GR Neutrino Transport and Neutrino Quantum Kinetics

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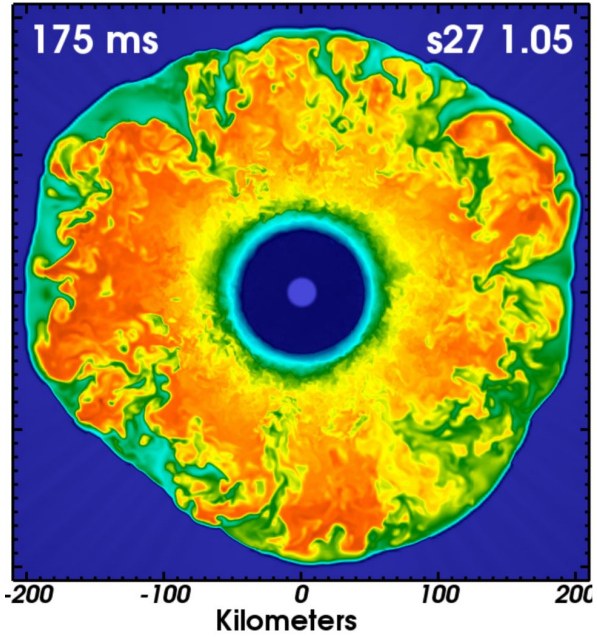
Gail McLaughlin, NCSU
Yonglin Zhu, NCSU
Alexey Vlasenko, NCSU
James Kneller, NCSU
Hiroki Nagakura, Princeton
(+more)

Neutrinos in Core-Collapse Supernovae

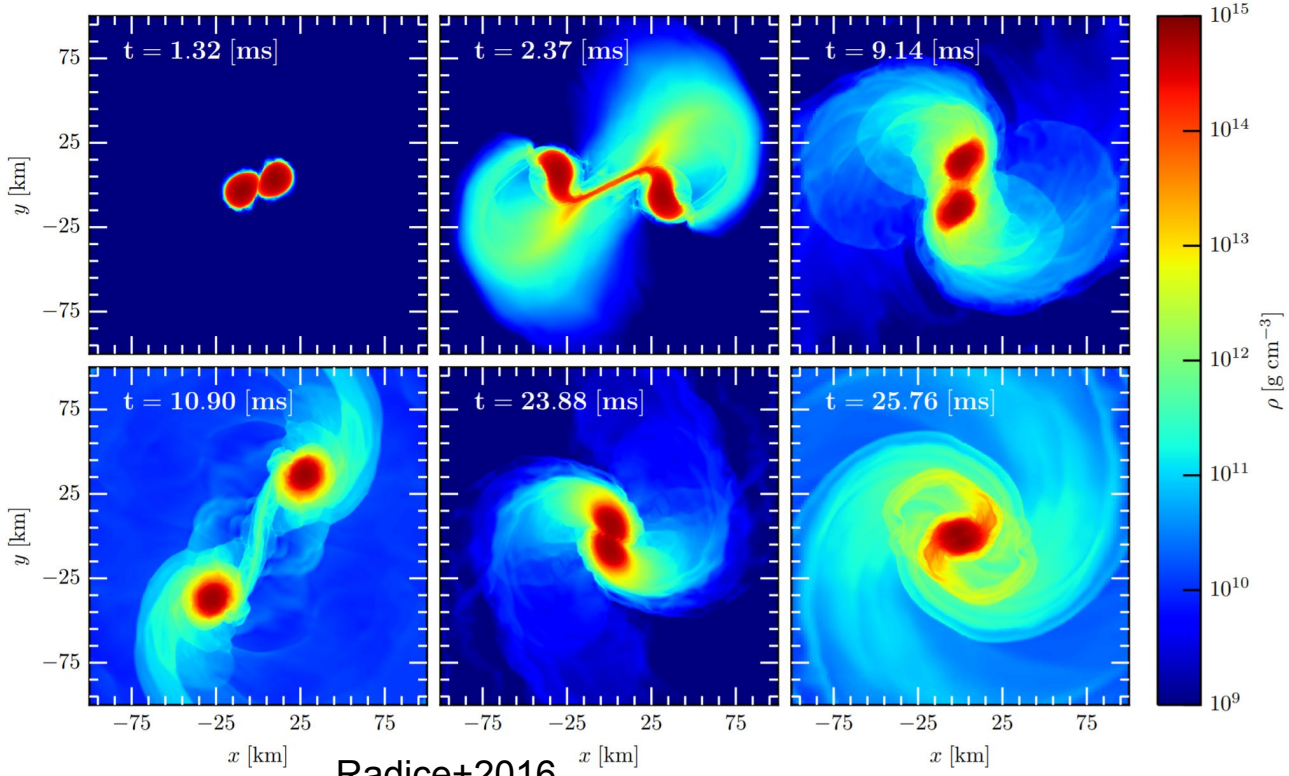


(Couch & O'Connor 2013)

Neutrinos drive explosion through **heating**, my drive neutron star **kick**, and may be observed soon.

- Turbulence/SASI drive asymmetric radiation
- Velocities few % speed of light
- Delicate balance determines explodability

Neutrinos in Neutron Star Mergers

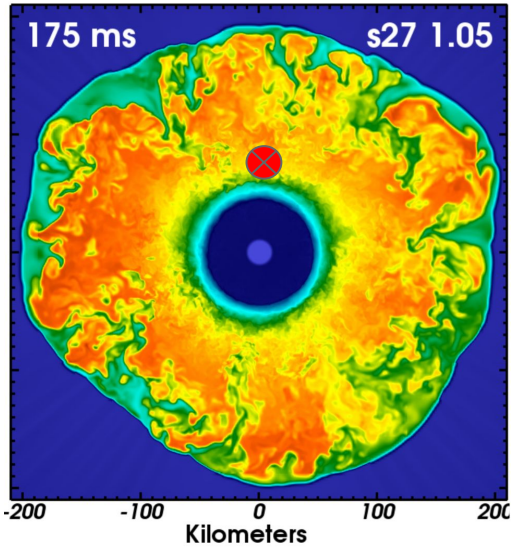


Radice+2016

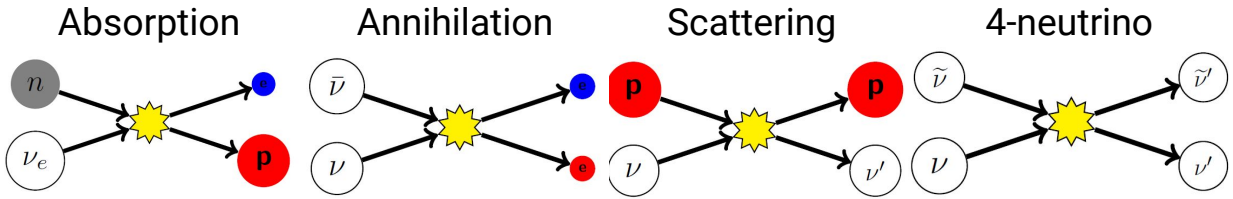
Neutrinos drive **outflows**, modify **composition**.

- Complex geometry
- Relativistic orbital velocity
- Strongly GR
- Smaller optical depth than CCSN (in disk)

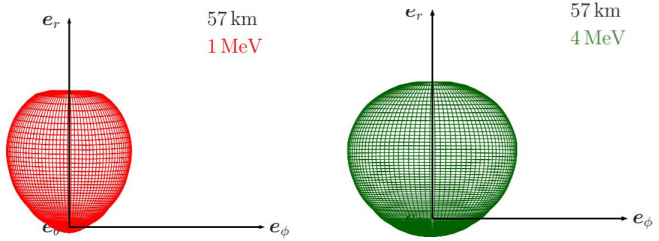
The distribution function is key!



Neutrinos transport the energy.



Nagakura+ (2019)

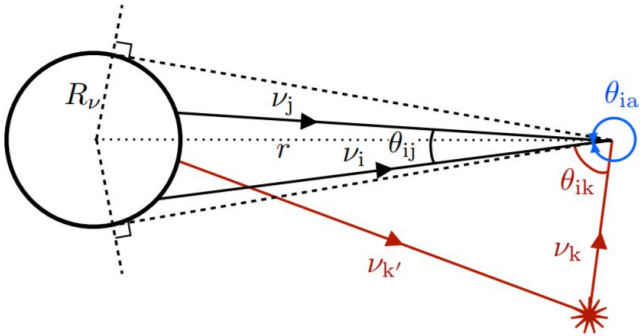


<u>Open Questions</u>

- Are the simulations resolved?
- Is an approximate method good enough?
- Are the interactions too approximate?
- Do flavor changes matter?
- (Are there unknown neutrino physics?)

The distribution function is key!

Neutrino Halo Effect

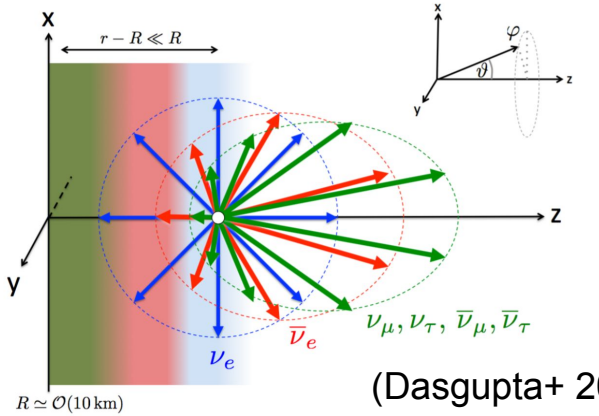


(Cherry+ 2012, 1203.1607)

Small number of reflected neutrinos can change neutrino flavors.

(Also, Cirigiliano+ 2018, Morinaga+ 2019)

Fast Oscillations

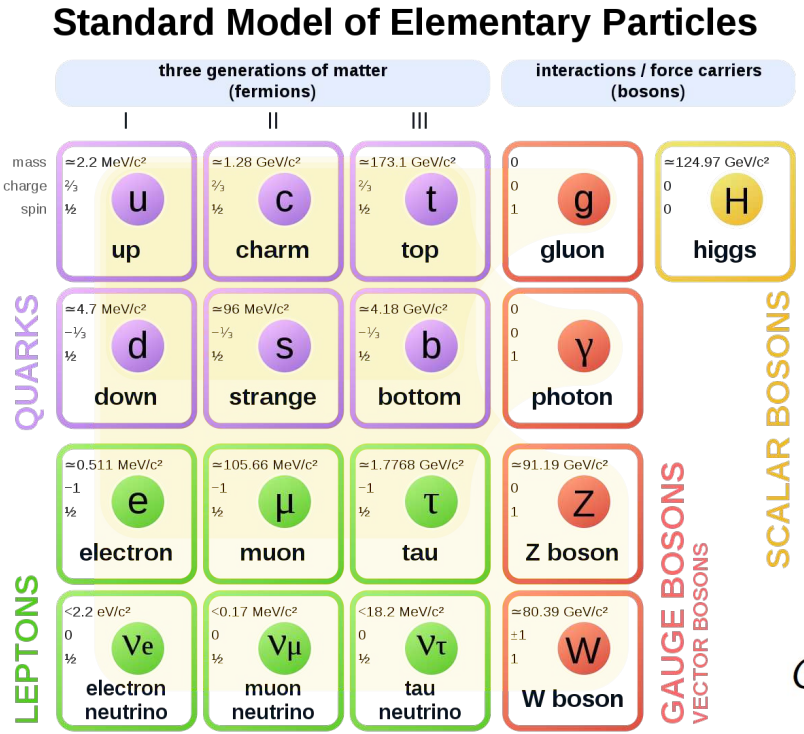


(Dasgupta+ 2017)

Neutrino/anti-neutrino angular dependence cause instabilities

(orig. Sawyer 2005, also Izaguirre+ 2016, Capozzi+ 2017, Dasgupta+ 2018, et al)

Neutrino Radiation Transport



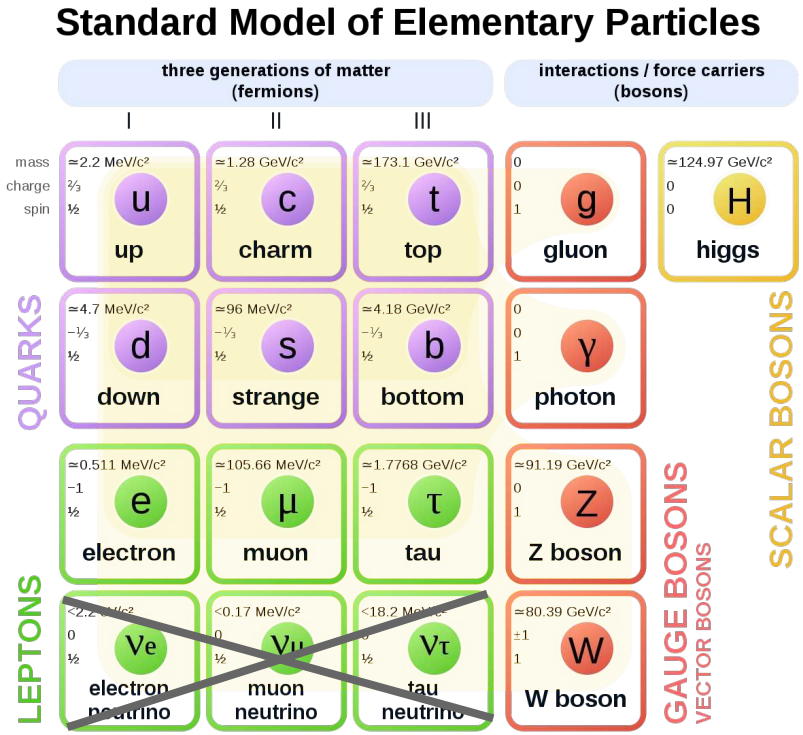
Treat each flavor separately:

$$\frac{d\,\boxed{\nu_e}}{d\lambda} = \mathcal{C}\left[\boxed{\nu_e}\,,\ \boxed{\nu_\mu}\,,\ \boxed{\nu_\tau}\right]$$

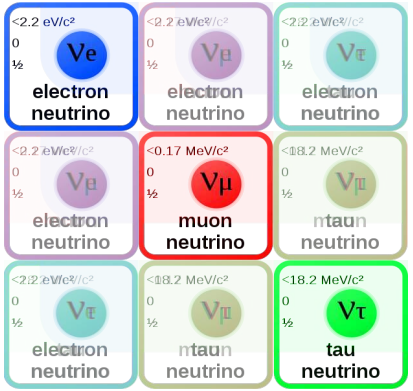
$$f_{(\nu_e)}(\mathbf{x}, \mathbf{p}, t)$$

$$\mathcal{C} \sim \int d^3\mathbf{p}_2 \int d^3\mathbf{p}_3 \int d^3\mathbf{p}_4\ \mathrm{R}(\mathrm{f}_1, \mathrm{f}_2, \mathrm{f}_3, \mathrm{f}_4, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)$$

