Feb. 28th. 2020

Quark-Hadron Phase transition by Color Molecular Dynamics and Lattice QCD simulations

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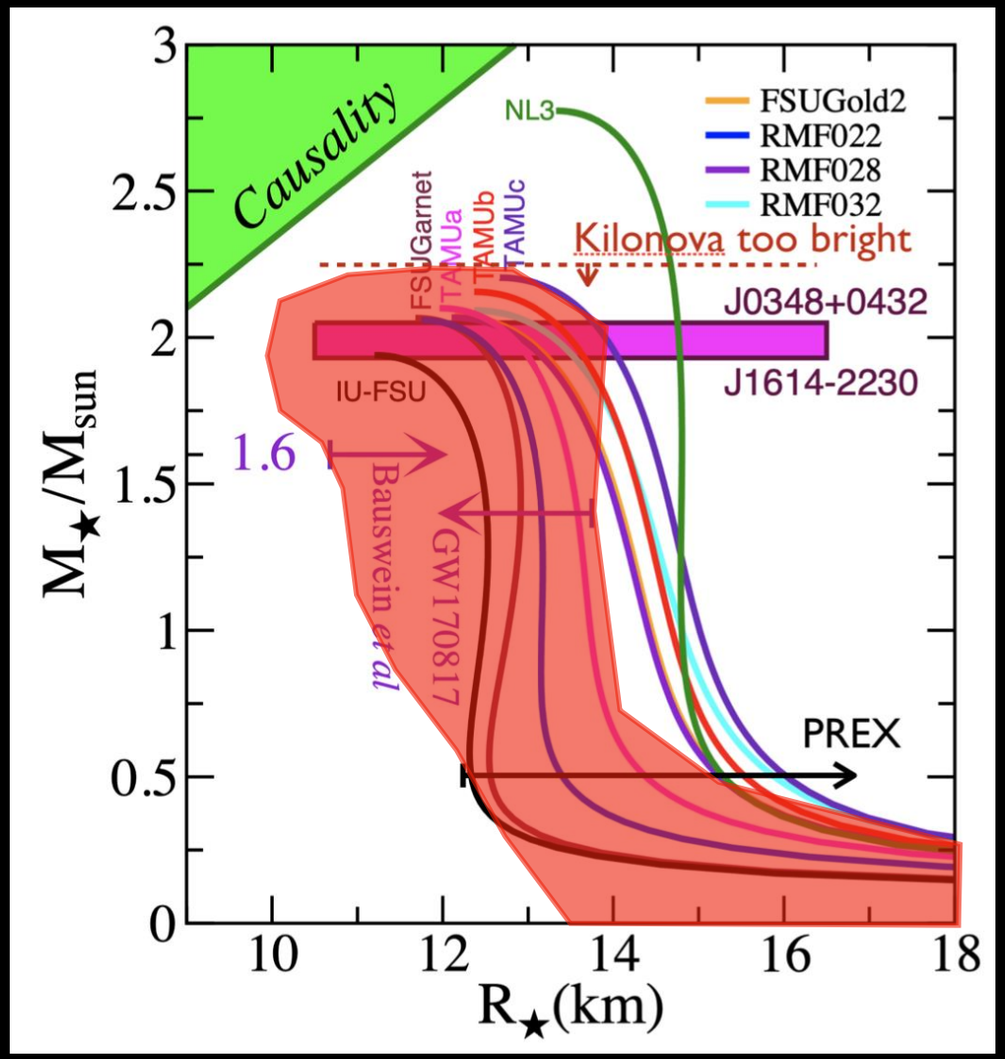
MR relation

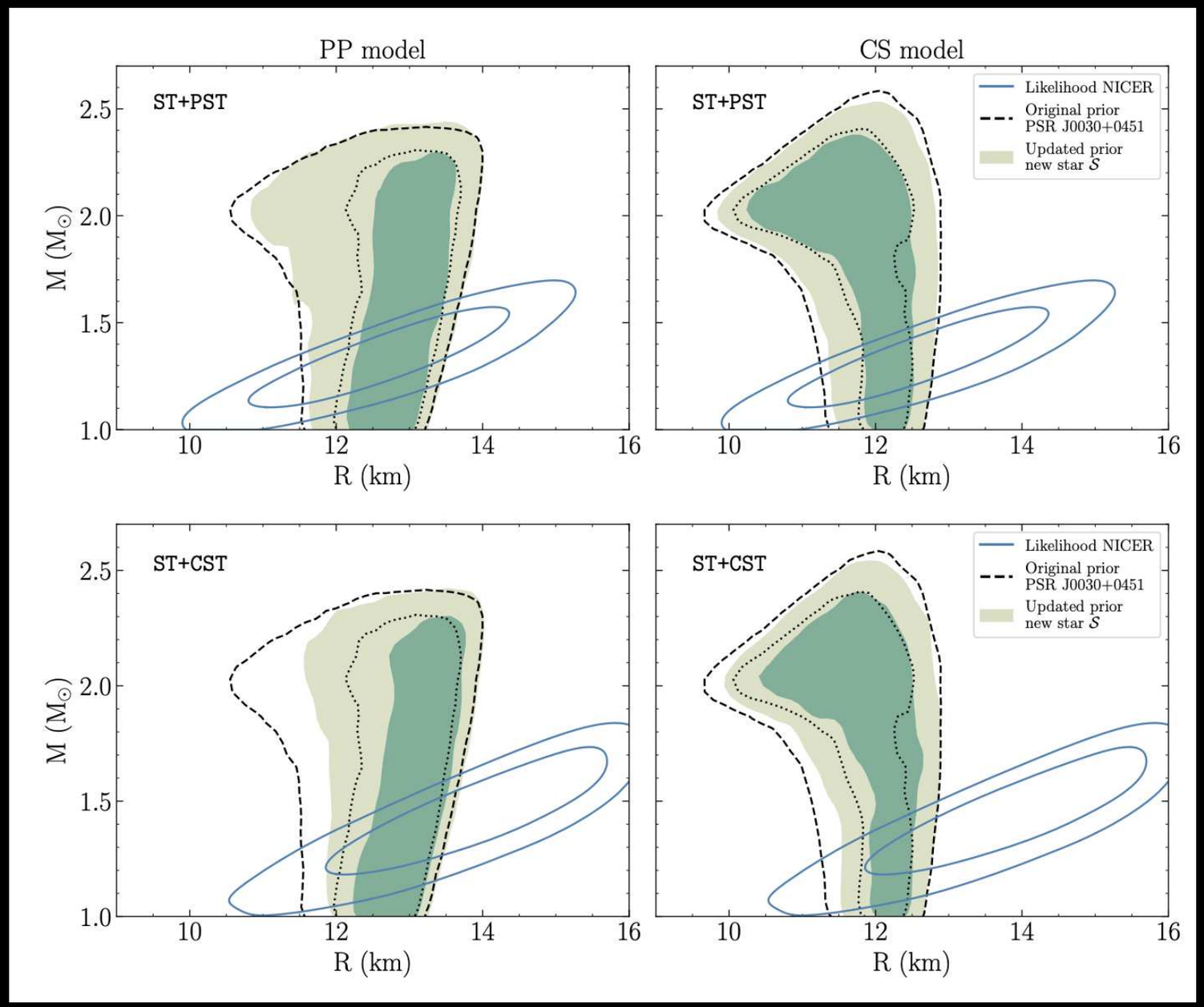
before NICER



Horowitz (arXiv:1911.00411)

