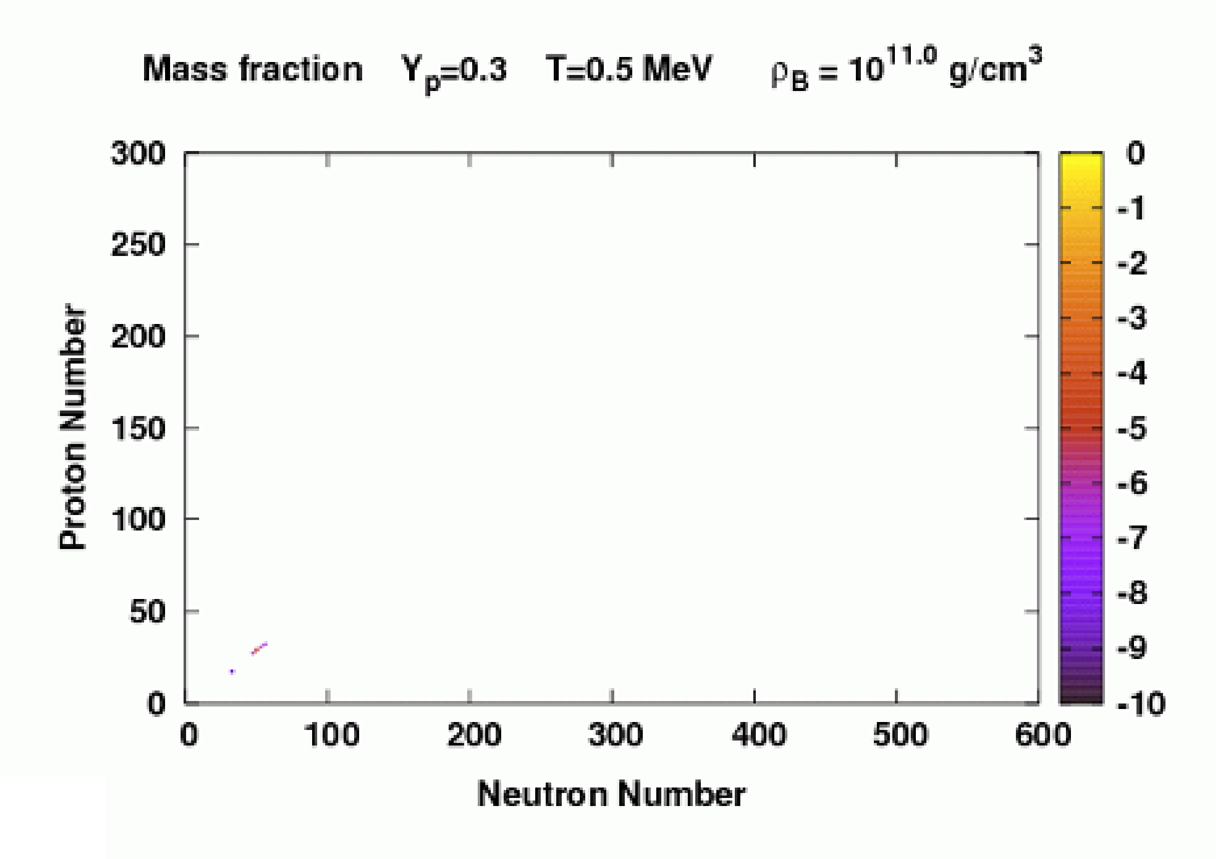
②baryon pressure



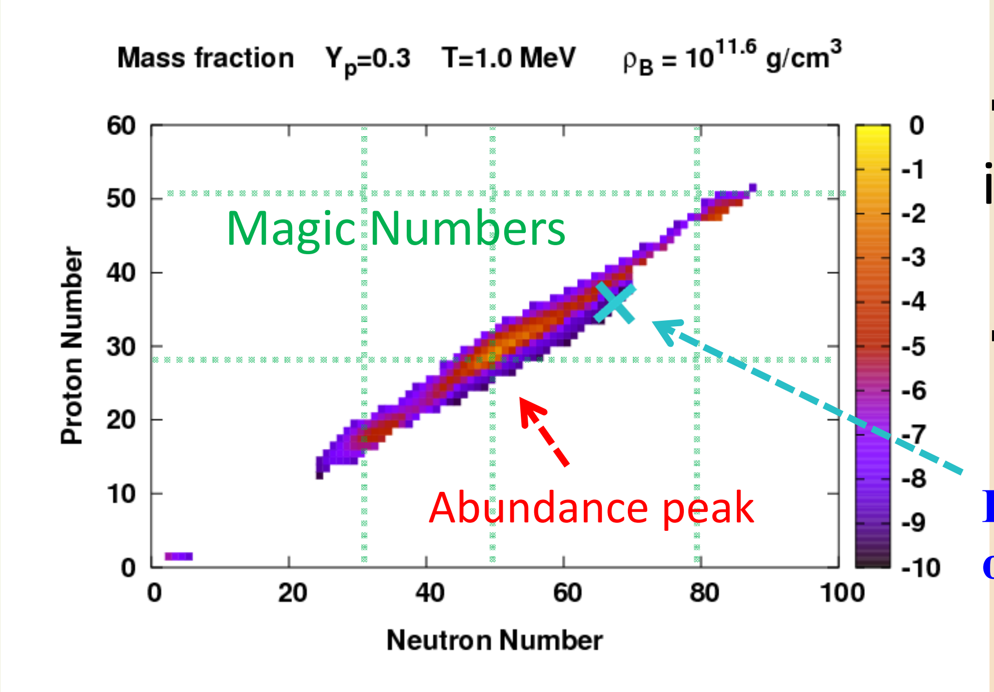
③ Mass fractions of nuclei in the (N, Z) plane along with adiabatic line ($S_{baryon} = 1\ [k_B]$)