Neutrino Quantum Kinetics



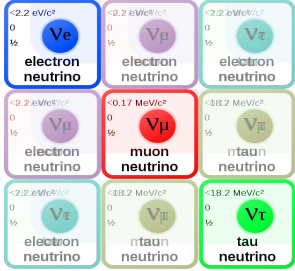
But the neutrino flavors are mixed!
(Pontecorvo 1968, Wolfenstein1978, Mikheev & Smirnov 1985)



Cardall 2008
Vlasenko et al. 2014
Blaschke+Cirigliano 2016

$$\frac{d}{d\lambda}\left[\;\right] = \mathcal{C}\left[\;\right] - \frac{i}{\hbar c}\left[\mathcal{H}, \;\right]$$

Flavor Oscillations in Supernova *Engines*

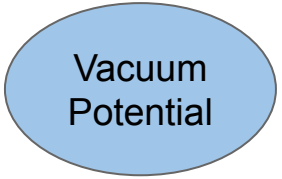


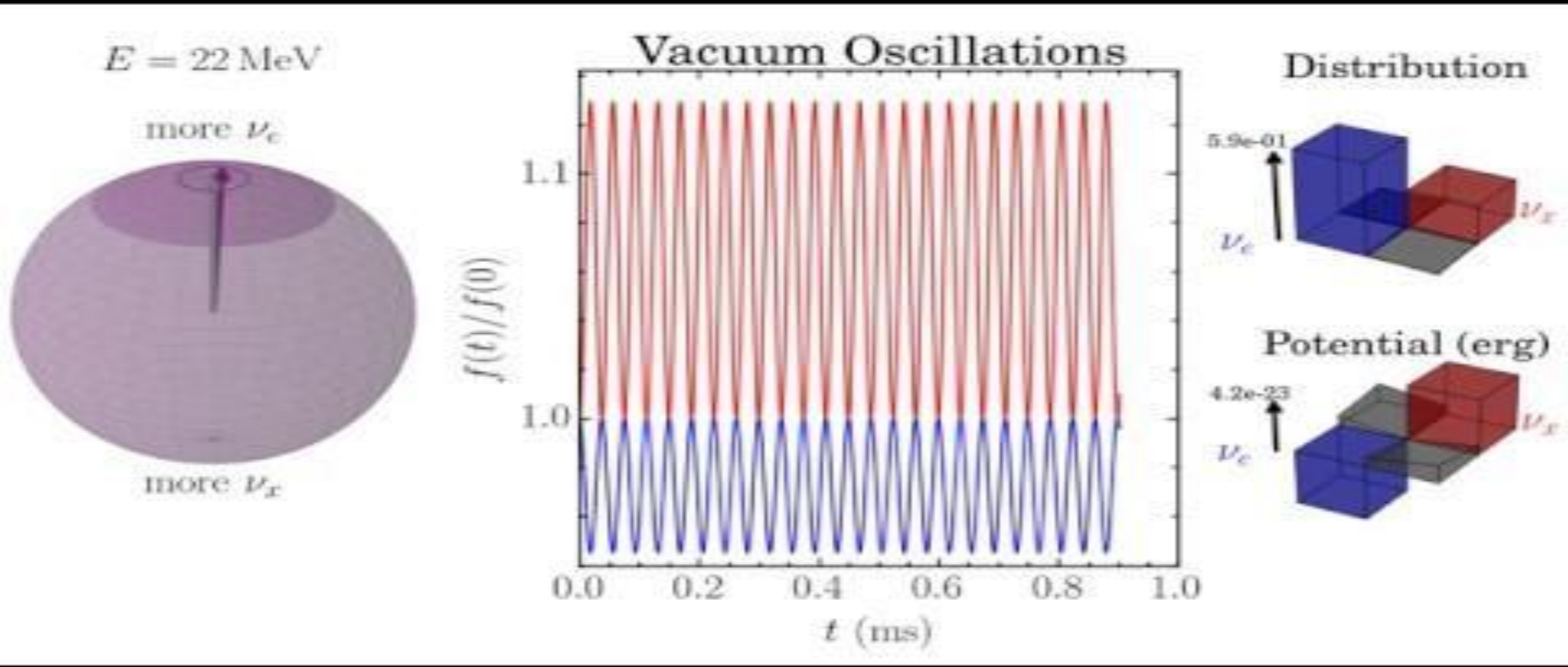
1968 - 2001

Oscillations Proposed and Discovered

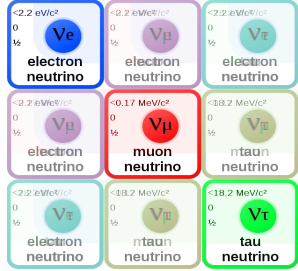
Excessively simplified: Pontecorvo (1968), Ahmad+ (2001)

Vacuum Oscillations




$E = 22\,\mathrm{MeV}$
more ν_e
more ν_x
Vacuum Oscillations
$f(t)/f(0)$
1.1
1.0
0.0
0.2
0.4
0.6
0.8
1.0
t (ms)
Distribution
5.9e-01
ν_e
ν_x
Potential (erg)
4.2e-23
ν_e
ν_x

Flavor Oscillations in Supernova *Engines*



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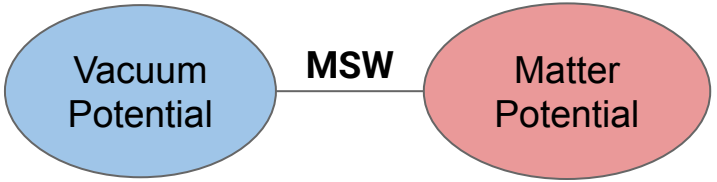
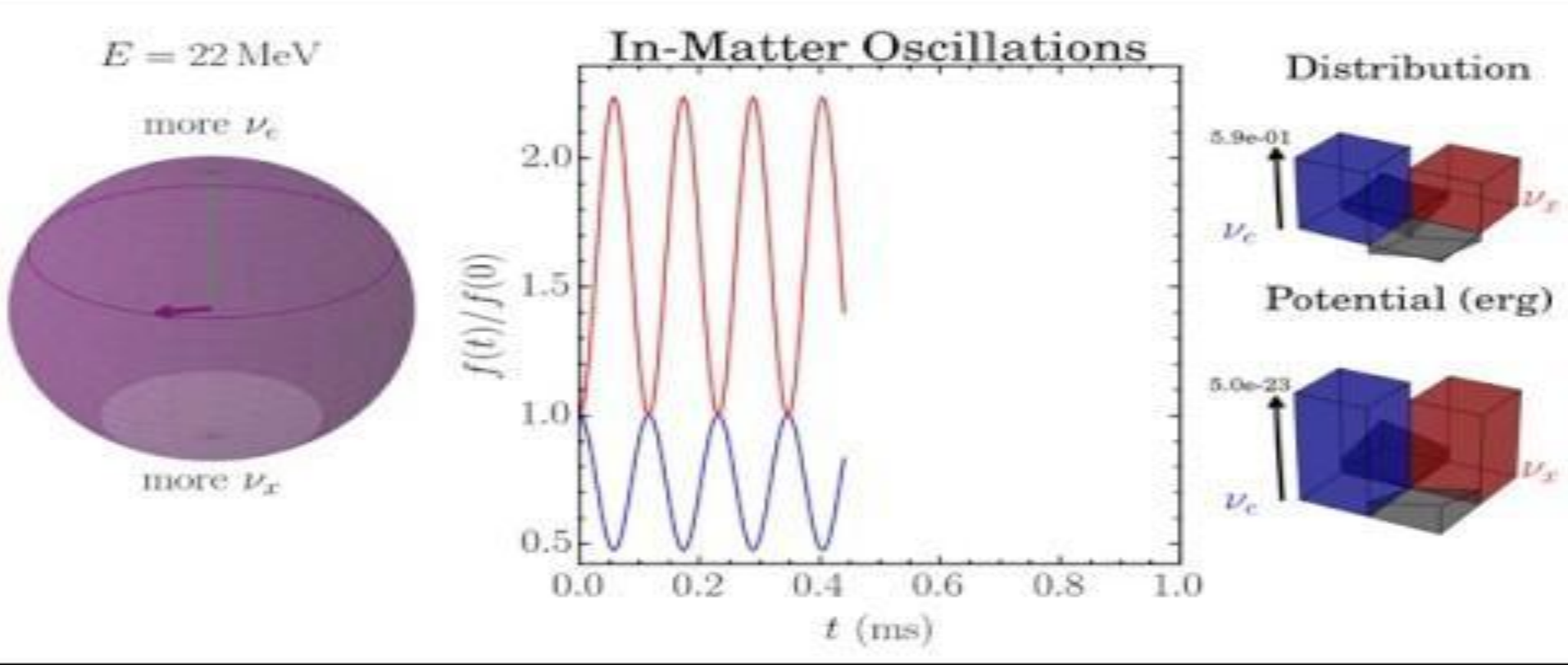
NO OSCILLATIONS: In-medium Effects

E.g., Wolfenstein (1978), Mkiheyev & Smirnov (1985), Fuller+ (1992), Quian+ (1993), Fogli+ (2007), Esteban-Pretel+ (2008)

Vacuum Oscillations — **MSW** — **Matter-Suppressed**




$E = 22\,\mathrm{MeV}$
more ν_e
more ν_x
In-Matter Oscillations
$f(t)/f(0)$
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0
t (ms)
Distribution
5.9e-01
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ν_x

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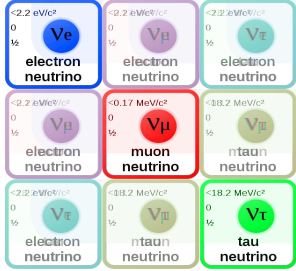

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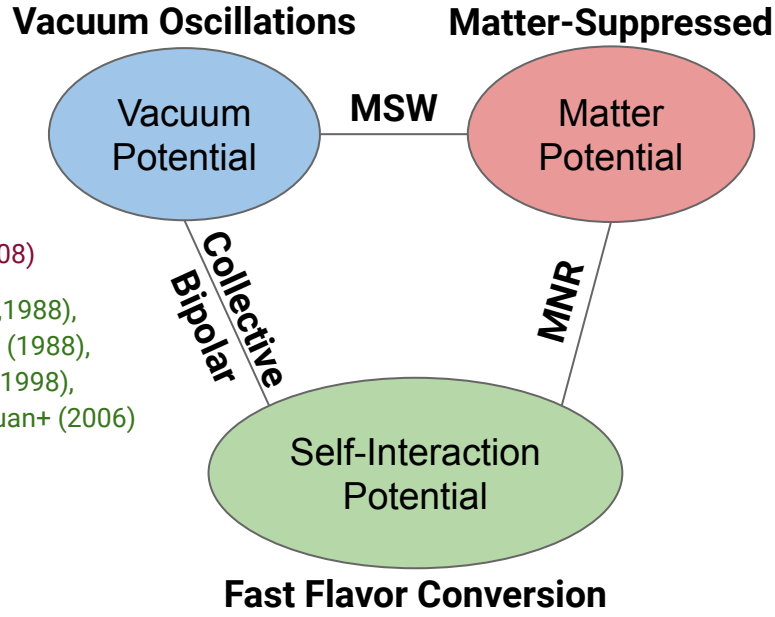
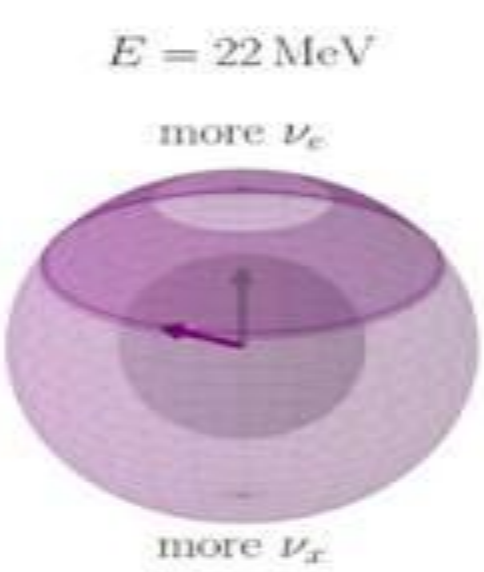
1987 - 2006

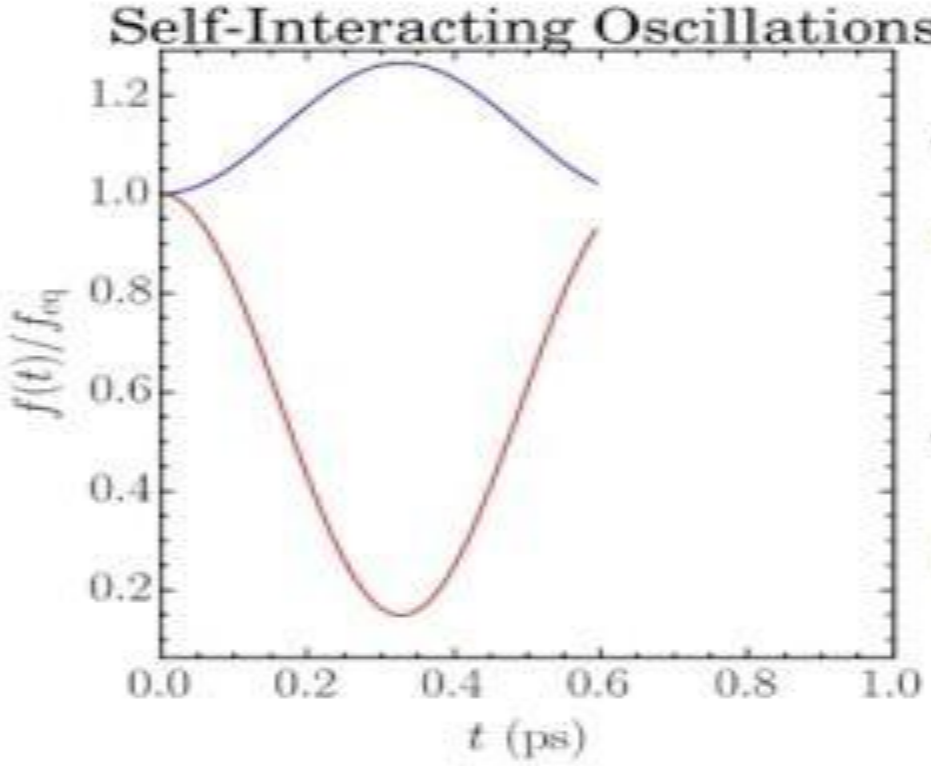
OSCILLATIONS: Neutrino-Neutrino Interactions