See also the results of
“PREX II & CREX”

with NICER constraint

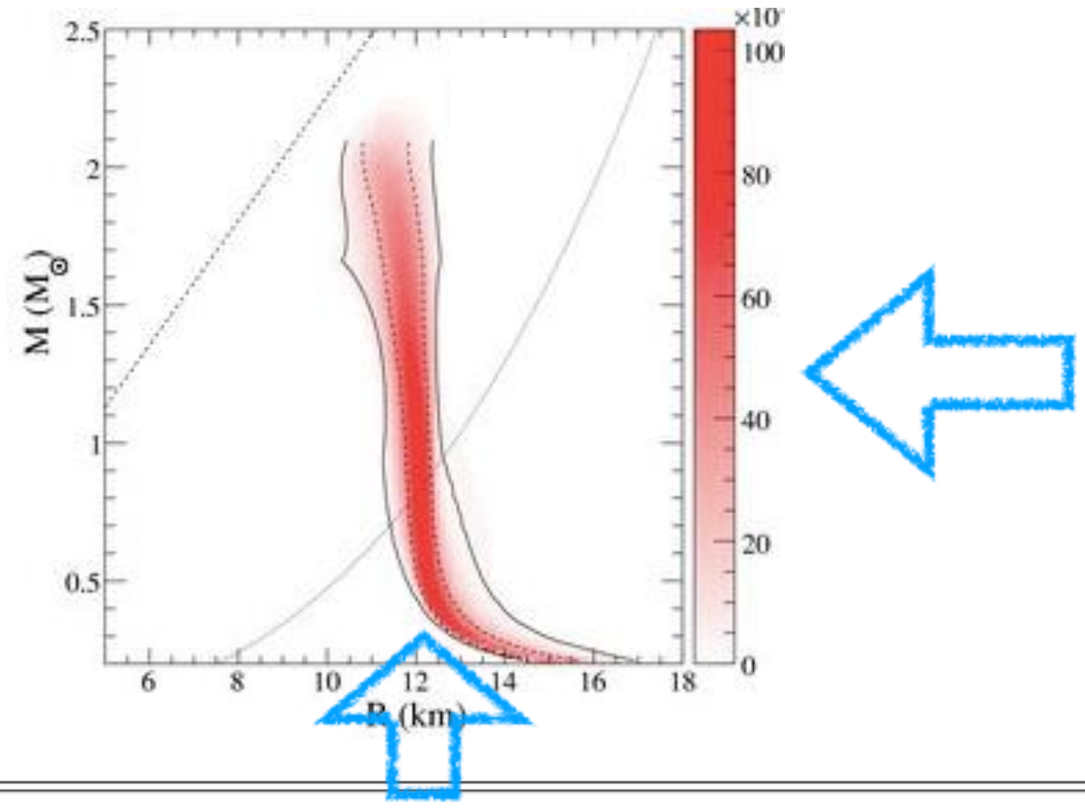


PSR J0030+0451

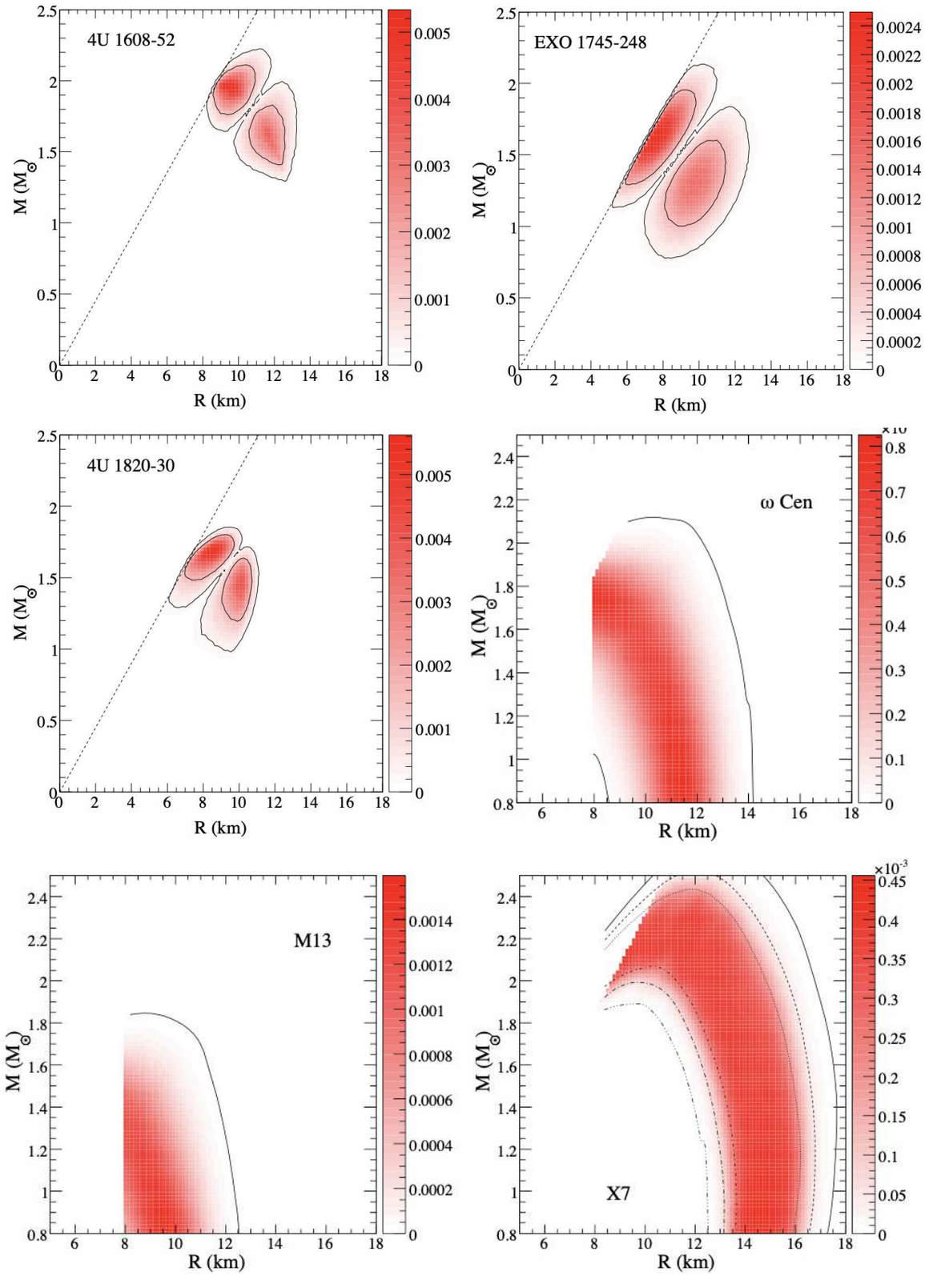
Future constraints for MR relation?

—— From historical view ——

A.W.Steiner, J.M.Lattimer, E.F.Brown
ApJ **722** (2010) 33



Quantity	$r_{ph} \gg R$	$r_{ph} = R$	Experiment
Schematic EOS parameters			
K (MeV)	216^{+43}_{-32}	$190^{+50}_{-7.2}$	230–250 (Colo et al. 2004)
$-K'$ (MeV)	280^{+410}_{-72}	500^{+290}_{-170}	
S (MeV)	$29^{+5.4}_{-0.9}$	$35^{+2.1}_{-6.3}$	28–34 (Klupfel et al. 2009)
γ	$0.26^{+0.15}_{-0.06}$	$0.26^{+0.36}_{-0.06}$	0.35–1.0 (Tsang et al. 2009)