Adiabatic line
Yp=0.3
($S_{baryon} = 1\ [k_B]$)





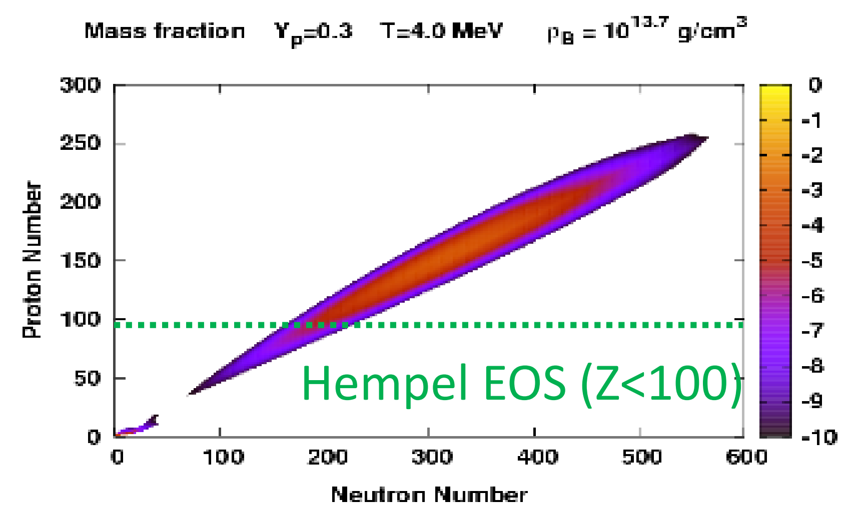
Mass fractions of nuclei （log10） in the (N, Z) plane



- Nuclei are abundant in Vicinities of Magic Numbers.

- Abundance peak is different from Shen EOS .

Representative nucleus of Shen's EOS (SNA)



Heavier than the limit of Hempel EOS nuclei are abundant

Comparison with multi nuclei EOS

(N. Buyukcizmeci, A. Botvina, M. Hempel , S. Furusawa et al. in progress)

Hempel :experimental and theorical mass data (Z<100)

Botvina :mass formula

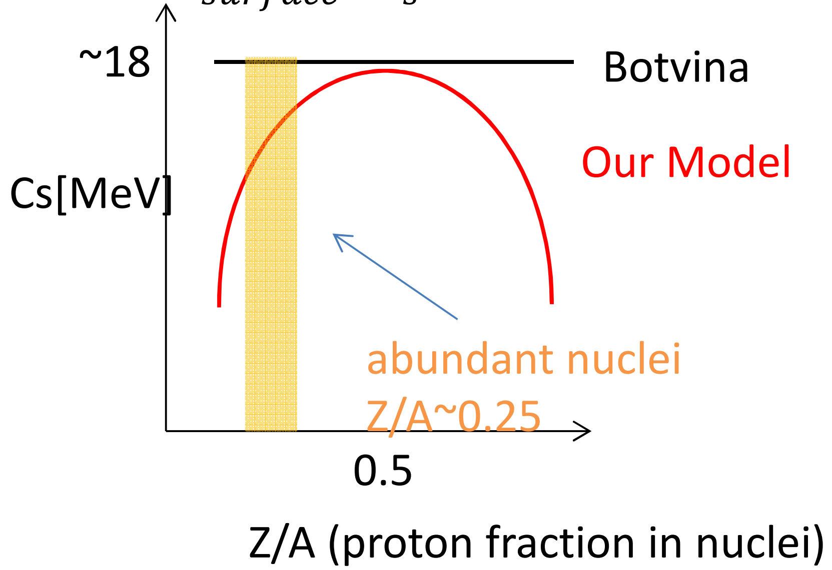
Our Model: mass formulae + experimental mass data