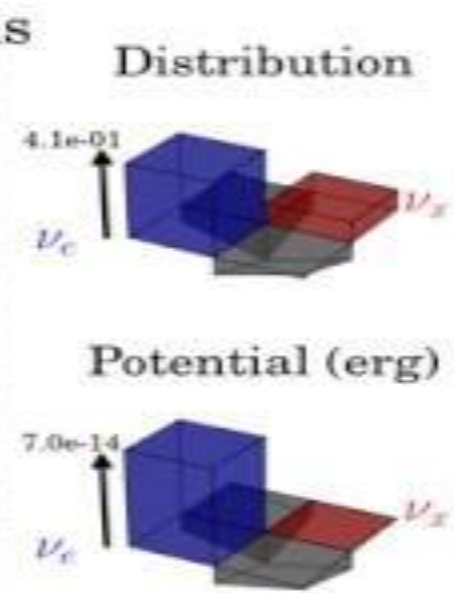
E.g., Fuller+ (1987,1988), Noetzold & Raffelt (1988), Pantaleone (1992,1998), Pastor+ (2002), Duan+ (2006)





Sherwood Richers
github.com/srichers/IsotropicSQA
arXiv:1903.00022





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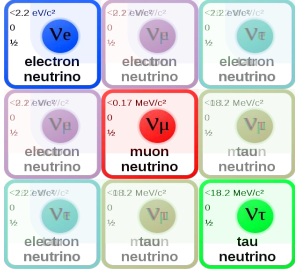

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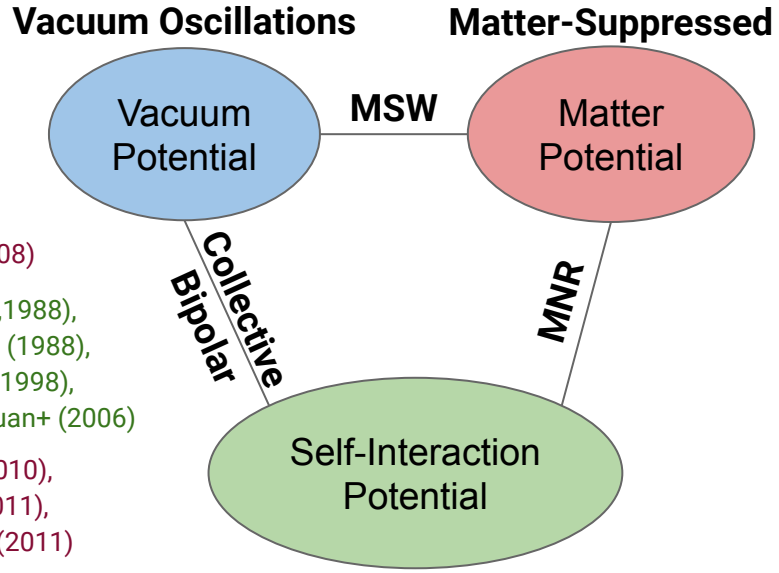
2011 - 2012

NO OSCILLATIONS: Direction-Dependent Calculations

E.g., Duan+ (2010), Dasgupta+ (2011), Chakrabotry+ (2011)



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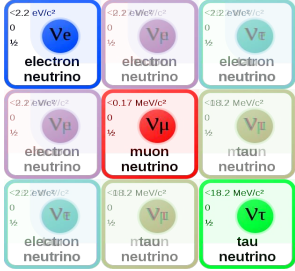

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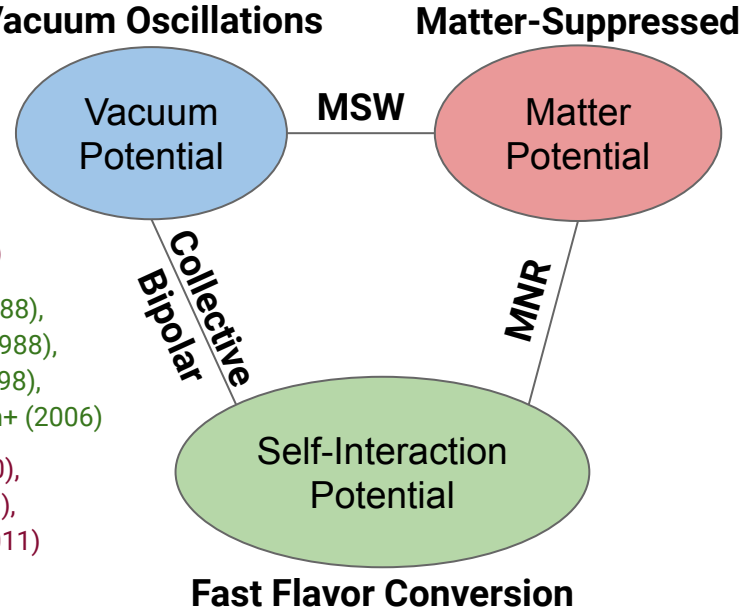
E.g., Duan+ (2010), Dasgupta+ (2011), Chakrabotry+ (2011)

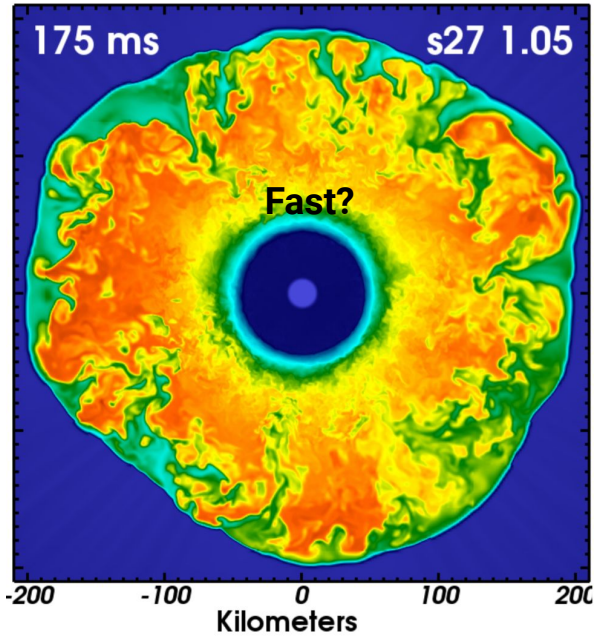
2005 - present

OSCILLATIONS: Direction-Dependent Calculations → New FAST Instability

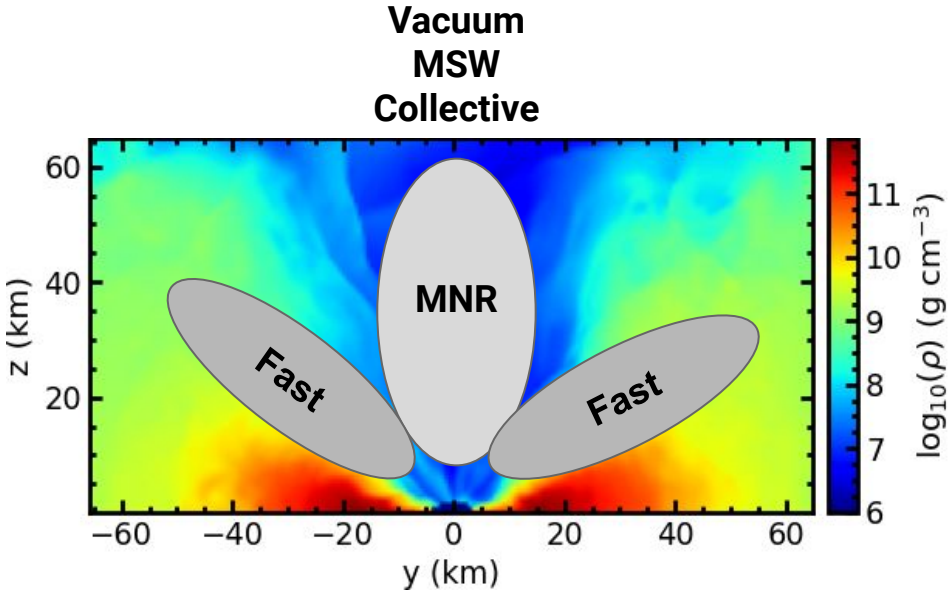
E.g., Sawyer (2005), Sawyer (2015), Capozzi+(2015), Chakrabotry+ (2016), Tamborra+ (2017), Abbar + Volpe (2019), Yi+ (2019), Capozzi+ (2019), Abbar+(2019), Azari+ (2019), Walk+(2019) AND OTHERS



Where do flavor changes occur?



Vacuum
MSW
Collective / Fast



Lots of places (possibly)!

Part I:

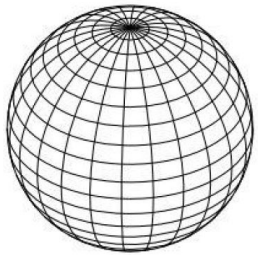
Understanding the Distribution Function

Neutrino Transport

$f(x^\mu, p^\mu)$ is (# of neutrinos) per (volume) per (energy) per (solid angle)

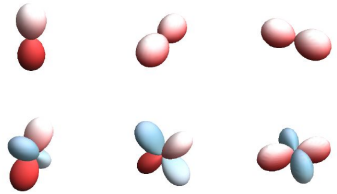
Classical Boltzmann Equation

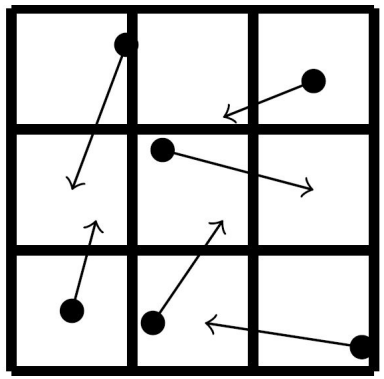
$$\frac{1}{c}\frac{\mathrm{df}}{\mathrm{d}\tau} = \mathcal{C}$$

$f =$ 

$f =$ (E) 

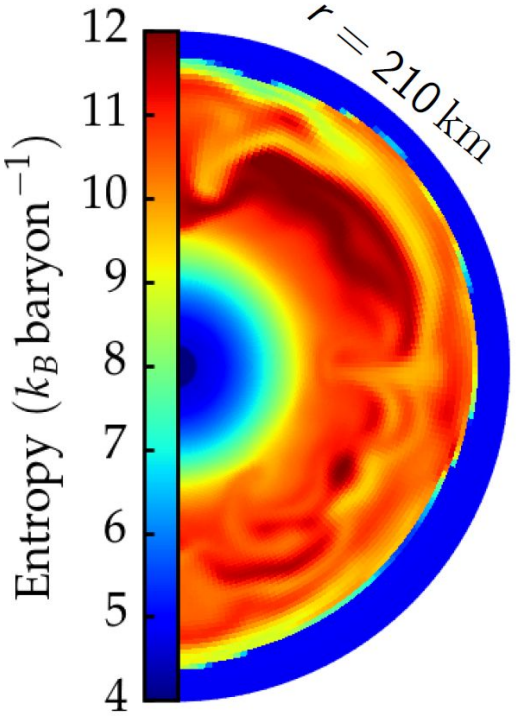
(F^i) 

(P^{ij}) 

$f =$ 

SEDONU: GR Monte Carlo Transport

1. Take fluid snapshot
2. Emit
3. Propagate & Absorb
4. Scatter



3D Interpolate:

- Metric (and derivatives)
- Opacities/Emissivities
- Scattering Kernels
- Velocity

See also: Foucart+(2018), Kato+(2020)

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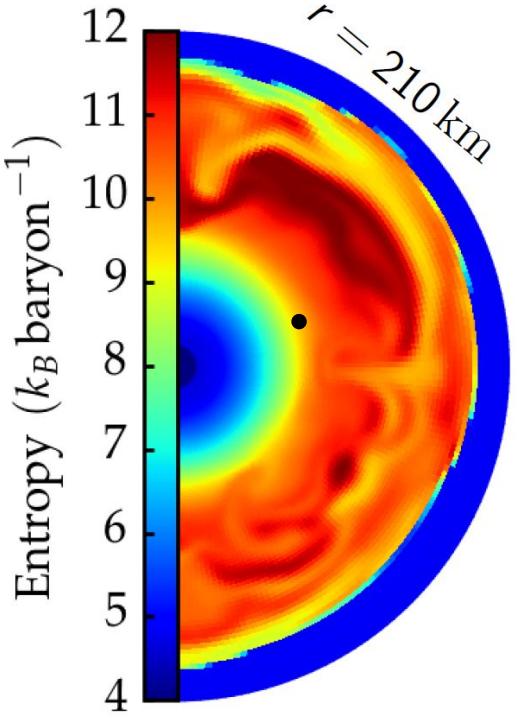

Emit:

- **Random** location
- **Random** direction
- **Random** energy

See also: Foucart+(2018), Kato+(2020)

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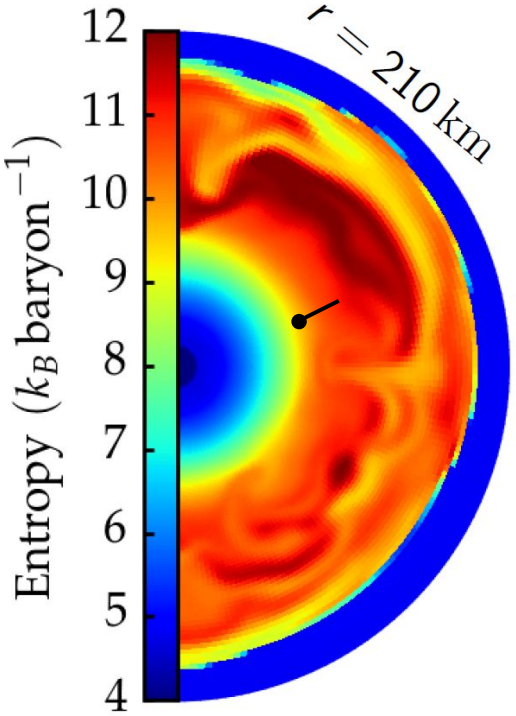

Propagate & Absorb:

- **Random** distance

$$\mathrm{PDF}(d) = \sigma_s e^{-\sigma_s d}$$

- Absorb continuously
- Accumulate f

See also: Foucart+(2018), Kato+(2020)