MASS-RADIUS RELATION

- No official information…

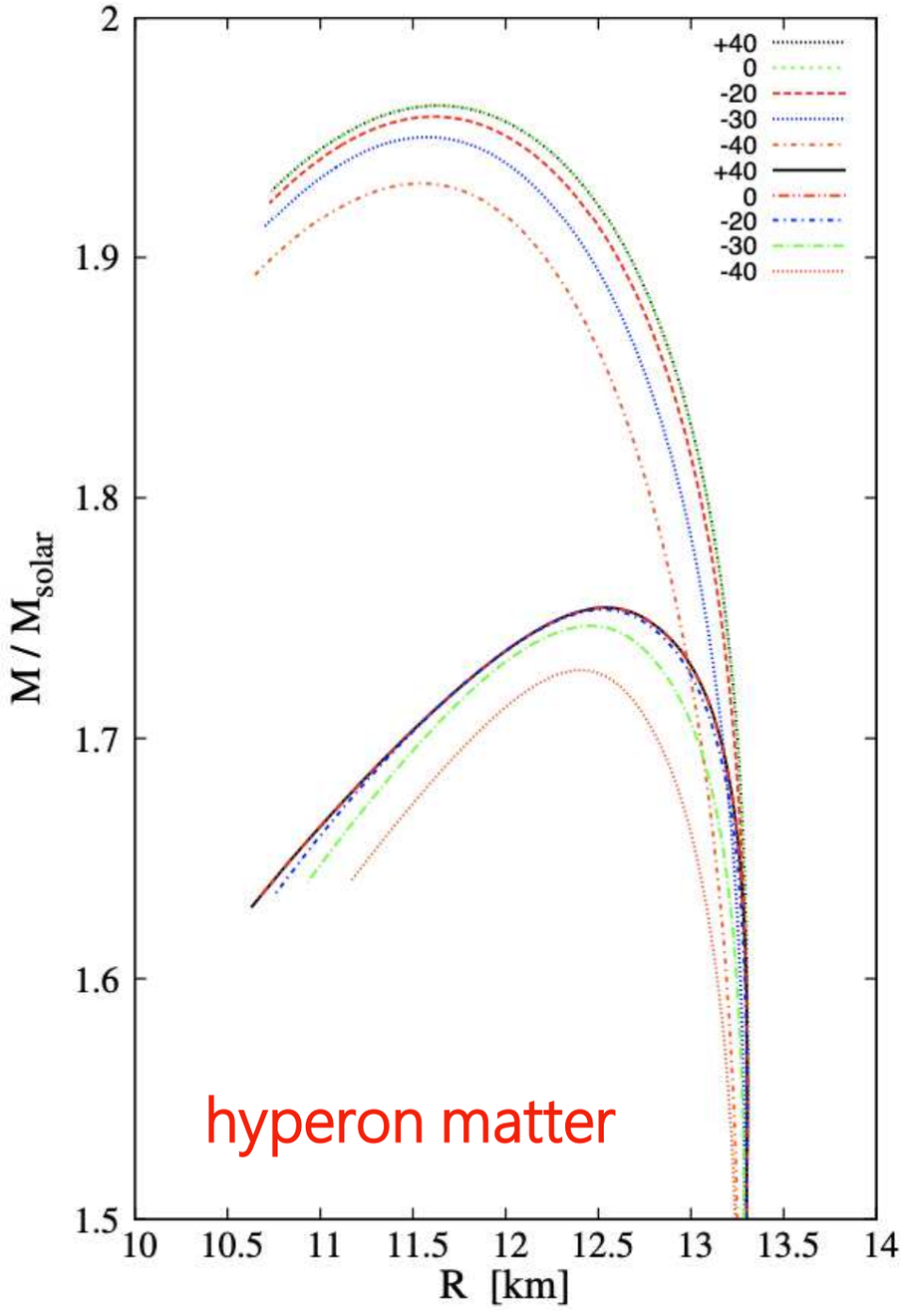
Targets & Exposures by Working Group

- https://heasarc.gsfc.nasa.gov/docs/nicer/science_team_investigations/

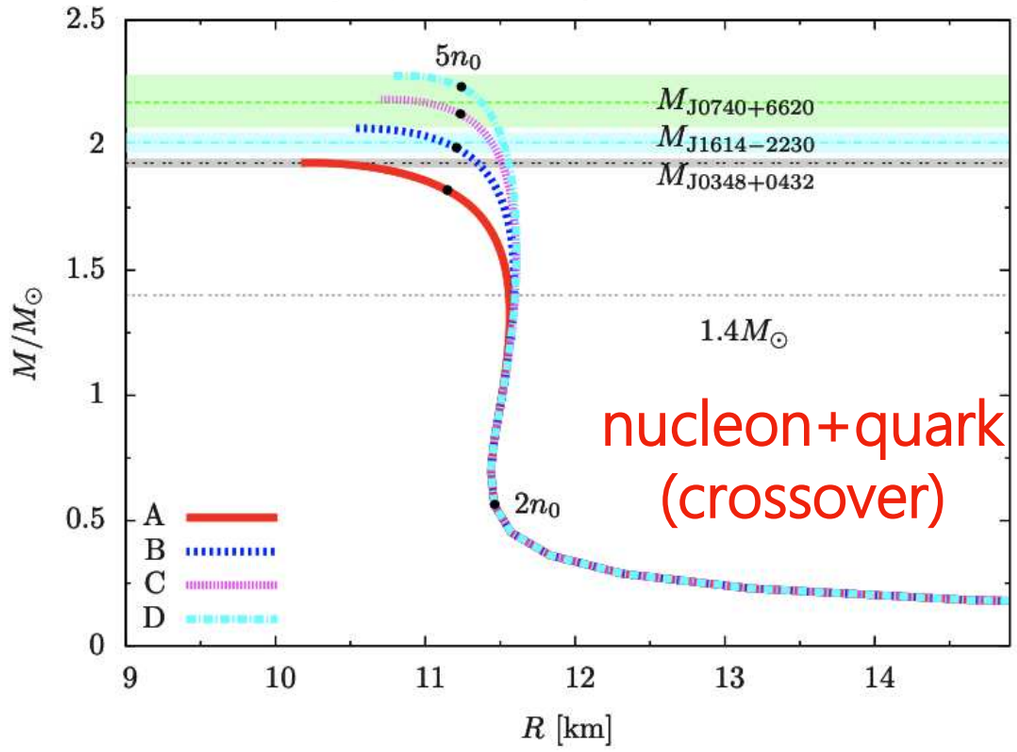
Target Name	Contact	Exposure (ksec)		Legacy	Purpose
		Plan	To date (approx)		
PSR J0030+0451	S. Bogdanov	1,600	1,936	Y	Radius, mass constraints from energy-dependent lightcurve modeling
PSR J0437-4715	S. Bogdanov	1,000	951	Y	Same
PSR J0740+6220	Z. Arzoumanian	1,600	75	Y	Same
PSR J1231-1411	P. Ray	1,600	1356	Y	Same
PSR J2124-3358	S. Bogdanov	1,600	1051	Y	Same
PSR J1614-2230	M. Wolff	500	400	Y	Radius lower limit from lightcurve modeling

Quiz: What is the component?

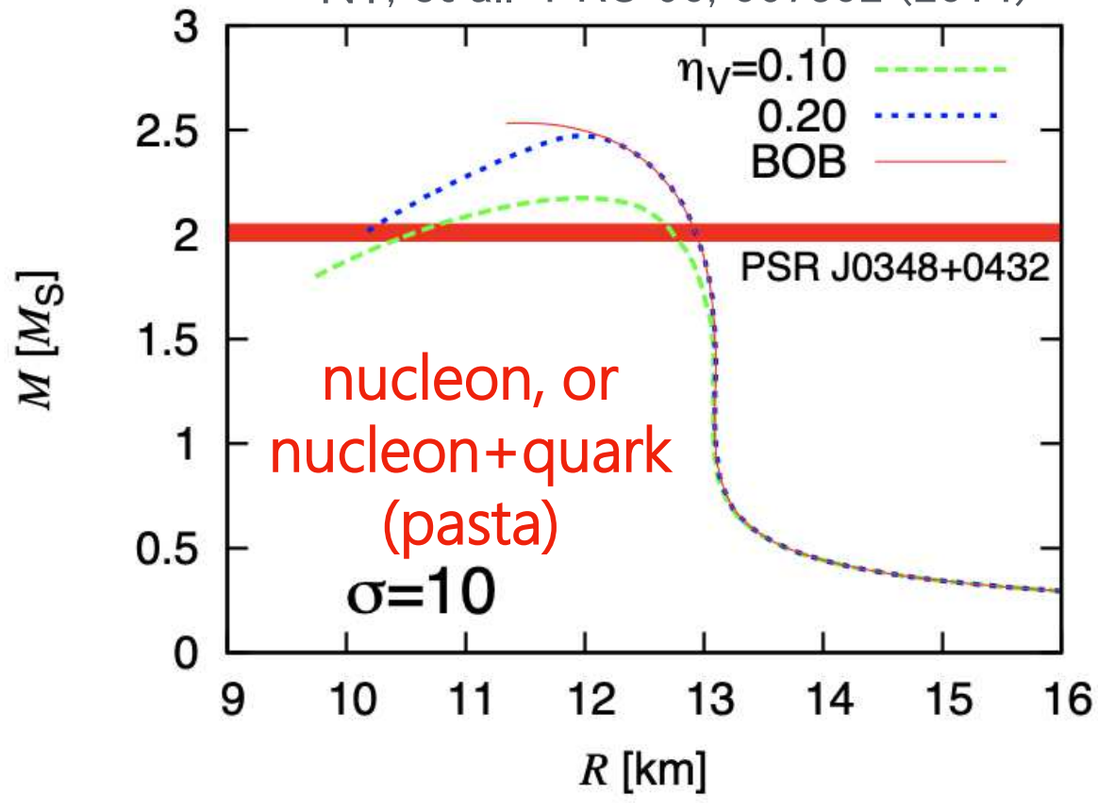
S. Weissenborn, et al. PRC 85, 065802 (2012)