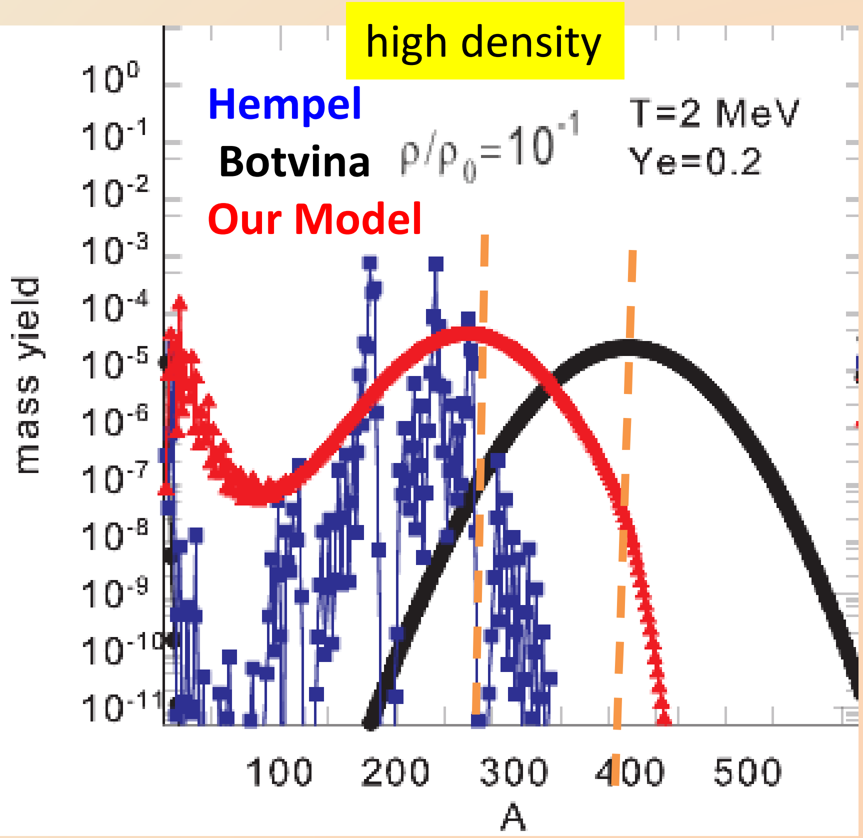
mass yield(A) $= \sum_Z N_{Z,A}/N_{baryon}$ N=number

Peak difference ⇐Difference of mass formulae

Surface Energy Coefficient C_s

$E_{surface} \sim C_s\ A^{2/3}$





4, Summary

Model: NSE

Free Nucleons: RMF

Nuclei: a mass formula based on Liquid Drop Model including

Shell Term ⇒ Ordinary NSE @low densities

Nuclear Pasta Phase ⇒continuous transition @ nuclear density

Results

● Thermodynamical quantities are not very different from other EOS.

● nuclear abundances are different.

(∵ multi nuclei or single nucleus approximations,
mass data or mass formula, difference of mass formula …)
⇒ It affects neutrino interactions in collapsing cores.

Next Steps

- Binding Energies of Light Nuclei (deuterons, triton, Helium 3)
- Other uniform matter EOS (RMF ⇒ other model)

Numerical method（Ordinal NSE and Our model)

How ρ、Ye=Yp、T ⇒ pressure, entoropy…& abundances of all nuclei ？

①Charge Neutrality

$$\boxed{n_p + \sum_i Z_i n_i = n_e = Y_e n_B}$$

②Baryon number conservation

$$\boxed{n_p + n_n + \sum_i A_i n_i = n_B = \rho / m_B}$$

We have to find the minimum of Free Energy Density on the conditions ①&②

$$\frac{\partial}{\partial n_i}(f - \alpha\text{①} - \beta\text{②}) = 0 \quad \Longrightarrow \quad \boxed{\mu_i = Z_i \mu_p + (A_i - Z_i)\mu_n}$$

Chemical potential

●Ordinal NSE ：constant M_i ⇒ n_i is determined from μ_p μ_n

We have to solve the equations ①、② for variables μ_p & μ_n

●Our model ：M_i depend on n_p n_n ⇒ n_i depend on μ_p μ_n n_p & n_n

We have to solve ①,② .③ (the relation between μ_p and n_p) & ④(μ_n between n_n) for μ_p, μ_n, n_p & n_n

	Lattimer	**H.Shen**	**Our model**
Model	Skyrme+ Compressible LDM	RMF + Thomas Fermi	RMF+ NSE+ +LDM +mass data
Component heavy nuc.	Single	Single	multi (Z<1000)
Shell term	×	×	△
Nuclear Shape	Droplet +bubble	Droplet only	Droplet +bubble (+other)
Esurface Correction	○	○	○