SEDONU: GR Monte Carlo Transport

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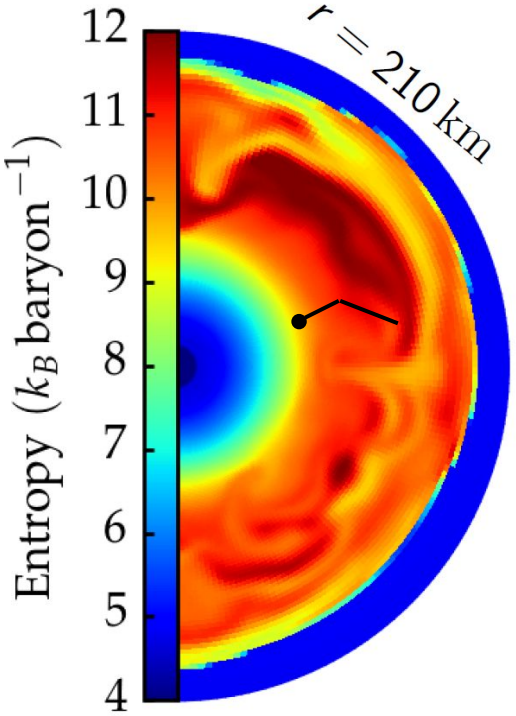

Scatter:

- **Random** direction & energy

See also: Foucart+(2018), Kato+(2020)

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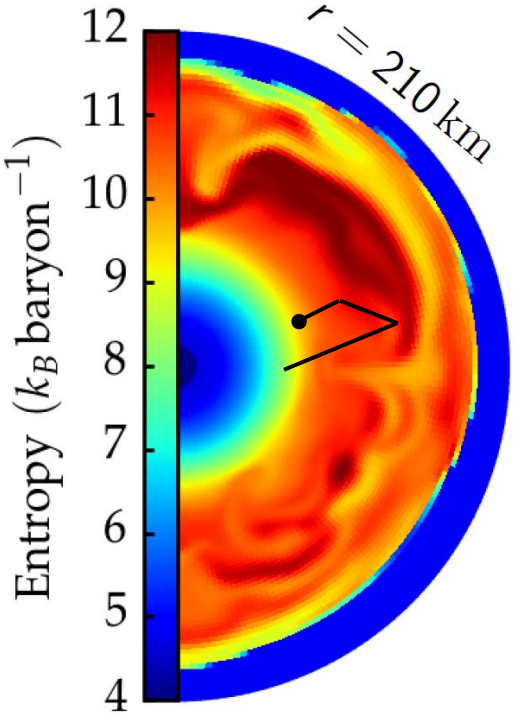

Scatter:

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See also: Foucart+(2018), Kato+(2020)

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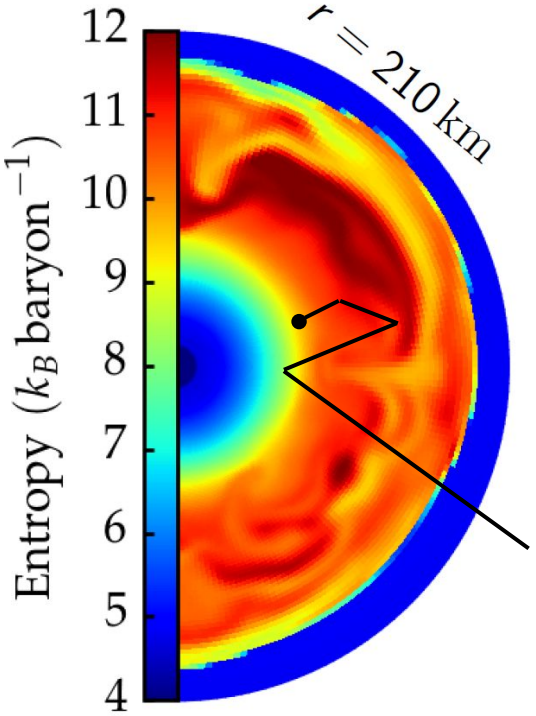
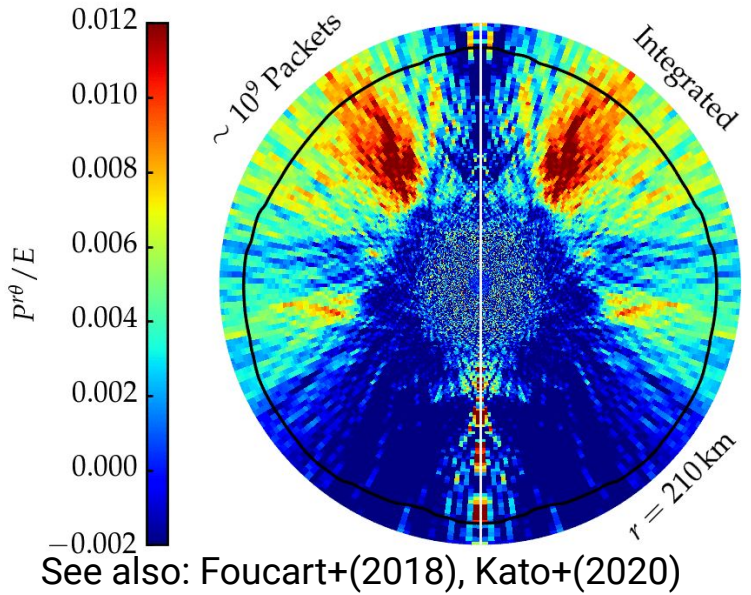
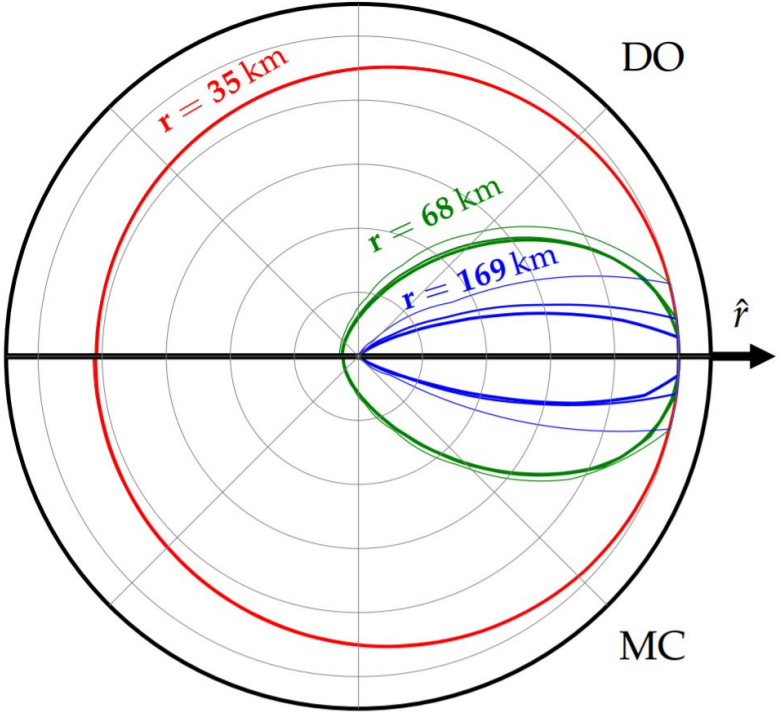


Repeat 10 billion times

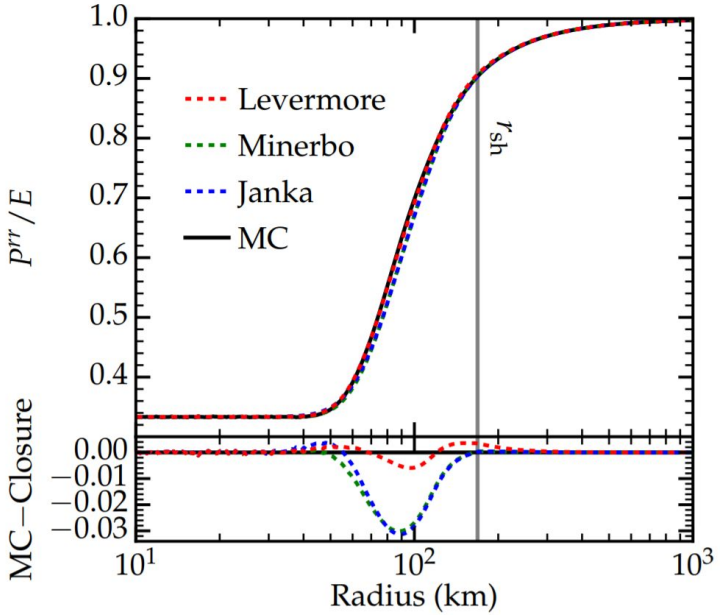


See also: Foucart+(2018), Kato+(2020)

CCSN: Neutrino Field

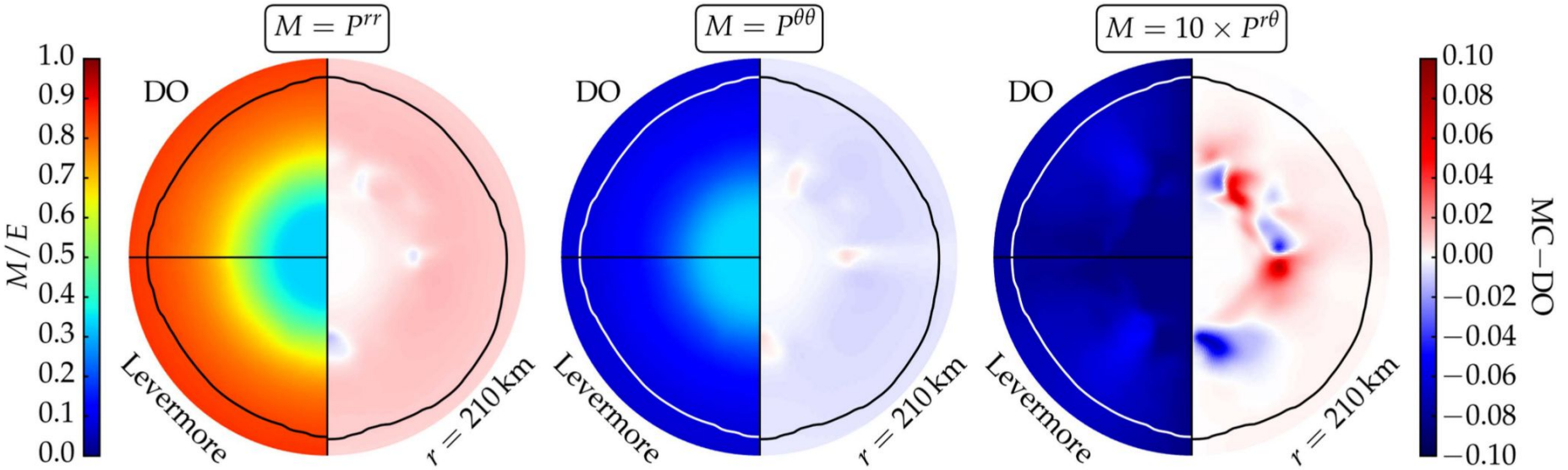


Richers+2017



Few % difference in 1D

CCSN: Neutrino Heating



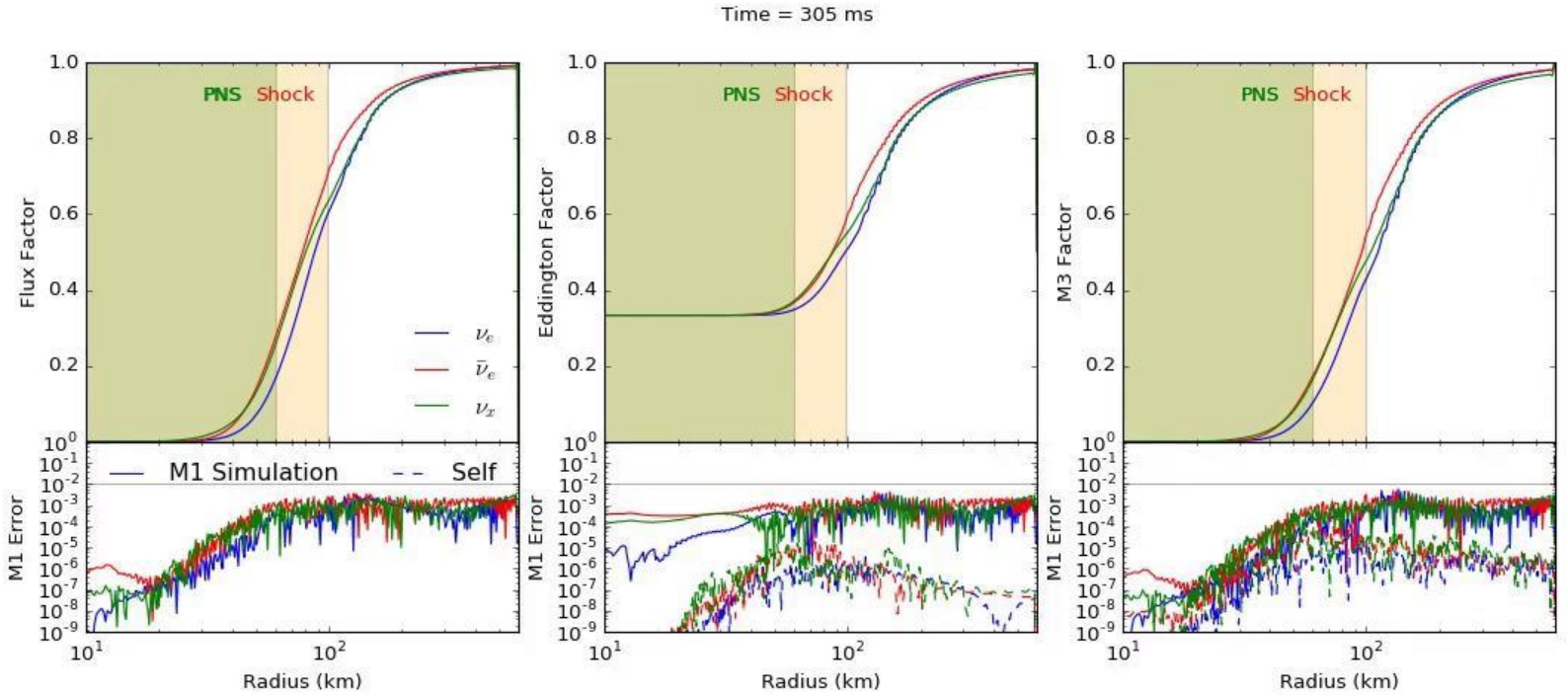
Richers+ (2017)

Few % difference in 2D as well!
can be ~20% (Harada+2019)