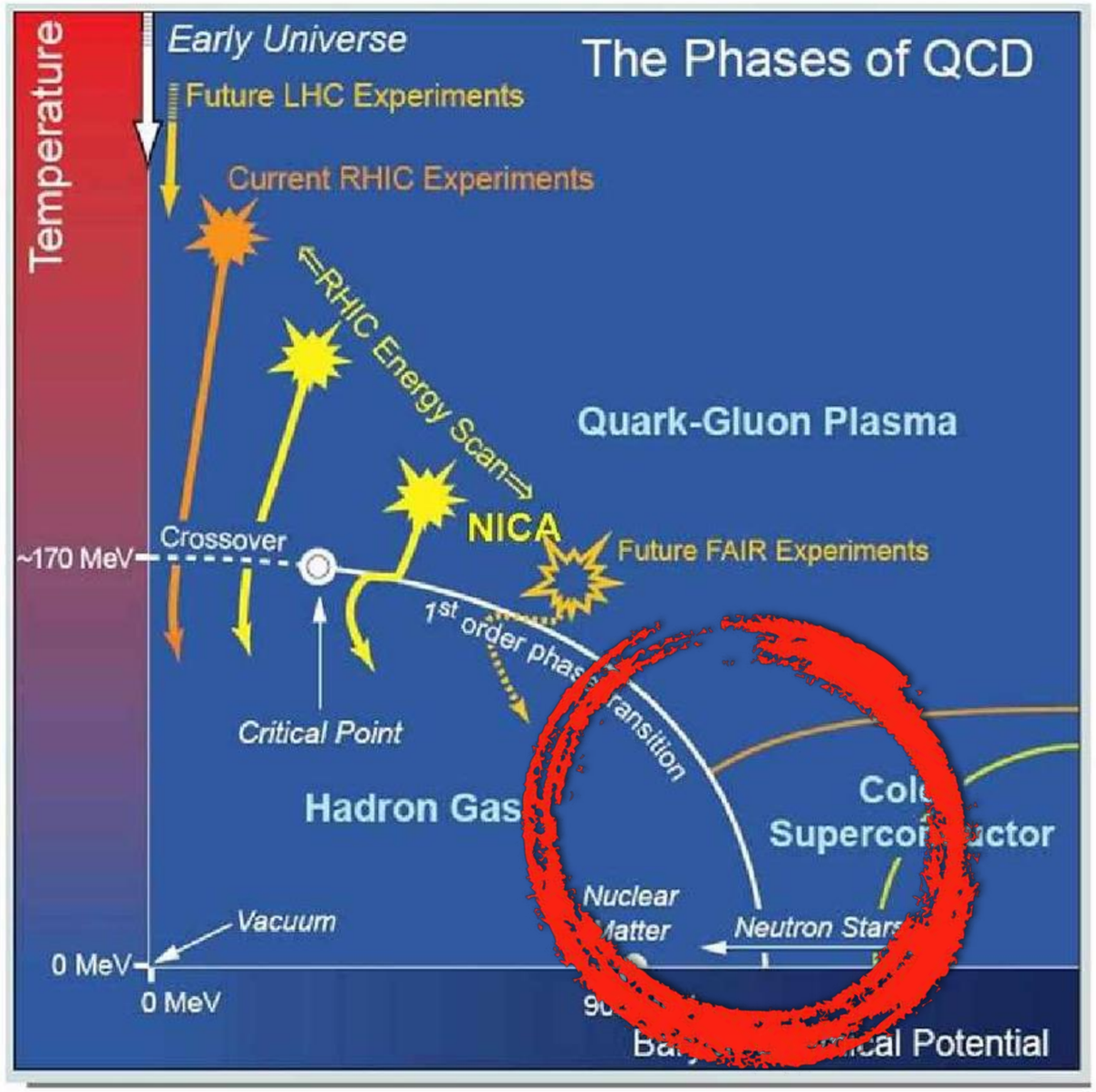
G. Baym, et al. ApJ 885, 42 (2019)



NY, et al. PRC 90, 067302 (2014)



Other properties?



Progress of the NICA project (2009)

- We will obtain “MR relation”.
- But what kind of material?

↓

✔ Pressure(energy) vs density.

△ Other physical properties.

e.g. thermal conductivity, heat capacity…

Lattice QCD
(sign problem)

+ Color Molecular dynamics

SITUATIONS OF NUCLEAR PHYSICS

BARYON-BARYON INTERACTIONS COULD BE CLEARED

*2(,3) bodies

→(Dirac)Brueckner-Hartree-Fock, Variational principle

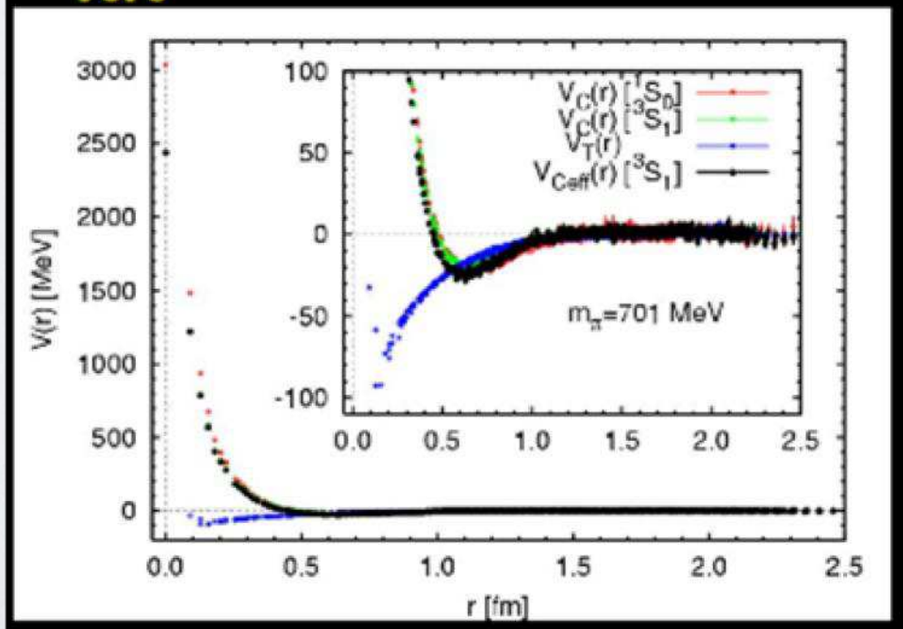




?

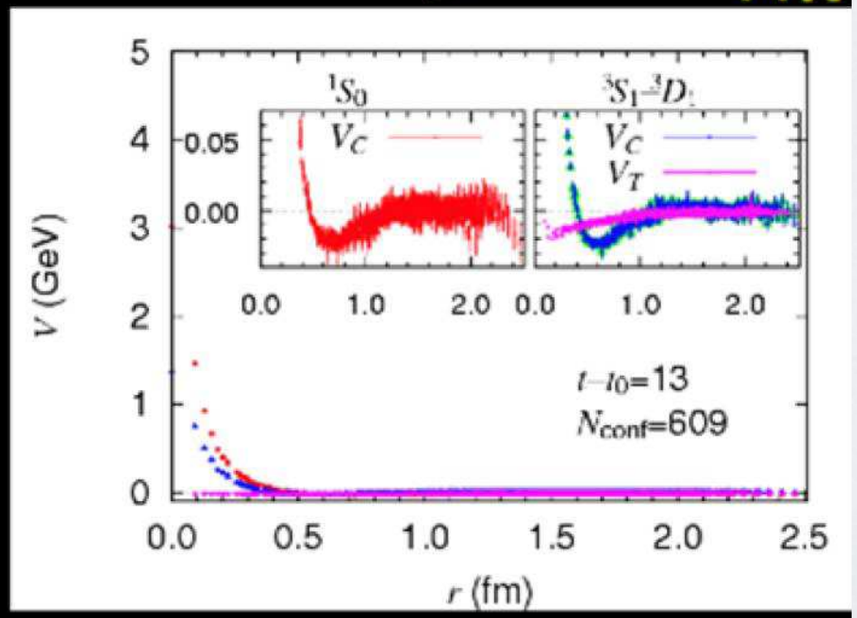
Full QCD (m_π=701 MeV, a=0.09 fm, L=2.9 fm)
using PACS-CS configuration