▪table

EOS table (Under Construction)

Density ρ :$10^{5} \sim 10^{16}$ $\mathrm{g/cm^{3}}$ 220 Mesh in log10

Temperature T :$10^{-1} \sim 10^{2.2}$ MeV 160 Mesh in log10

Proton fraction Yp : 0.01 ∼0.65 128 Mesh

	Botvina	**Hempel**	**Our model**
Model	NSE +LDM	RMF + NSE+ mass data (Theoretical +Experimental)	RMF+ NSE+ +LDM +mass data(Experimental)
Component heavy nuc.	multi (Z<1000)	multi (Z<100)	multi (Z<1000)
Shell term	×	○	△
Nuclear Shape	Droplet only	Droplet only	Droplet +bubble (+other)
*E*symmetry **of nuclei**	25 MeV	mass data	38 MeV
*E*surface	△ depending on T	mass data	△ depending on density &symmetry Z/A

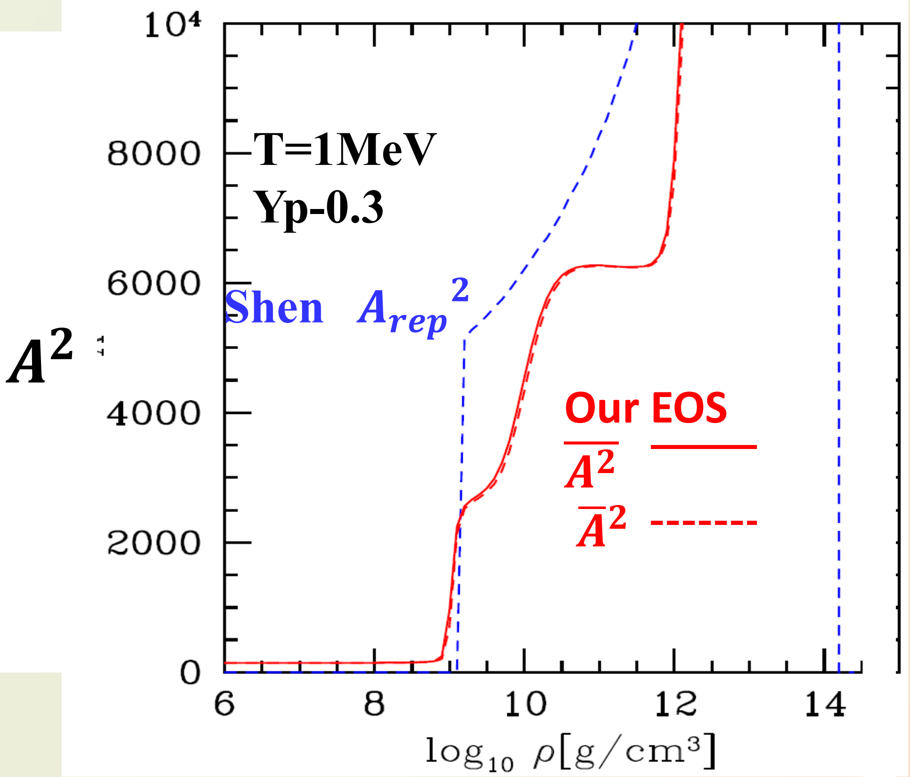
③Nuclear average mass number

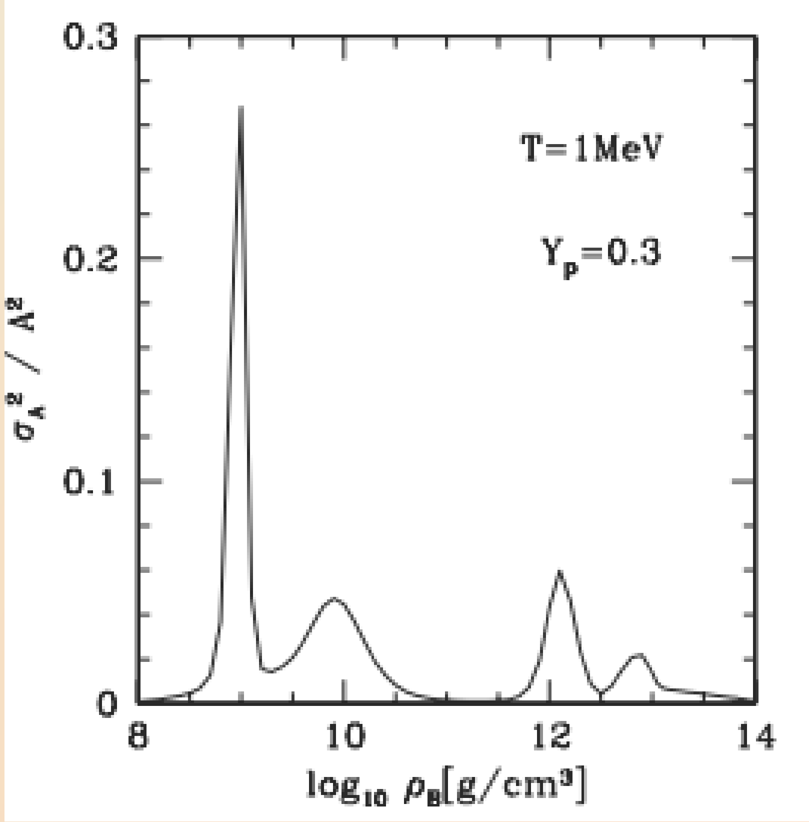
$N(A,Z) + \nu \longrightarrow N(A,Z) + \nu$ **depend on A^2**