Outline

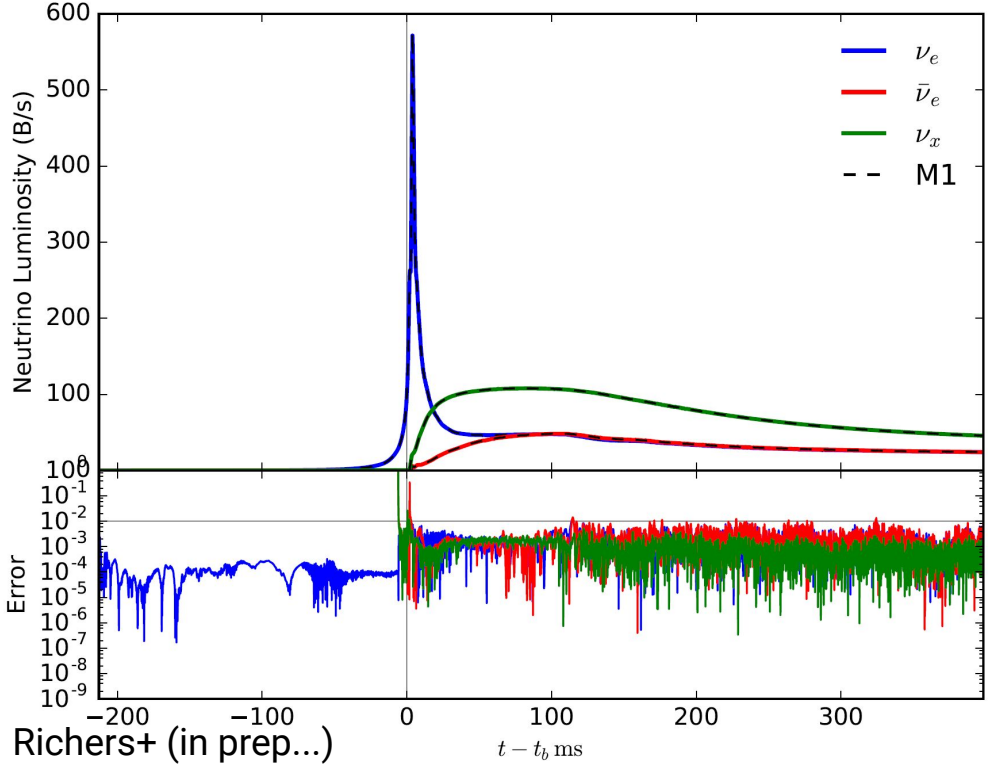
	Supernovae	Neutron Star Mergers
Standard Model	**Moments** work well in 1D (worse in multi-D)	
Quantum Kinetics	**Proof of concept** MC closure does little in 1D	

Using Monte Carlo to close moment scheme



Richers+ (in prep...)

Using Monte Carlo to close moment scheme



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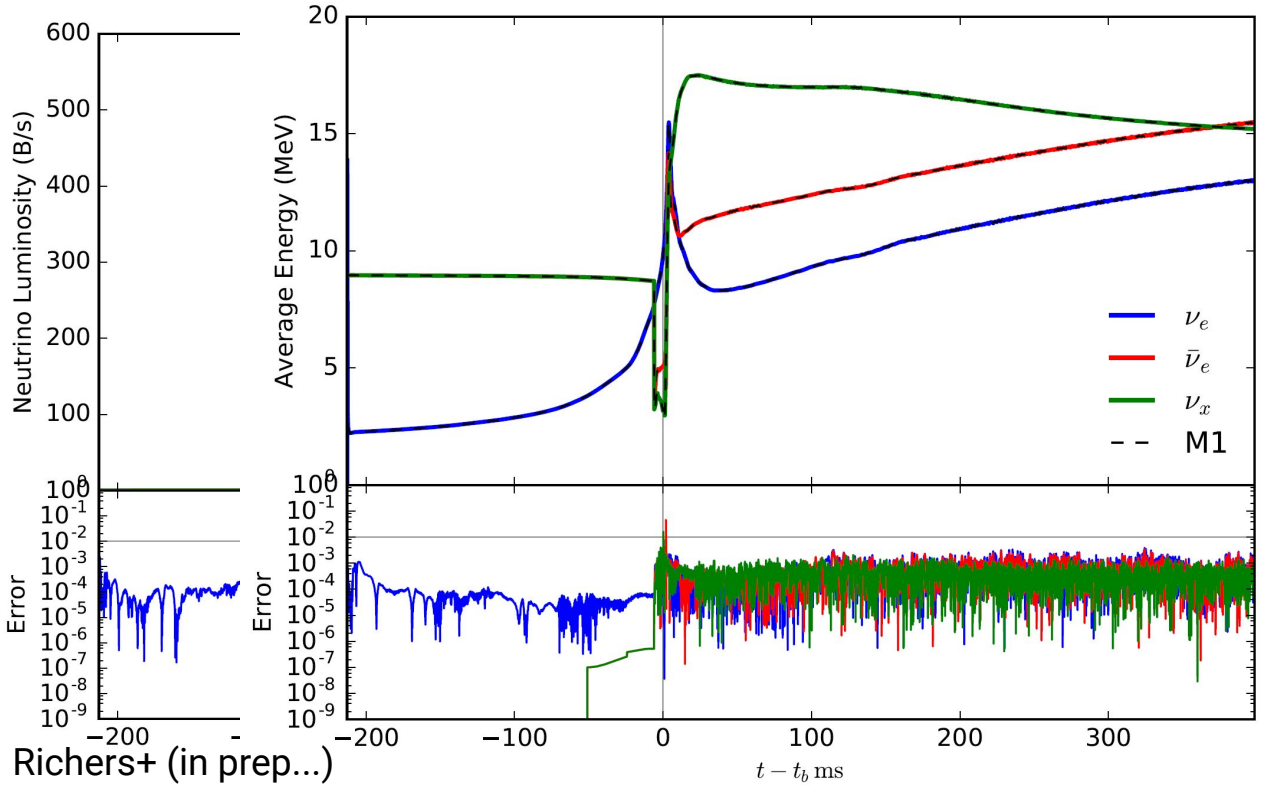

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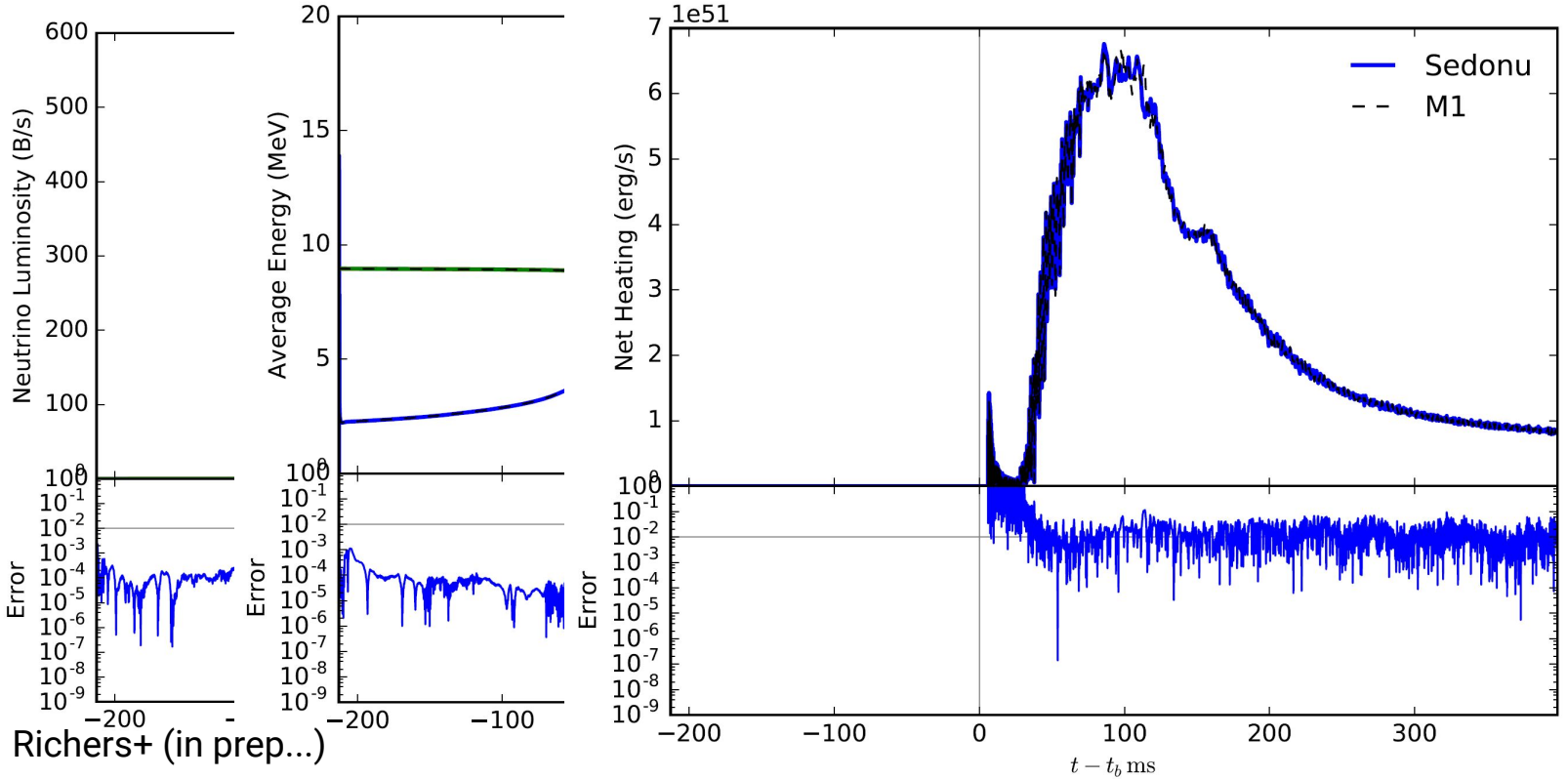

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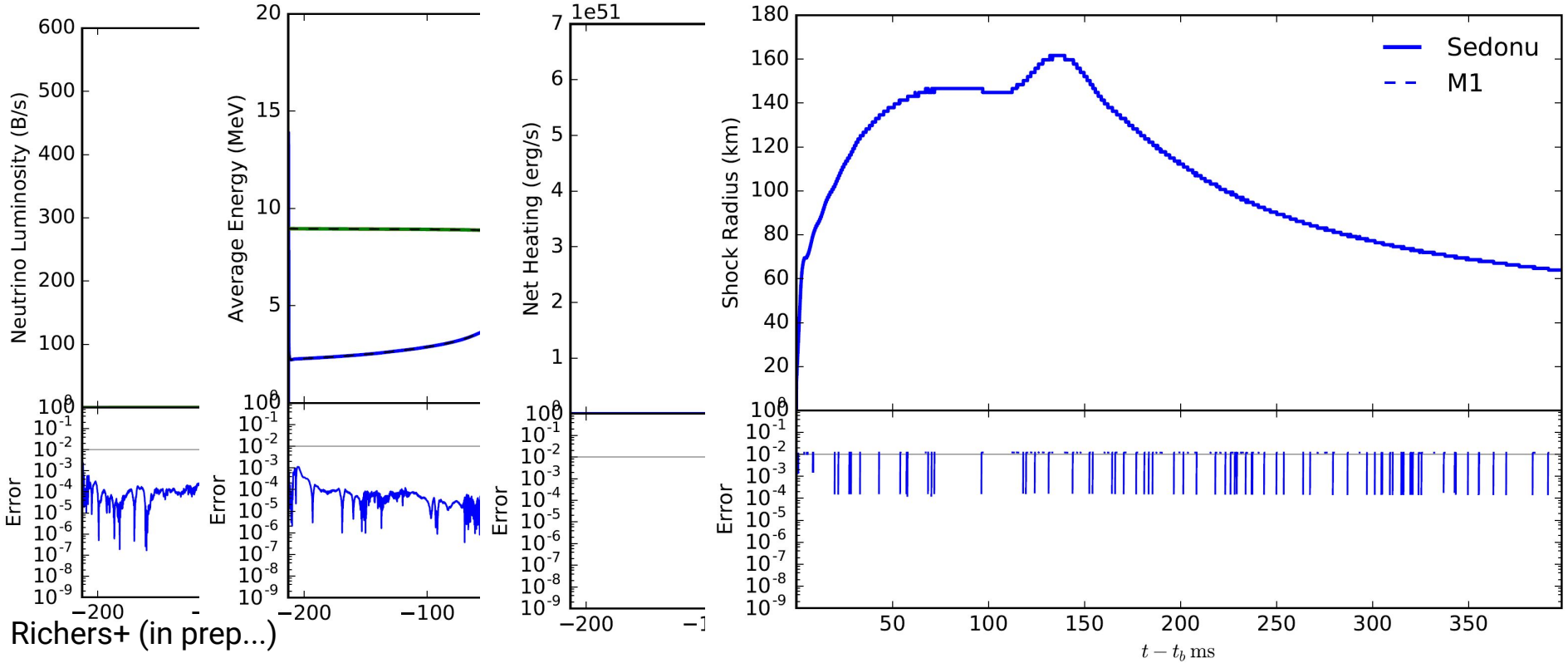

Richers+ (in prep...)