NN



Ishii, Aoki & Hatsuda (2009)

ΛN



Nemura, Ishii, Aoki & Hatsuda (2009)

FROM QCD TO NS MATTER

京 (Kei)

size	～ 6 fm
number	6 (, 9)

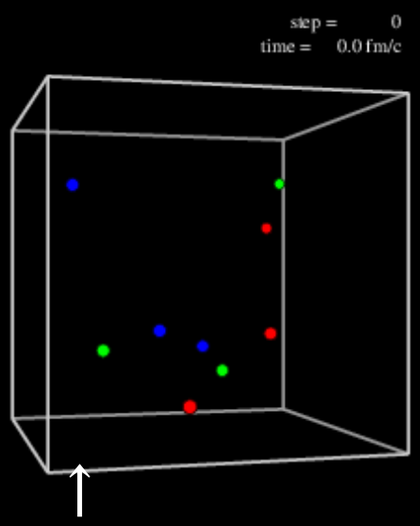
富岳 (Fugaku)

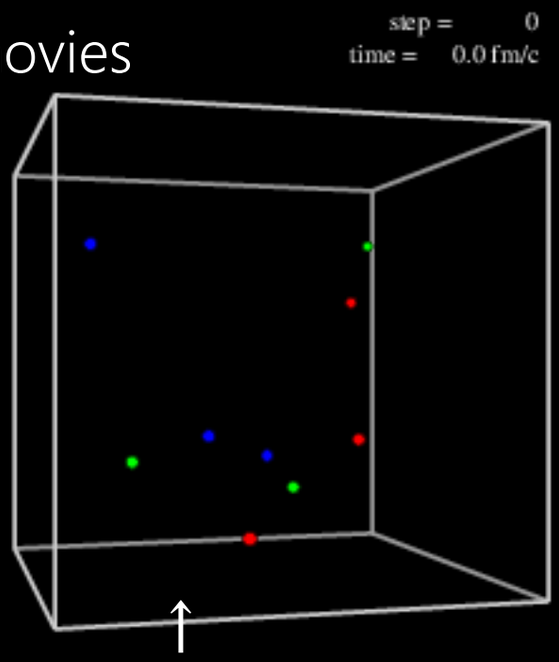
size	～ 10 fm
number	6 (, 9)

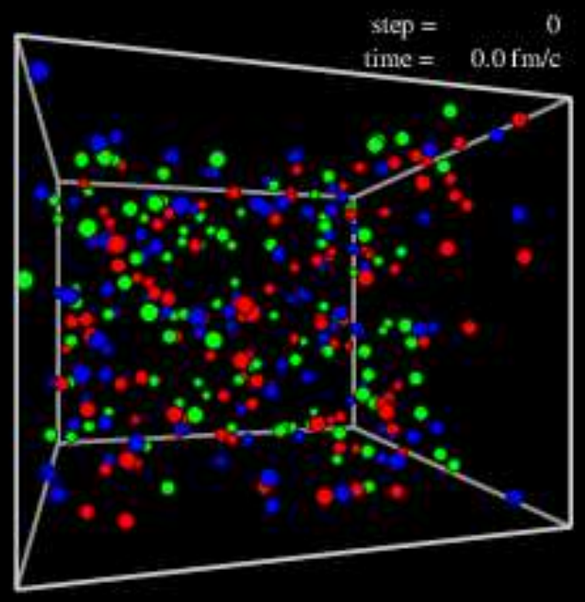
Neutron Star matter

1～10 n_0

* schematic movies







These are the limits of Lattice QCD(LQCD) calculations, because of the sign problem.

Using the results (interactions) from LQCD, we conduct the Color molecular dynamics(CMD) simulations.
MD simulations are ab-initio methods, but they work well to study the medicines and/or DNA, etc..

COLOR QMD

See, for example,
Maruyama, Hatsuda (2000) PRD

number of variables

$$\{x, y, z, Px, Py, Pz, \alpha, \beta, \theta, \varphi\}_i$$

10 variables on each particle

time evolution

$$\frac{\partial \mathcal{L}}{\partial q} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} \qquad \mathcal{L} = \left\langle \Psi \left| i\hbar\frac{d}{dt} - \hat{H} \right| \Psi \right\rangle$$

wave functions

$$\Psi = \prod_{i=1}^{3A} \phi_i(\mathbf{r})\chi_i \qquad \chi_i = f_i s_i c_i$$

f_i ... flavors (fixed)

s_i ... spins (fixed now)

$$c_i \equiv \begin{pmatrix} \cos\alpha_i e^{-i\beta_i}\cos\theta_i \\ \sin\alpha_i e^{+i\beta_i}\cos\theta_i \\ \sin\theta_i e^{i\varphi_i} \end{pmatrix} \begin{matrix} \text{R} \\ \text{G} \\ \text{B} \end{matrix}$$

$$\phi_i(\mathbf{r}) \equiv (\pi L^2)^{-\frac{3}{4}} \exp[-(\mathbf{r}-\mathbf{R}_i)^2/2L^2 - i\mathbf{P}_i\mathbf{r}]$$

cooling of the system

$$\dot{\mathbf{R}}_i = \frac{1}{3}\sum_{j\in\{i\}}\left[\frac{\partial H}{\partial \mathbf{P}_j} + \mu_P\frac{\partial H}{\partial \mathbf{R}_j}\right]$$

$$\dot{\mathbf{P}}_i = \frac{1}{3}\sum_{j\in\{i\}}\left[-\frac{\partial H}{\partial \mathbf{R}_j} + \mu_P\frac{\partial H}{\partial \mathbf{P}_j}\right]$$

confinement conditions

$$\begin{cases} |\mathbf{R}_i - \mathbf{R}_j| < d_{cluster} \quad (i,j=1,2,3) \\ \sum_{a=1}^{8}\left[\sum_{i=1}^{3}\langle\chi_i|\lambda^a|\chi_i\rangle\right]^2 < \varepsilon \end{cases}$$

λ^a being the Gell-Mann matrices

HOW TO TAKE INTO ACCOUNT LQCD

Baryon-baryon interactions at short distances – constituent quark model meets lattice QCD –

Aaron Park,[1,*] Su Houng Lee,[1,†] Takashi Inoue,[2,3,‡] and Tetsuo Hatsuda[4,3,§]

arXiv:1907.06351

$$H = \sum_{i=1}^{N}(m_i + \frac{\mathbf{p}_i^2}{2m_i}) - \frac{3}{16}\sum_{i<j}^{N}(V_{ij}^C + V_{ij}^{CS}),$$

interactions

$$V_{ij}^C = -\lambda_i^c\lambda_j^c\left(-\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0} - D\right),$$

$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2}\frac{1}{r_{ij}}e^{-(r_{ij}/r_{0ij})^2}\lambda_i^c\lambda_j^c\sigma_i\sigma_j,$$

↑ or ↓ for us and fixed.

$$r_{0ij} = (\alpha + \beta\mu_{ij})^{-1}.$$

$$\mu_{ij} = m_i m_j/(m_i + m_j)$$

Optimized parameters

κ	κ'	a_0	D	α	β	$m_{u,d}$	m_s	m_c	m_b
0.59	0.5	5.386 GeV^{-2}	0.96 GeV	2.1 fm^{-1}	0.552	0.343 GeV	0.632 GeV	1.93 GeV	5.3 GeV

Comparison with experimental data

(I,S)	$(\frac{1}{2},\frac{1}{2})$ N, P	$(\frac{1}{2},\frac{3}{2})$ Δ	$(0,\frac{1}{2})$ Λ	$(1,\frac{1}{2})$ Σ	$(1,\frac{3}{2})$ Σ^*	$(\frac{1}{2},\frac{1}{2})$ Ξ	$(\frac{1}{2},\frac{3}{2})$ Ξ^*
M_B	0.977	1.23	1.12	1.2	1.38	1.324	1.52
Expt.	0.938	1.232	1.115	1.189	1.382	1.315	1.532