We need average of squared mass number $\overline{A^2}$

Shen EOS (SNA) ⇒ square of average mass number ${A_{rep}}^2$

We investigated $\overline{A^2} \neq \bar{A}^2$





$\overline{A^2} \sim \bar{A}^2$ (Dispersion is small)

⇒ Calculation from $\bar{A}^2$ is no problem

$$f = \underbrace{f_{p,n}}_{\text{Free nucleons}} + \sum_i n_i \{ \underbrace{E_i^{trans}}_{\text{Translational}} + \underbrace{E_i^{bulk} + E_i^{surf} + E_i^{coul}}_{\text{Nuclear Mass}} \}$$

⇒RMF theory
+ excluded volume effect

⇒Boltzmann gas
+ approximate exclude volume

⇒Modified Liquid Drop Model

Modified Liquid Drop Model for Nuclei (1 < Z < 1000)

- **Bulk Energy**: RMF and Mass data (at low densities)

$$\boxed{E_i^B = M_i^{\text{data}} - [E_i^C]_{\text{vacuum}} - [E_i^S]_{\text{vacuum}}}$$

- **Surface**: Surface tensions σ_i (Lattimer 1991) +high density correction

$$\boxed{E_i^s = \sigma_i \times (surface\ area) \times (1 - n_{nucleon}/n_{si})^2}$$

n_{si}: nuclear saturation density

- **Coulomb** :Integration of Coulomb force in WS cell

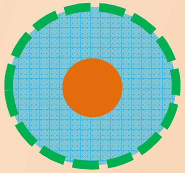
The cell volume V_i of each nucleus ⇐ the charge neutrality $(e^-, p \& Z_i)$

$$\boxed{u_i = V_i^N / V_i} \qquad \boxed{V_i^N = \frac{A_i}{n_{si}}}$$

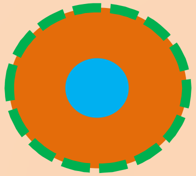
u<0.3 :Droplet(D)
0.3<u<0.7 : other Pasta (interpolation)
u>0.7 :Bubble(B)

V_i:Cell V_i^N :nucleus Free nucleons

(D)



(B)