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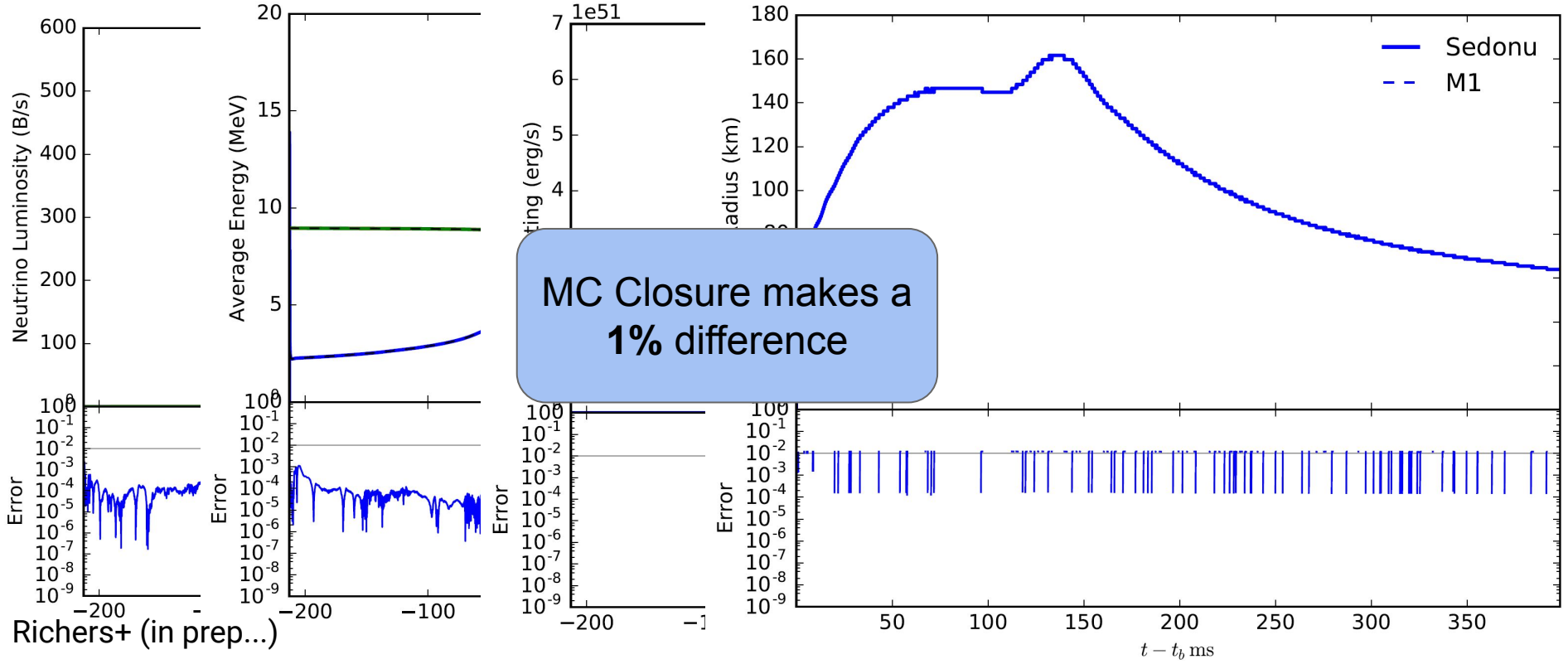


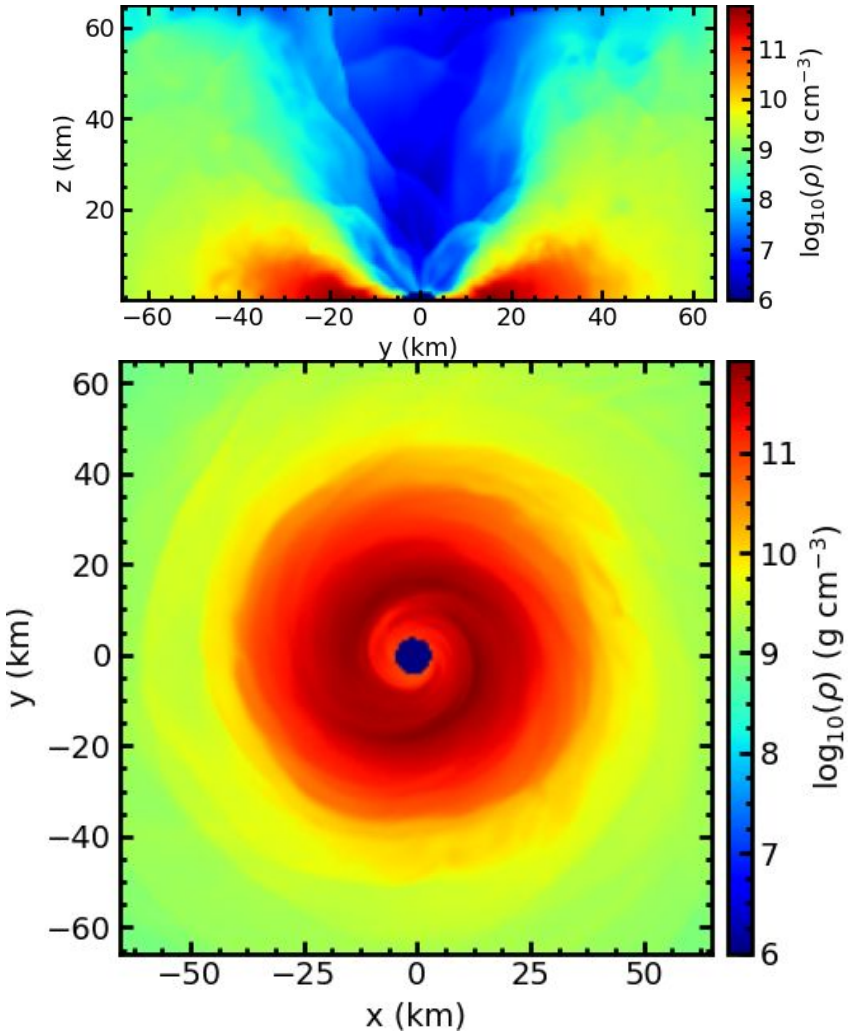
Richers+ (in prep...)

Outline

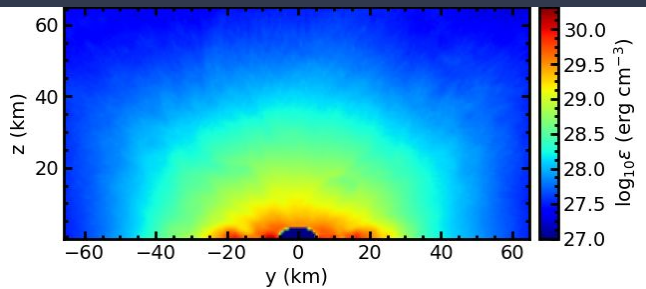
	Supernovae	Neutron Star Mergers
Standard Model	**Moments** work well in 1D (worse in multi-D)	
Quantum Kinetics	**Proof of concept** MC closure does little in 1D	

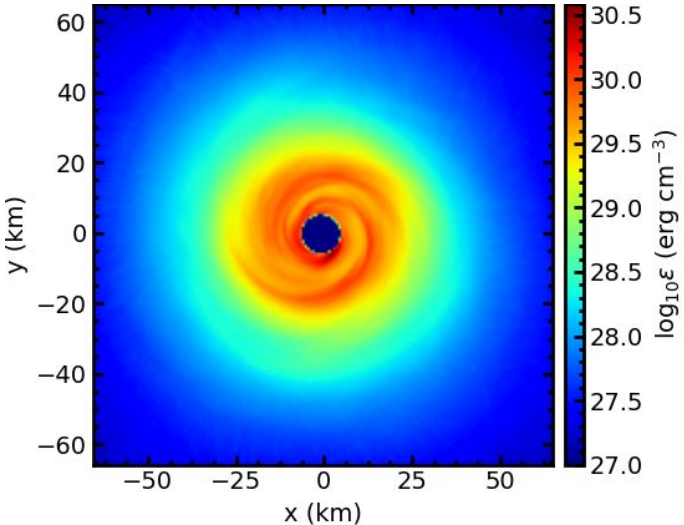
Mergers are more challenging!

Hydro: Radice+2018



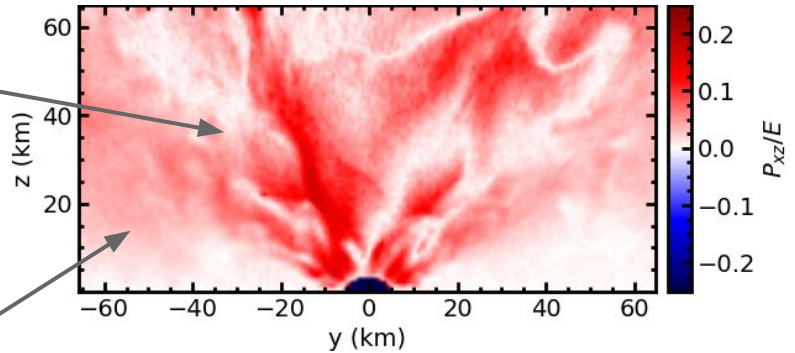
Lots of structure in higher moments.

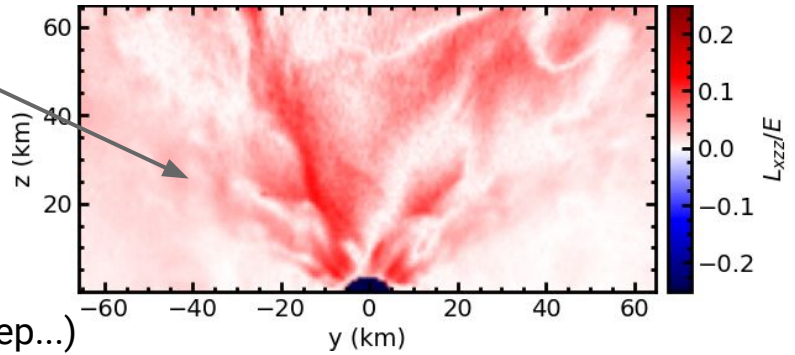




Compare to
0.01 for CCSN

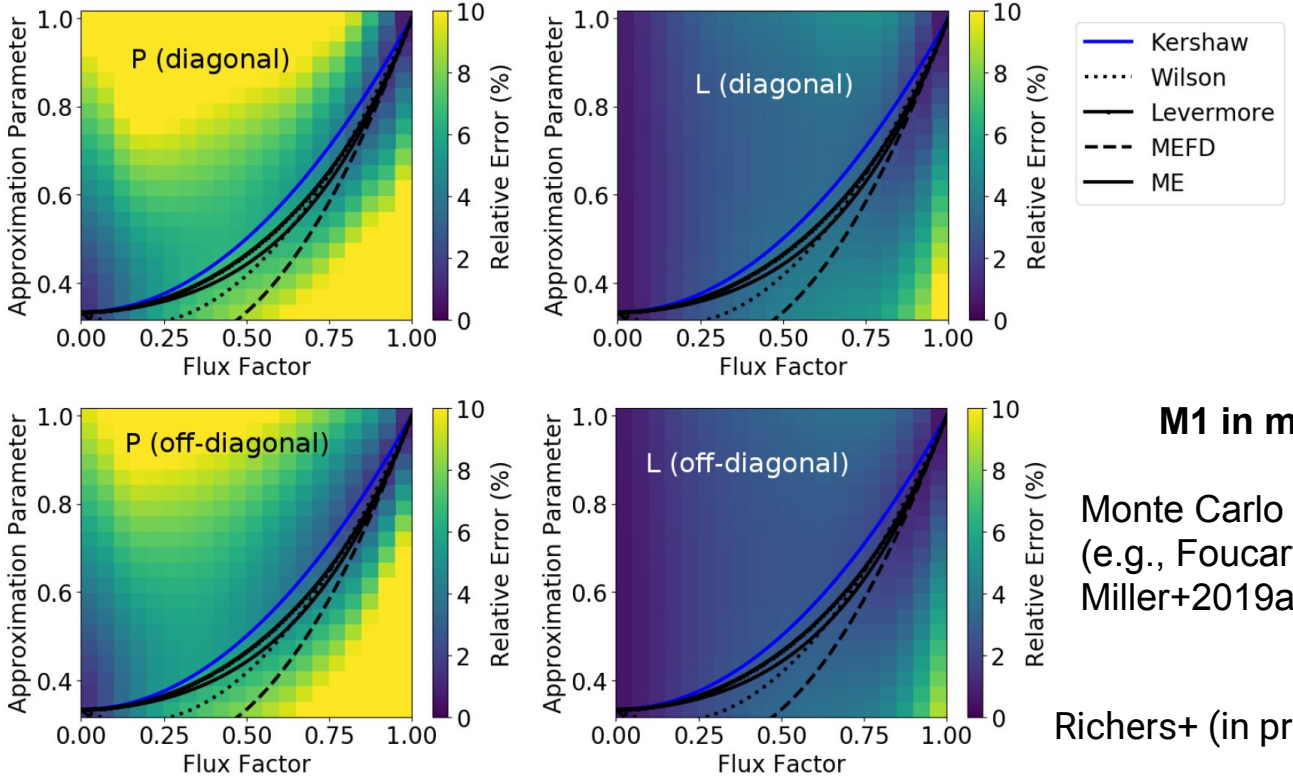
0 for
spherical
radiation





Richers+ (in prep...)

MERGERS: How well does M1 do?



Unlike CCSNe,
M1 in mergers has sizable errors

Monte Carlo transport may be able to fix it (e.g., Foucart+2018, Miller+2019a, Miller+2019b, Miller+2019c)

Richers+ (in prep...)

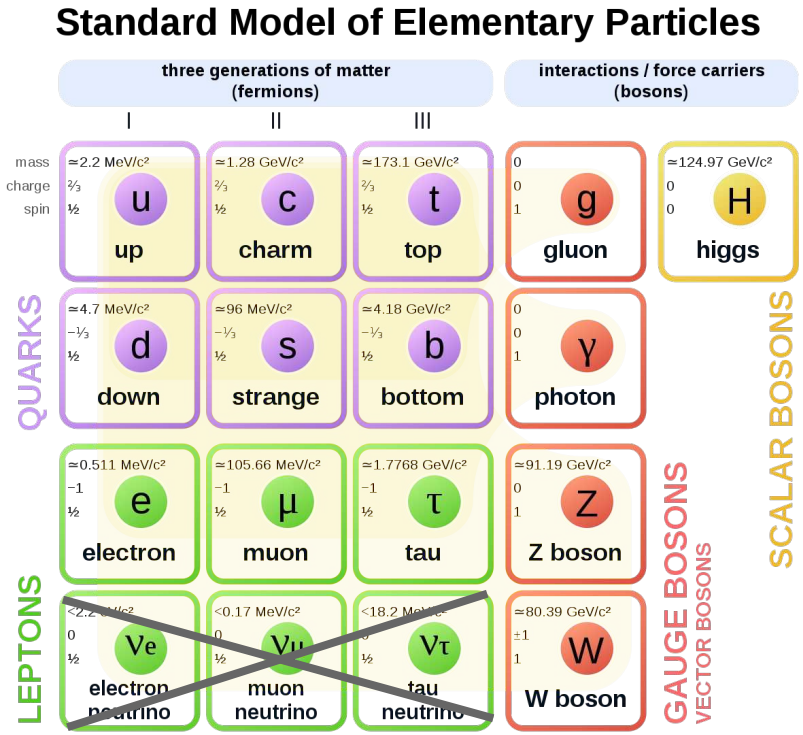
Outline

	Supernovae	Neutron Star Mergers
Standard Model	**Moments** work well in 1D (worse in multi-D) **Proof of concept** MC closure does little in 1D	**Analytic closures** are inadequate for merger simulations
Quantum Kinetics		