INTERACTIONS IN CMD

$$\hat{H} = \sum_i \sqrt{m^2 + \hat{\mathbf{p}}_i^2} + \frac{1}{2}\sum_{i,j\neq i} \hat{V}_{ij}$$

$$\hat{V}_{ij} = -\sum_{a=1}^{8} t_i^a t_j^a V_C(\hat{r}_{ij}) + V_M(\hat{r}_{ij}) + V_{Pauli}(r) + V_{coul}(r)$$

$$V_C(r) \equiv Kr - \alpha_s \frac{e^{-\mu r}}{r} + V_{spin}(r)$$

$$V_{spin}(r) = C_{\mathrm{spin}} \frac{e^{-\mu r}}{r} s_i s_j$$

$$V_M(r) \equiv -\frac{g_{\sigma q}^2}{4\pi}\frac{e^{-\mu_\sigma r}}{r} + \frac{g_{\omega q}^2}{4\pi}\frac{e^{-\mu_\omega r}}{r}$$

$$V_{Pauli} = \frac{C_p}{(q_0 p_0)^3} \exp\left[-\frac{(\mathbf{R}_i - \mathbf{R}_j)^2}{2q_0^2} - \frac{(\mathbf{P}_i - \mathbf{P}_j)^2}{2p_0^2}\right] \delta_{\chi i \chi j}$$

$t^a = \lambda^a/2$ with λ^a being the Gell-Mann matrices

preliminary

V_C ..quark–quark interactions

V_{spin} ..spin–spin interactions

V_M quark–meson interactions

V_{Pauli} ..Pauli interactions

New interactions we added.

Ours

(I,S)	$(\frac{1}{2},\frac{1}{2})$ N,P	$(\frac{1}{2},\frac{3}{2})$ Δ	$(0,\frac{1}{2})$ Λ	$(1,\frac{1}{2})$ Σ	$(1,\frac{3}{2})$ Σ^*	$(\frac{1}{2},\frac{1}{2})$ Ξ	$(\frac{1}{2},\frac{3}{2})$ Ξ^*
M_B	0.932	1.232	1.114	1.146	1.446	1.316	1.616
Expt.	0.938	1.232	1.115	1.189	1.382	1.315	1.532

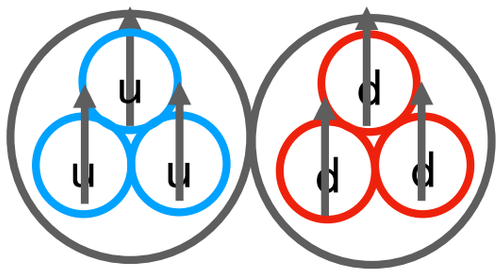
Aaron et al.

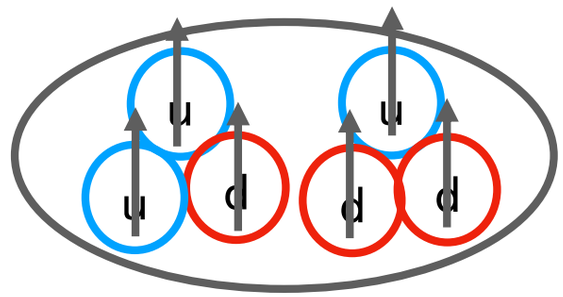
(I,S)	$(\frac{1}{2},\frac{1}{2})$ N, P	$(\frac{1}{2},\frac{3}{2})$ Δ	$(0,\frac{1}{2})$ Λ	$(1,\frac{1}{2})$ Σ	$(1,\frac{3}{2})$ Σ^*	$(\frac{1}{2},\frac{1}{2})$ Ξ	$(\frac{1}{2},\frac{3}{2})$ Ξ^*
M_B	0.977	1.23	1.12	1.2	1.38	1.324	1.52
Expt.	0.938	1.232	1.115	1.189	1.382	1.315	1.532