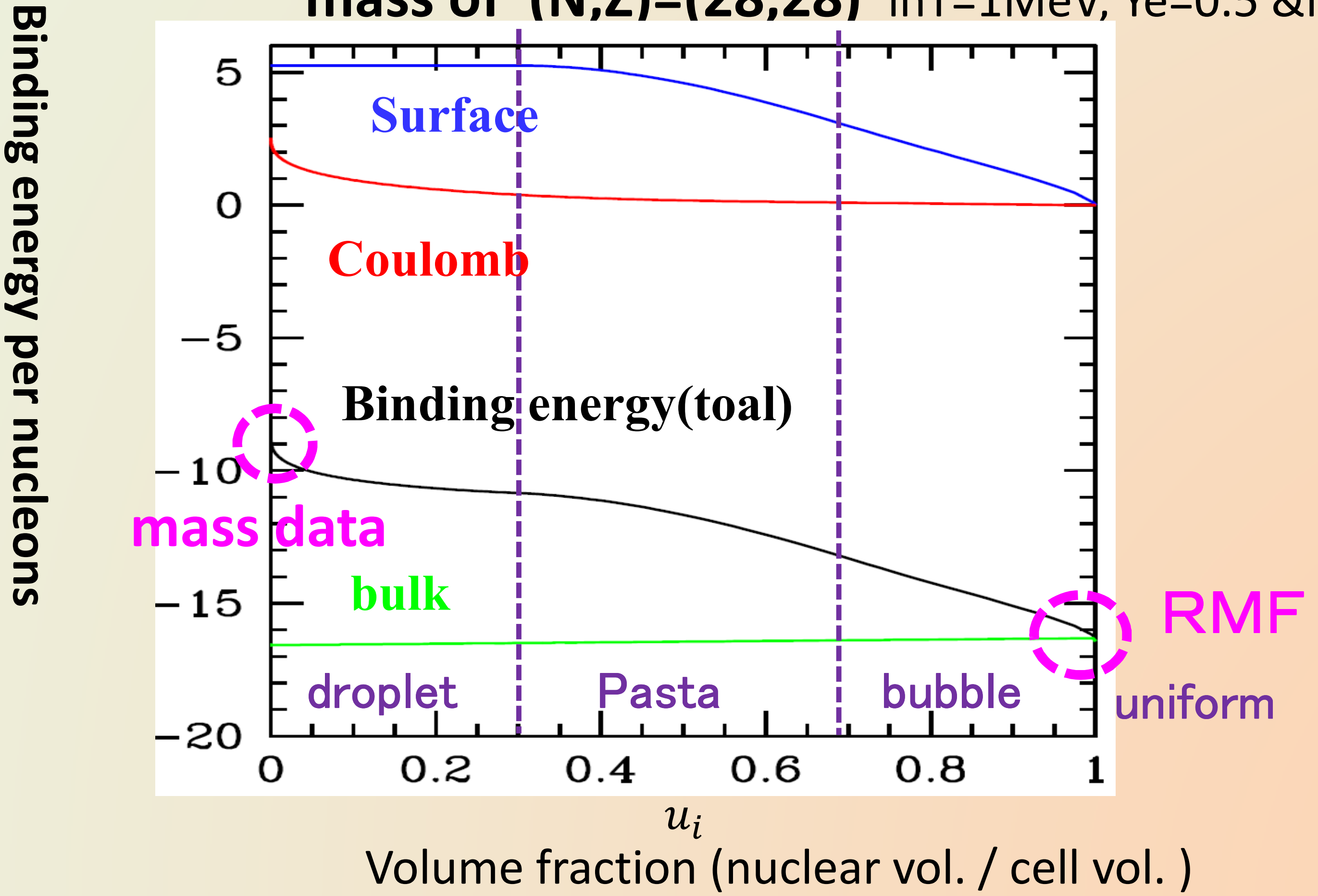
Nuclear mass formula of our model

$$M_i = E_i^{bulk} + E_i^{surf} + E_i^{coul}$$

@low densities limit @saturation density

$$= \begin{cases} M_i^{\rm data} \cancel{- [E_i^{surf}]_{\rho,n'_p,n'_n=0} - [E_i^{coul}]_{\rho,n'_p,n'_n=0} + E_i^{surf}(n'_n,n'_p,n_e) + E_i^{coul}(n'_p,n_e)} \\ A_i F^{RMF}(T,n_s,Z_i/A_i) \cancel{+ E_i^{surf}(n'_n,n'_p,n_e) + E_i^{coul}(n'_p,n_e)} \end{cases}$$

mass of (N,Z)=(28,28) inT=1MeV, Ye=0.5 &no free nucleons



①Free nucleons

$$f = \boxed{f_{p,n}} + \sum_i n_i\{E_i^{trans} + E_i^{bulk} + E_i^{surf} + E_i^{coul}\}$$

- exclude volume effect

  V:total volume **V**

V:total volume **V_{nuclei} :volume occupied by nuclei**

V’: Volume for free nucleons   V’ =V -V_{nuclei}

- **Relativistic Mean Field(RMF) in V’**

$$n'_{p/n} = \frac{N_{p/n}}{V'} \qquad \boxed{f'_{p,n} = f^{RMF}(n'_p, n'_n, T)} \qquad \eta = \frac{V'}{V} \qquad \boxed{f_{p,n} = \eta f'_{p,n}}$$

Free Energy Density in V’

Free Energy Density in V

②Translational Energy

$$f = f_{p,n} + \sum_i n_i\{\boxed{E_i^{trans}} + E_i^{bulk} + E_i^{surf} + E_i^{coul}\}$$

- Boltzmann gas
- g_i : internal degrees of freedom (including excited state) **Fai & Randrup(1982)**
- **approximate exclude volume effect Lattimer**

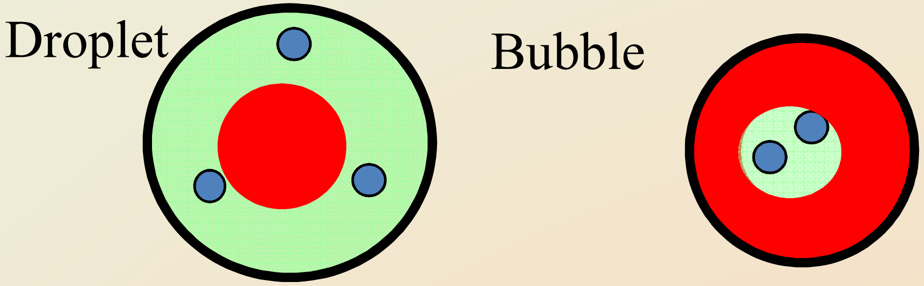
$$\boxed{E^i_{\mathrm{kine}} = \underbrace{kT\left\{\log\left(\frac{n_i}{g_i(T)n_i^Q}\right) - 1\right\}}_{\text{Boltzmann Gas}}\underbrace{\left(1 - \frac{n_b}{n_s}\right)}_{\text{Approximate Volume efect}}}$$