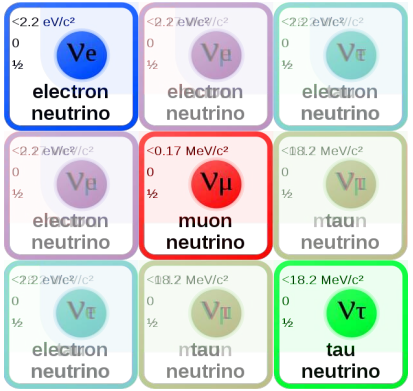
Part II:

Quantum Kinetics

Neutrino Quantum Kinetics



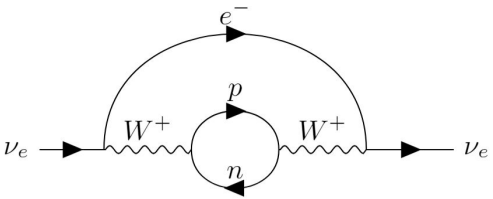
But the neutrino flavors are mixed!
(Pontecorvo 1968, Wolfenstein1978, Mikheev & Smirnov 1985)



Cardall 2008
Vlasenko et al. 2014
Blaschke+Cirigliano 2016

$$\frac{d}{d\lambda}\left[\;\right] = \mathcal{C}\left[\;\right] - \frac{i}{\hbar c}\left[\mathcal{H}, \;\right]$$

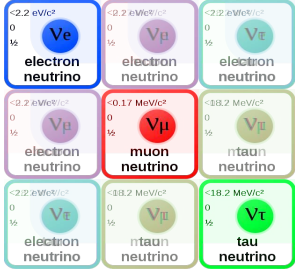
Absorption and Emission



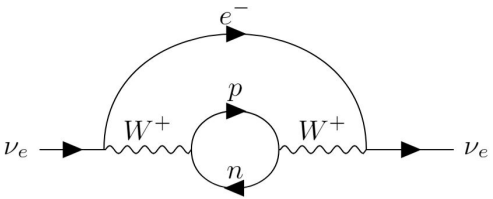
1) Read neutrino structure from diagram

$$C^+ \sim \begin{bmatrix} (1-f_{ee}) & -f_{e\mu}/2 & -f_{e\tau}/2 \\ -f_{\mu e}/2 & 0 & 0 \\ -f_{\tau e}/2 & 0 & 0 \end{bmatrix}$$

$$C^- \sim \begin{bmatrix} f_{ee} & f_{e\mu}/2 & f_{e\tau}/2 \\ f_{\mu e}/2 & 0 & 0 \\ f_{\tau e}/2 & 0 & 0 \end{bmatrix}.$$



Absorption and Emission



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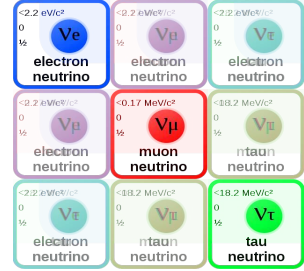
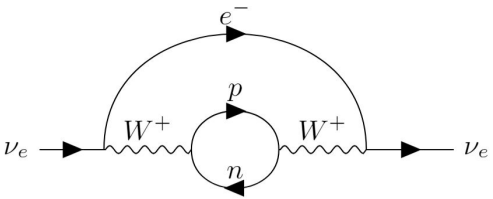
2) Require diagonals are the same as **known rates**

$$C^+_{ee} = j_{(\nu_e)}(1-f_{ee})$$
$$C^-_{ee} = \kappa_{(\nu_e)} f_{ee} \, ,$$

Collision Rates → QKE Terms





QKE terms
from straightforward combinations of
ordinary terms

1) Read neutrino structure from diagram

$$C^+ \sim \begin{bmatrix} (1-f_{ee}) & -f_{e\mu}/2 & -f_{e\tau}/2 \\ -f_{\mu e}/2 & 0 & 0 \\ -f_{\tau e}/2 & 0 & 0 \end{bmatrix}$$

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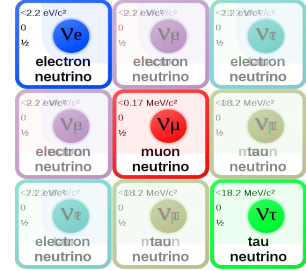
2) Require diagonals are the same as **known rates**

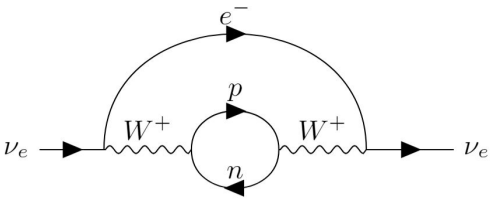
$$C^+_{ee} = j_{(\nu_e)}(1-f_{ee})$$
$$C^-_{ee} = \kappa_{(\nu_e)} f_{ee} \, ,$$

3) Profit.

$$C_{ab} = j_{(\nu_a)}\delta_{ab} - (\langle j \rangle_{ab} + \langle \kappa \rangle_{ab}) f_{ab}$$

Collision Rates → QKE Terms





QKE terms
from straightforward
combinations of
ordinary terms

We also include:

- Electron scattering/annhilation
- Neutrino scattering/annihilation
- Nucleon Scattering
- Effective bremsstrahlung

1) Read neutrino structure from diagram

$$C^+ \sim \begin{bmatrix} (1-f_{ee}) & -f_{e\mu}/2 & -f_{e\tau}/2 \\ -f_{\mu e}/2 & 0 & 0 \\ -f_{\tau e}/2 & 0 & 0 \end{bmatrix}$$

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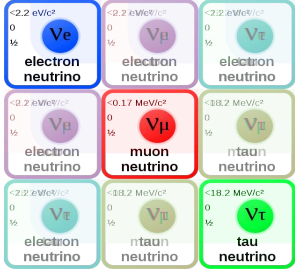
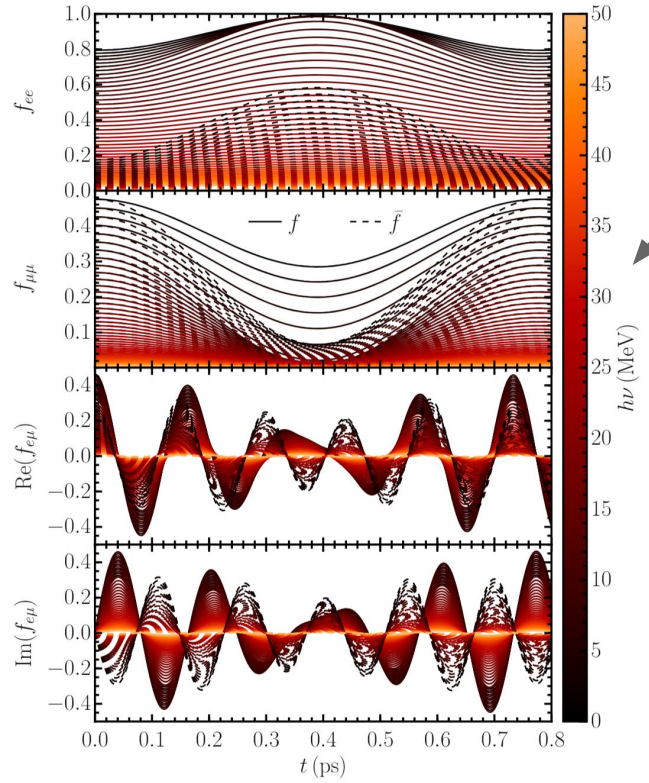
$$C^-_{ee} = \kappa_{(\nu_e)} f_{ee} \, ,$$

3) Profit.

$$C_{ab} = j_{(\nu_a)}\delta_{ab} - (\langle j \rangle_{ab} + \langle \kappa \rangle_{ab}) f_{ab}$$

Why hasn't this been solved yet?



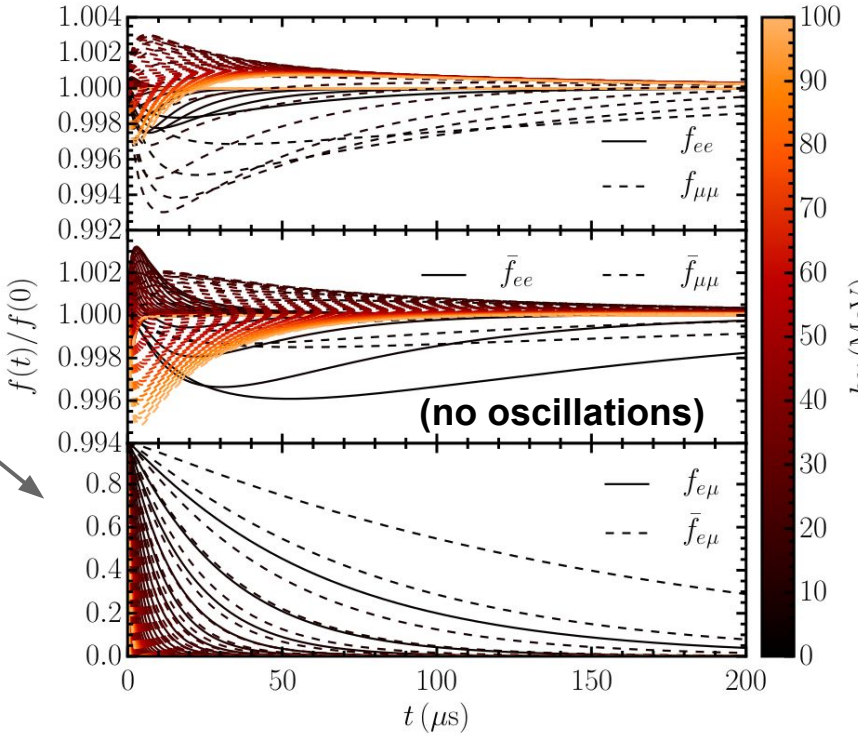


Oscillations are on extremely short timescales

Interactions take 10^7 times longer!

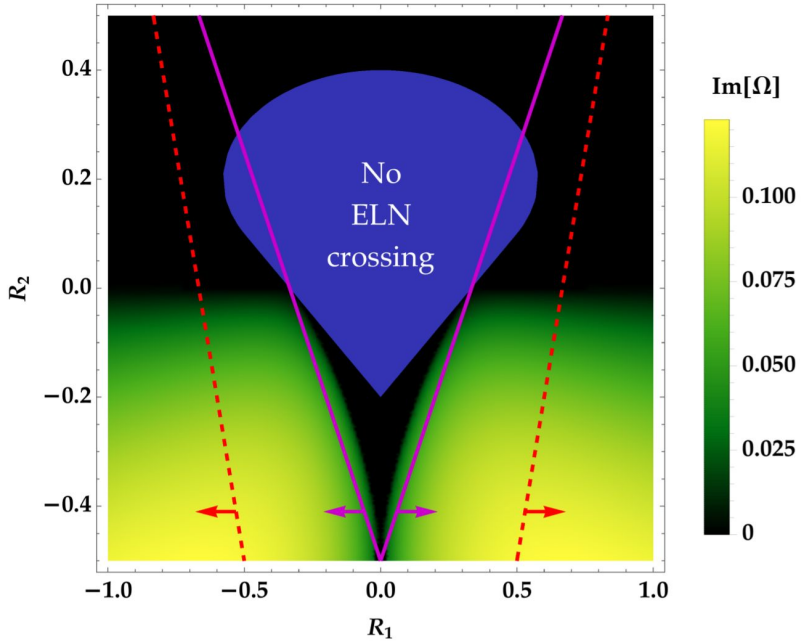
Not possible to evolve directly.

Richers+ (2019)



The Fog of War

- Linear stability analysis
- Searching in simulation snapshots
- Simulating toy problems
- Simulating in reduced dimensions (Early universe)



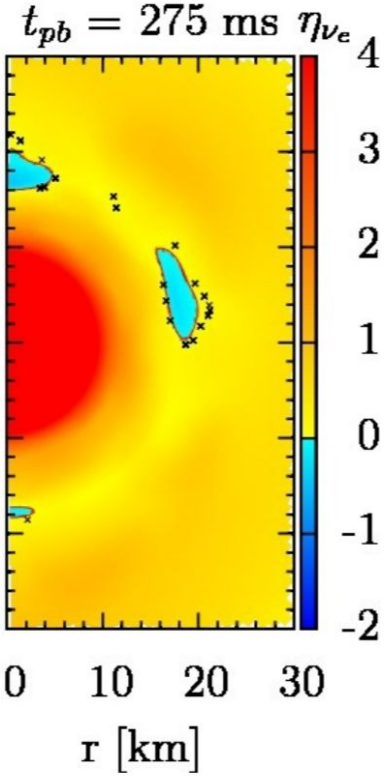
E.g., Mirizzi+(2012), Cherry+(2012), Cherry+(2013), Dasgupta+(2016), Cirigliano+(2018), Capozzi+(2017), Capozzi+(2019), Zaizen+(2019), Johns+(2020)

The Fog of War

- Linear stability analysis
- Searching in simulation snapshots
- Simulating toy problems
- Simulating in reduced dimensions (Early universe)

YES:
e.g.,
Abbar+(2018),
Nagakura+(2019),
Morinaga+(2019),
Azari+(2020)

NO:
e.g.,
Azari+(2019),
Tamborra+(2017)