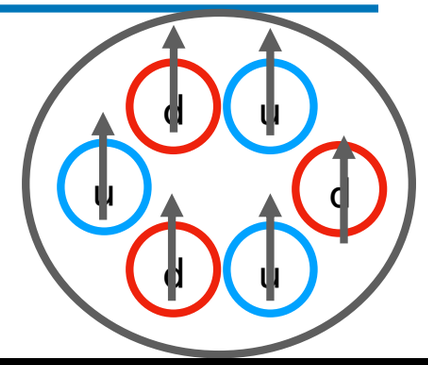
case①: dibaryons

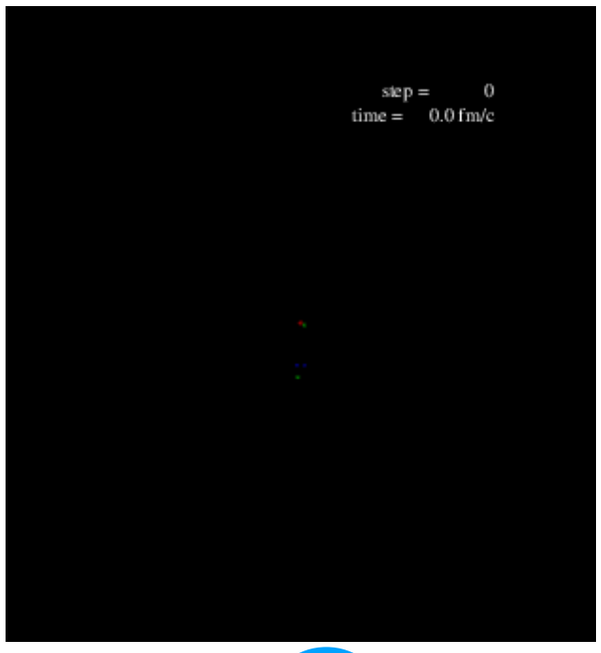
Mukobara

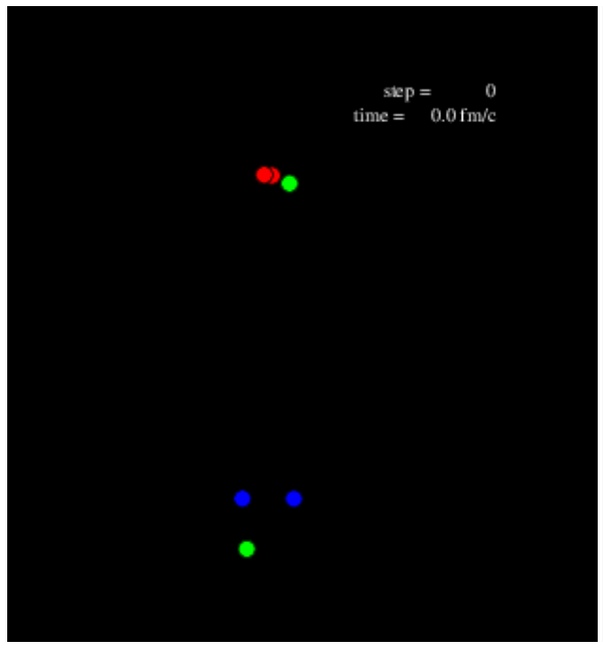
schematic pictures of Δ−dibaryon state

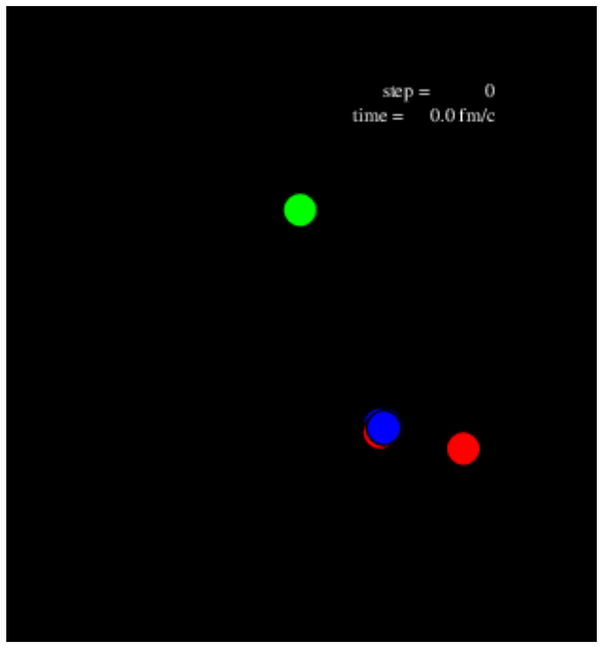


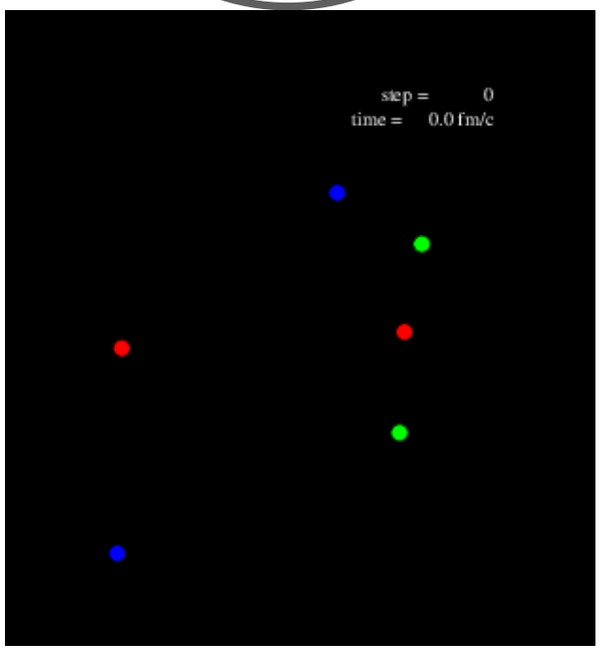




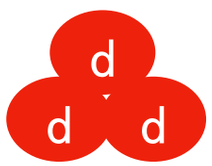




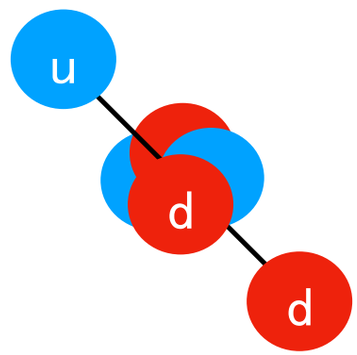




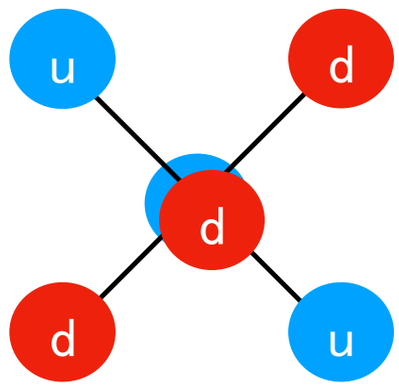




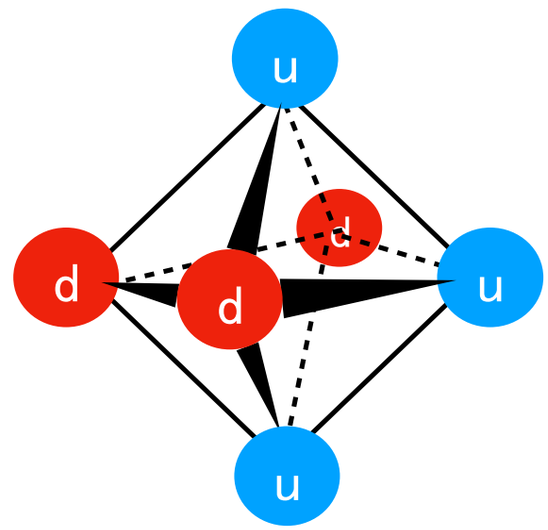
(semi−)independent



rod



cross

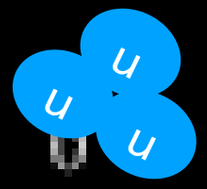


octahedron

case②: Δ tetra baryons

Mukobara

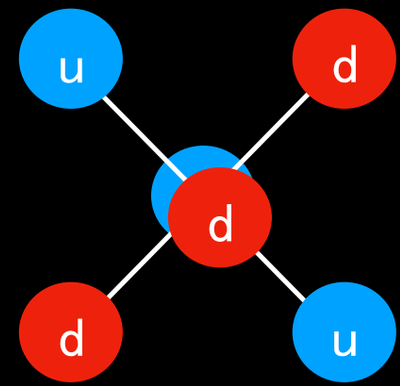
There is no bound state.