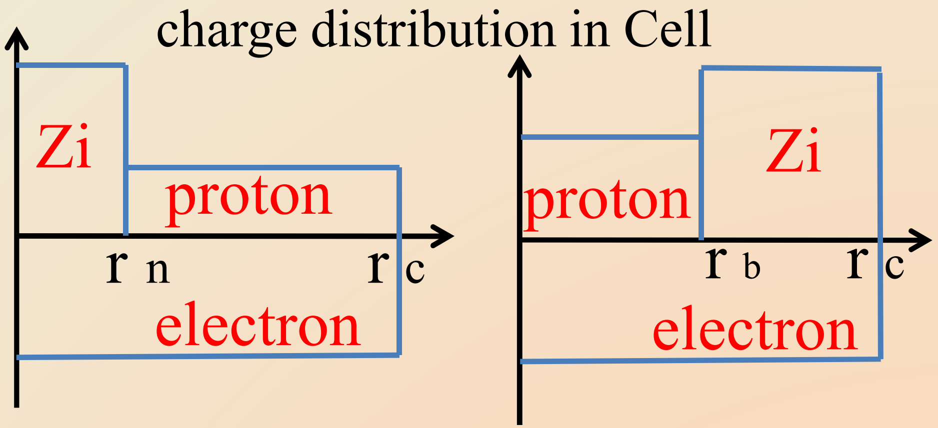
Boltzmann Gas

Approximate Volume efect

③Coulomb Energy

$$M_i = E_i^B + E_i^S + \boxed{E_i^C}$$





$$E_i^{coul} = \int_{\text{cell}} \frac{q(r)\,dq(r)}{r}$$

$$E_i^{coul} = \begin{cases} \frac{3}{5}\left(\frac{3}{4\pi}\right)^{-1/3} \frac{e^2}{n_{si}^2}\left(\frac{Z_i - \boxed{n_p' V_i^N}}{A_i}\right)^2 {V_i^N}^{5/3} \boxed{D(u_i)} & (u_i \leqslant 0.3 \\ \frac{3}{5}\left(\frac{3}{4\pi}\right)^{-1/3} \frac{e^2}{n_{si}^2}\left(\frac{Z_i - \boxed{n_p' V_i^N}}{A_i}\right)^2 {V_i^B}^{5/3} \boxed{D(1-u_i)} & (u_i \geqslant 0.7 \end{cases}$$

the contribution of Free proton **electron**

$$D(u_i) = (1 - \frac{3}{2}u_i^{1/3} + \frac{1}{2}u_i)$$

$$f = \underline{f_{p,n}} + \sum_i n_i \{ \underline{E_i^{trans}} + \underline{E_i^{bulk} + E_i^{surf} + E_i^{coul}} \}$$

Free nucleons

⇒RMF theory

\+ excluded volume effect

Translational

⇒Boltzmann gas

\+ approximate exclude volume

Nuclear Mass

⇒Modified Liquid Drop Model

Modified Liquid Drop Model for Nuclei (1 < Z < 1000)

- **Bulk Energy**: RMF and Mass data (at low densities)

- **Surface**: Surface tensions σ_i (Lattimer 1991) +high density correction

  n_{si}: nuclear saturation density

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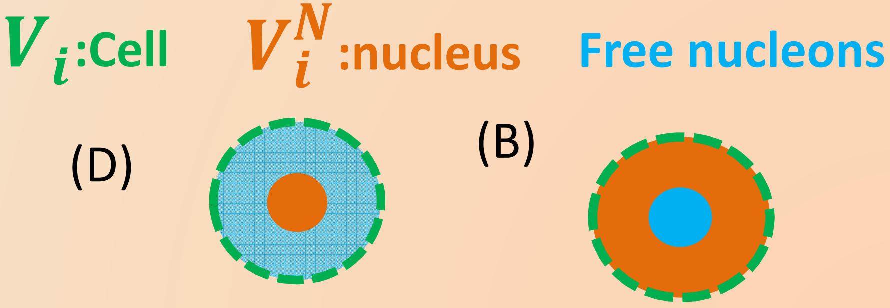
The cell volume $V_i$of each nucleus ⇐ the charge neutrality $(e^-, p \& Z_i)$

☆Pasta phase

u<0.3 :Droplet(D)

0.3<u<0.7 : other Pasta (interpolation)

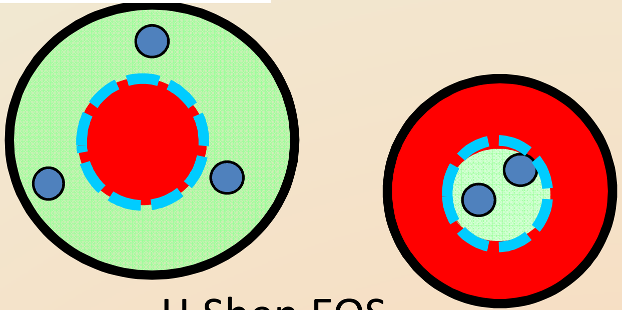
u>0.7 :Bubble(B)



④Surface Energy

$$M_i = E_i^B + \boxed{E_i^S} + E_i^C$$

$$E_i^{surf} = \boldsymbol{\sigma_i} \times \underline{(\boldsymbol{surface\ area})} \times \underline{(\mathbf{1} - \boldsymbol{n_{nucleon}/n_{si}})^2}$$



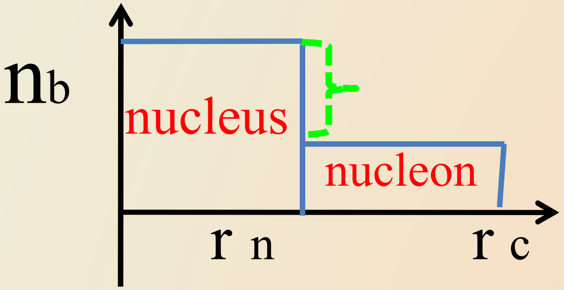
H.Shen EOS

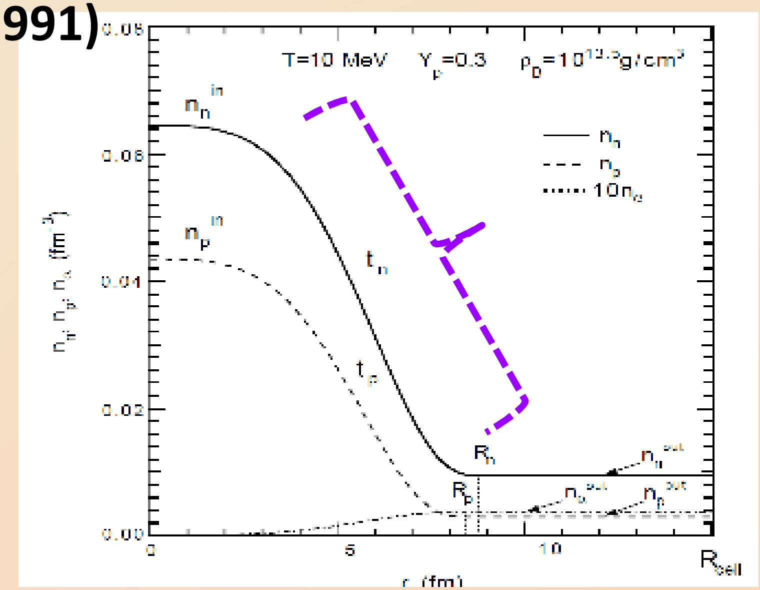
- $\boldsymbol{\sigma}_\mathrm{i}$ **: Surface tensions**

including **<u>surface symmetry energy</u> (Lattimer 1991)**

- **<u>high density correction by free nucleons</u>**

($E_i^{surf} \propto \int \mathrm{d}r\ \underline{|\nabla n_b(r)|^2}$ in H.Shen EOS)





$$E_i^S = \begin{cases} 4\pi {r_i^N}^2 \sigma_i \left(1 - \dfrac{n'_p + n'_n}{n_{si}}\right)^2 = 4\pi \left(\dfrac{3}{4\pi} V_i^N\right)^{2/3} \sigma_i \left(1 - \dfrac{n'_p + n'_n}{n_{si}}\right)^2 & (u_i < 0.3) \\ 4\pi {r_i^B}^2 \sigma_i \left(1 - \dfrac{n'_p + n'_n}{n_{si}}\right)^2 = 4\pi \left(\dfrac{3}{4\pi} V_i^B\right)^{2/3} \sigma_i \left(1 - \dfrac{n'_p + n'_n}{n_{si}}\right)^2 & (u_i > 0.7) \end{cases}$$

$$g_i(T) = g_i^0 + \frac{c_1}{A_i^{5/3}} \int_0^\infty dE e^{-E/T} \exp\left(\sqrt{2a(A_i)E}\right)$$

$$a(A_i) = \frac{A_i}{8}(1 - c_2 A^{-1/3}) \text{ MeV}^{-1}$$

$$c_1 = 0.2 \text{ MeV}^{-1}, \;\; \mathrm{c}_2 = 0.8 \;.$$