step = 0

time = 0.0 fm/c

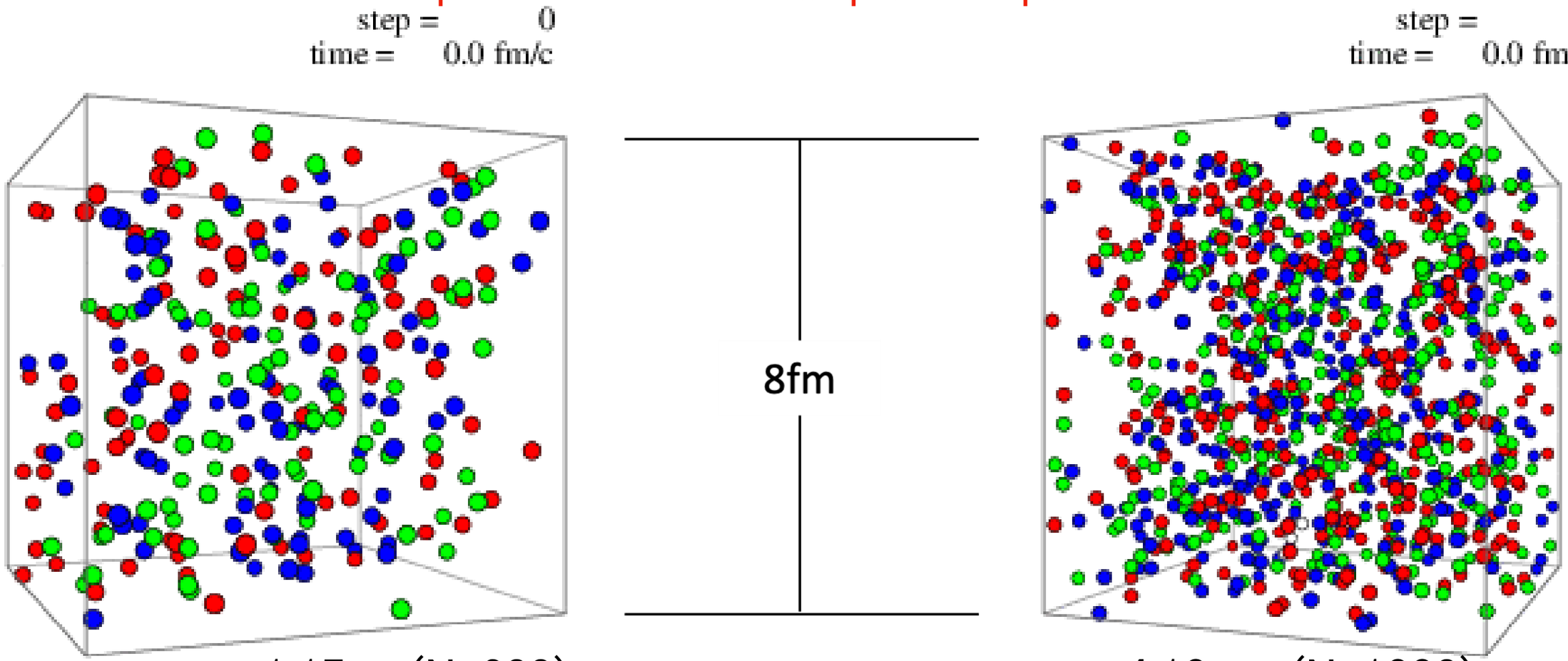




case③: many bodies

Hoshi

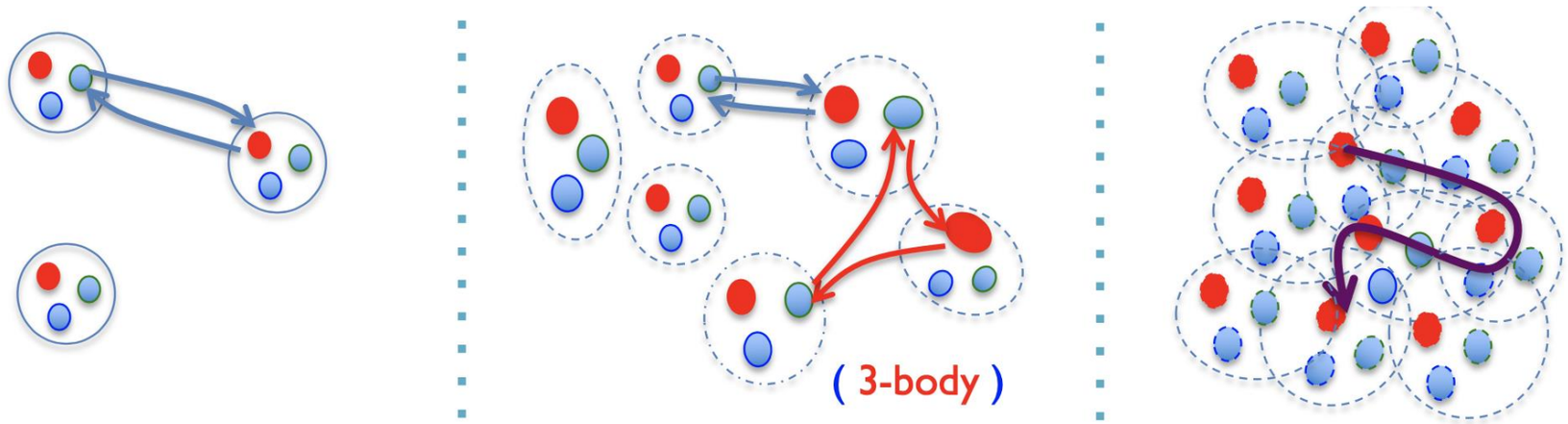
phase transition as quantum percolation



$\rho = 1.17\,\rho_0$ (N=288)

$\rho = 4.10\,\rho_0$ (N=1008)

low density ← ρ → high density

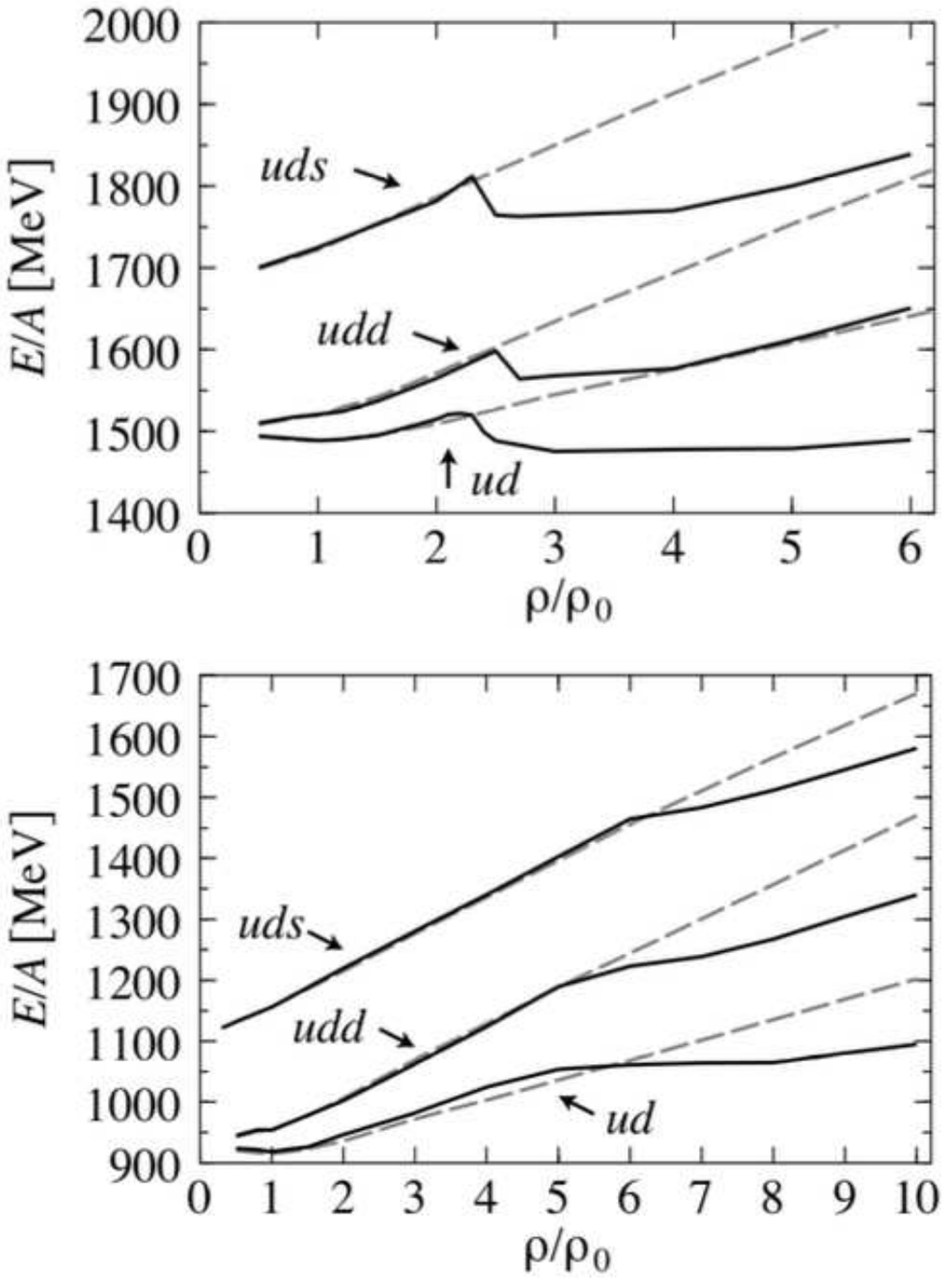


Kojo-Powell-Song-Baym 2014

Let’s get inside of the phase transition !

Note: many bodies

Note that interactions decide
"1st oder or crossover".



Akimura et al. 2006

We can obtain physical quantities by
Green–Kubo formula.

$$\alpha = \int_0^\infty \left\langle \dot{U}(t)\dot{U}(0) \right\rangle dt.$$

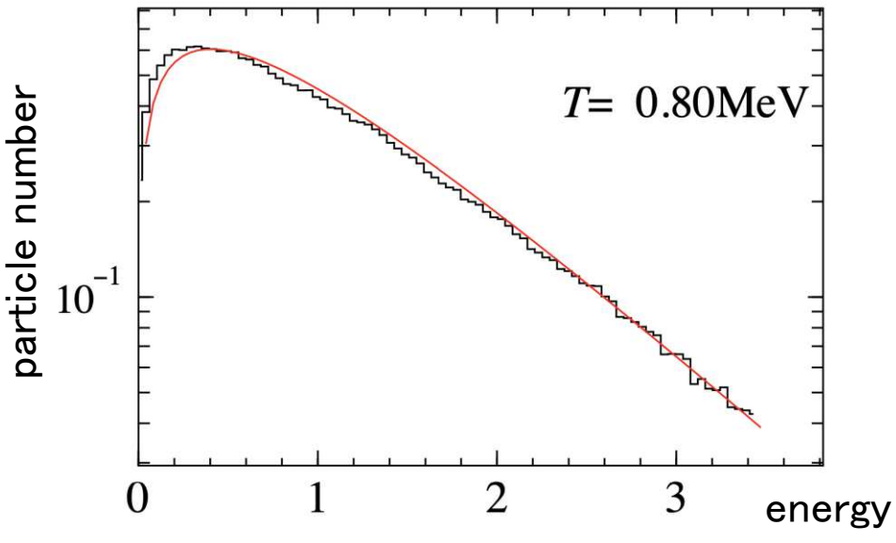
e.g. thermal conductivity λ

$$\alpha = VT^2\lambda,$$

$$U_x(t) = \sum_{i=1}^{N} x_i(t)E_i(t),$$

$$\dot{U}_x(t) = \frac{d}{dt}\sum_{i=1}^{N} x_i(t)E_i(t).$$

Statistical temperature from CMD is
consistent with theoretical distribution.



Summary and Discussion

Summary

- We have obtained di-baryon states in CMD, but not tetra-baryons.
- Many-body calculations suggest the quark-hadron phase transitions as percolation(cross over).
- We also found the bound sates of “K^-pp”, “Ω^--dibaryons”Gongyo+ (2018).
- How about the cases with more sophisticated interactions?

(the interactions consistent with LQCD)

Discussion

- Our calculations include Coulomb interactions. —> pasta structures ?
- The quark clusters are physical？
- How to take into account the vacuum effect (chiral condensation).