The Fog of War

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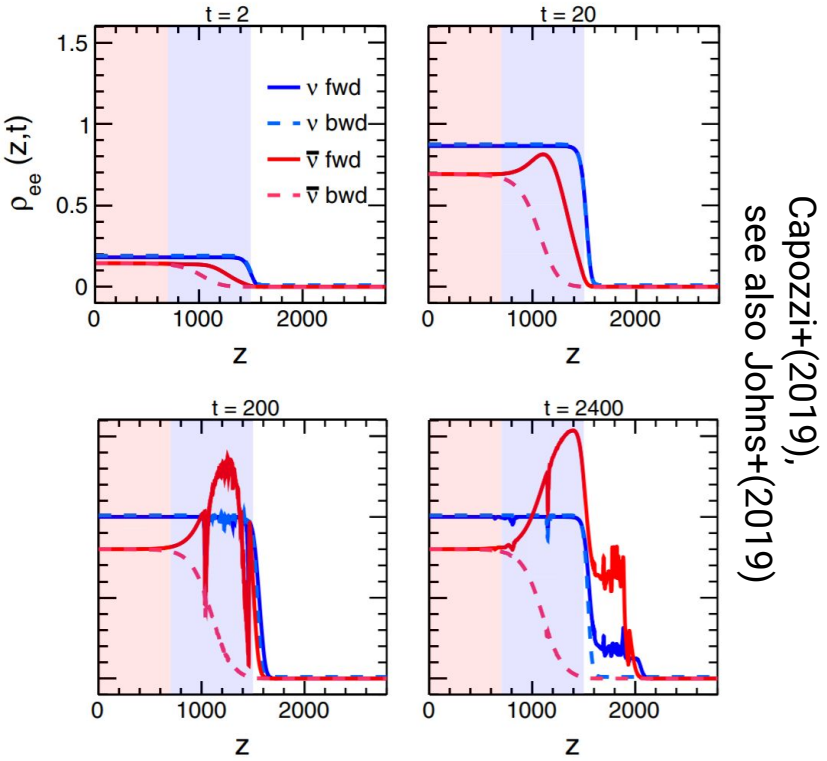
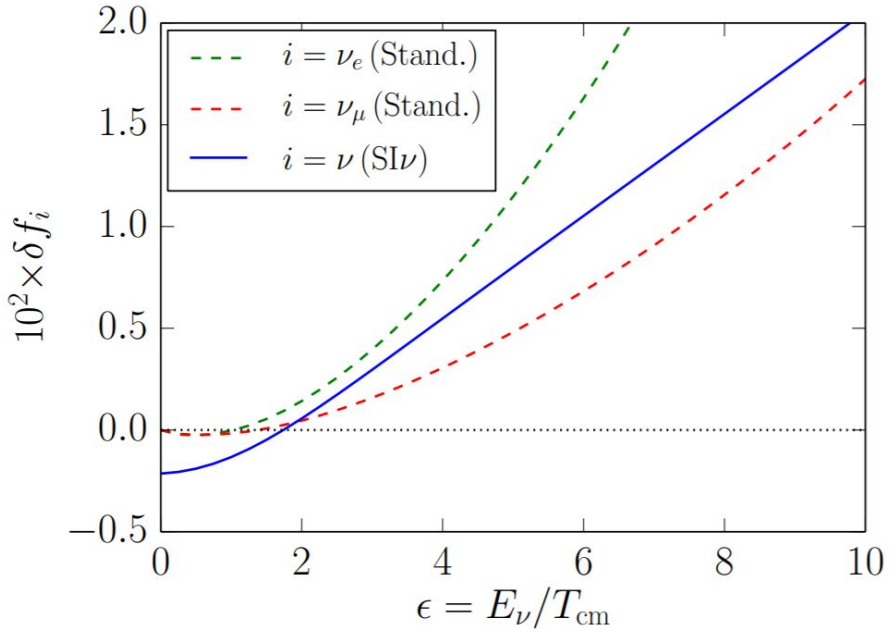


Capozzi+(2019), see also Johns+(2019)

The Fog of War

- Linear stability analysis
- Searching in simulation snapshots
- Simulating toy problems
- Simulating in reduced dimensions

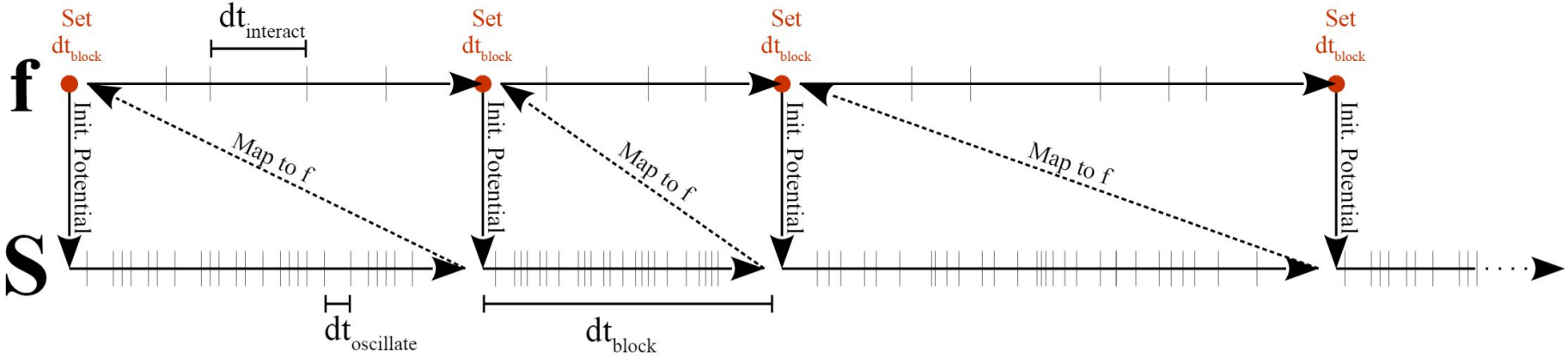
  (Early universe)



Grohs+(2020)

IsotropicSQA

github.com/srichers/IsotropicSQA



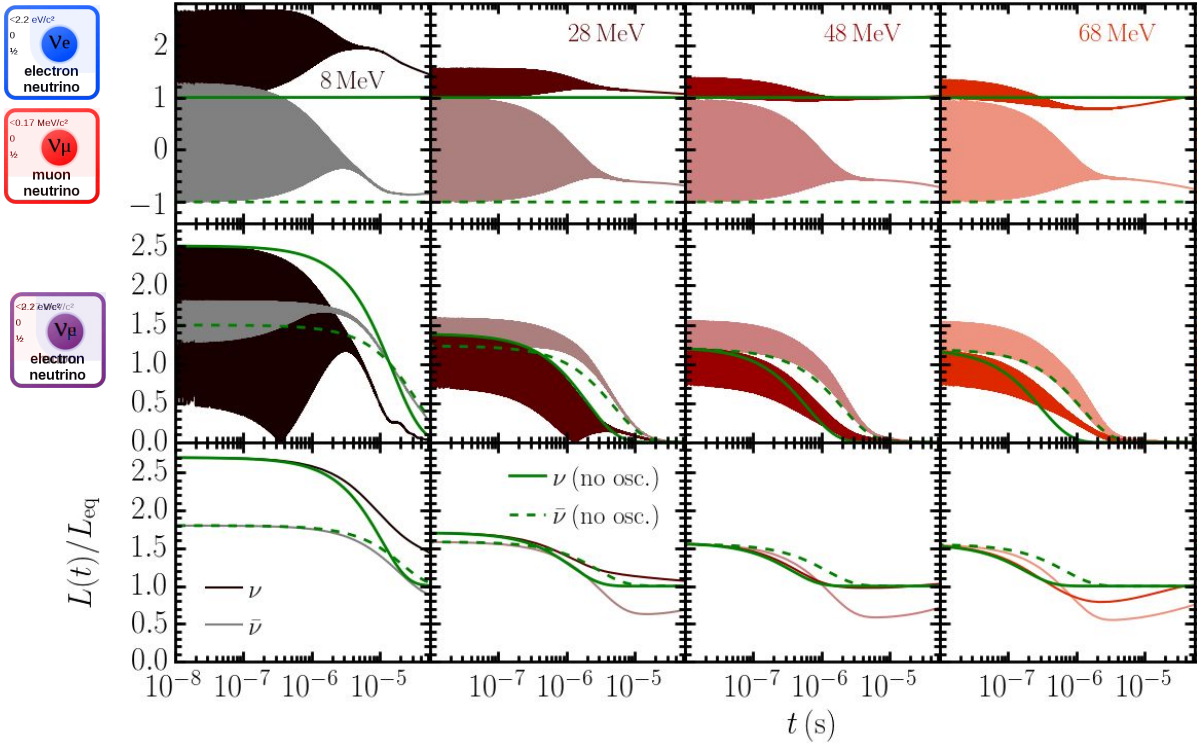
$$\frac{d\,\square}{d\lambda} = \mathcal{C}[\,\square\,] - \frac{i}{\hbar c}\,[\mathcal{H}, \square\,]$$

1) Evolve **oscillations only** for some “block” time
2) Evolve **interactions only** for same amount of “block” time
3) Check the impact of the interactions and adjust “block” time

Details:

- Must randomize block time to accurately sample distribution
- Must evolve oscillations with high accuracy (many many timesteps)

Everything Together

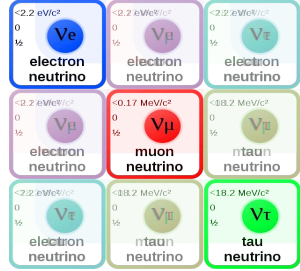


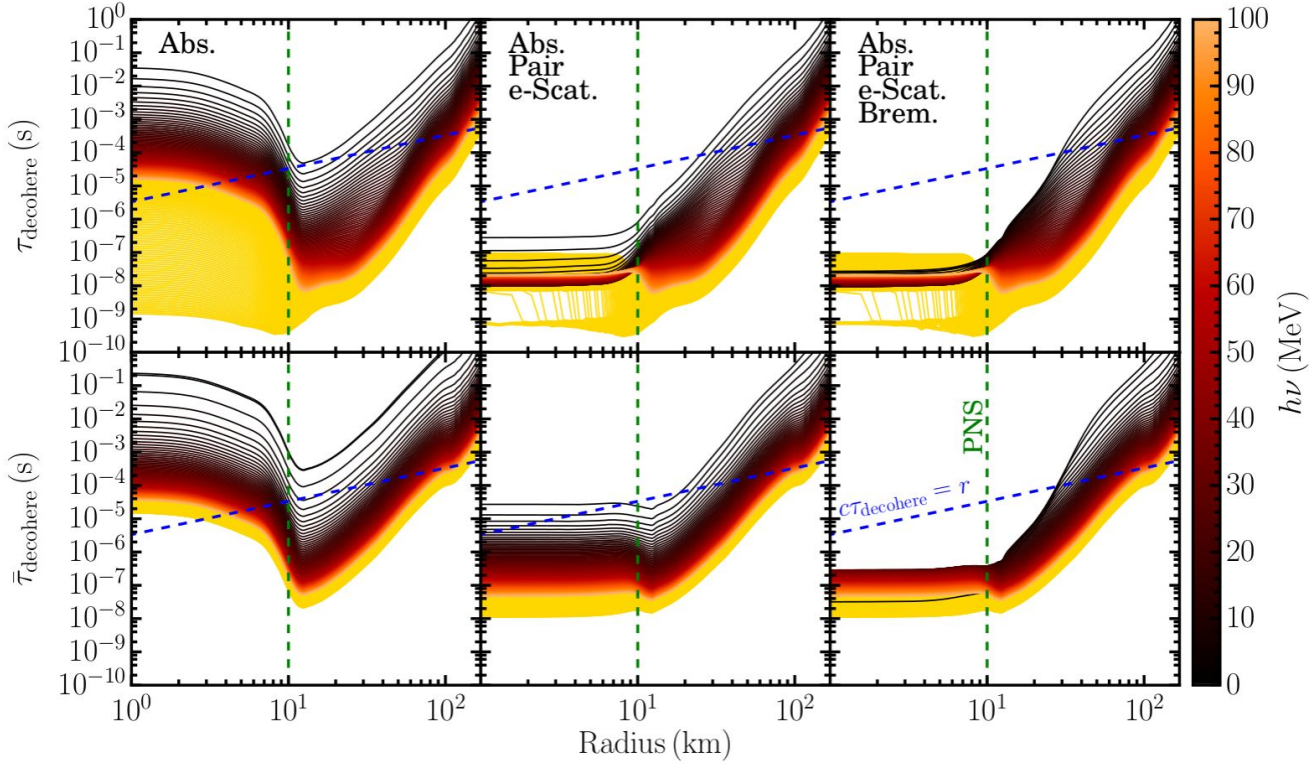
Decoherence with/without oscillations is **similar**

Separate timescales:

1) Damping oscillations
2) Relaxing to equilibrium

Many collision processes are important!





Core:

- Absorption/Emission
- Pair Processes
- Scattering
- Bremsstrahlung
- Neutrino-Neutrino

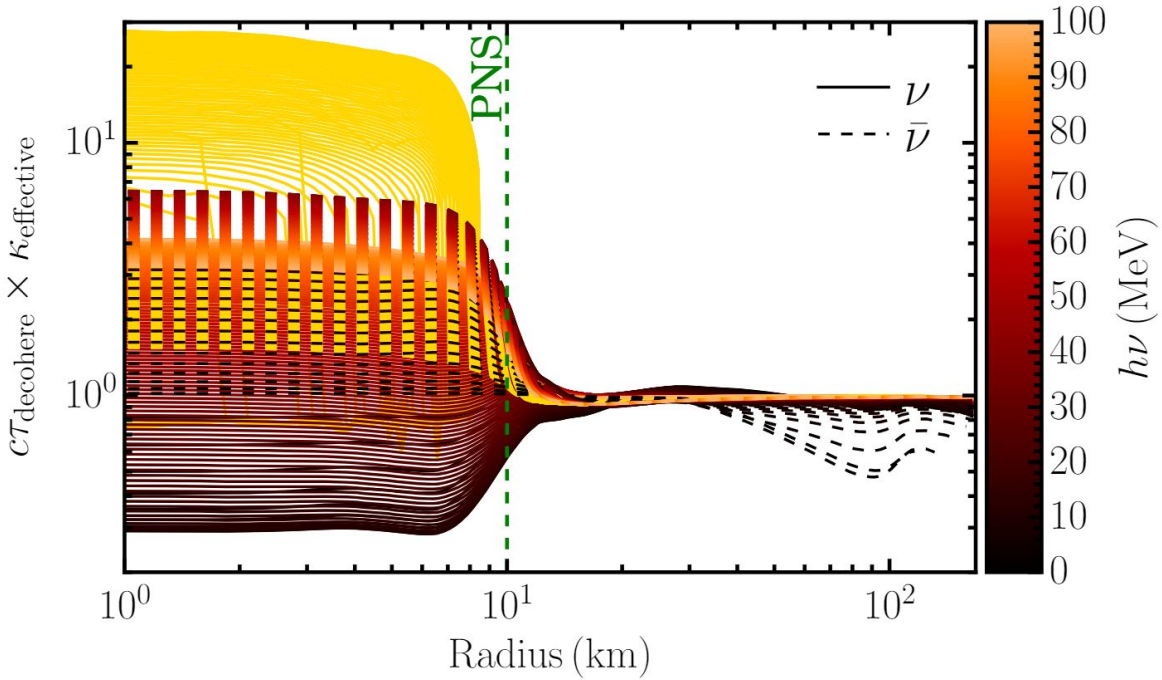
Heating Region:

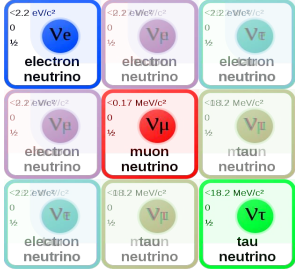
- Absorption/Emission

Simple “effective opacity”

$$\kappa_{\text{effective}} := \kappa_{\text{abs},e\mu} + \frac{1}{2}\widetilde{\kappa}_{\text{scat},e\mu}$$

More effective than
mean free path or
absorption opacity

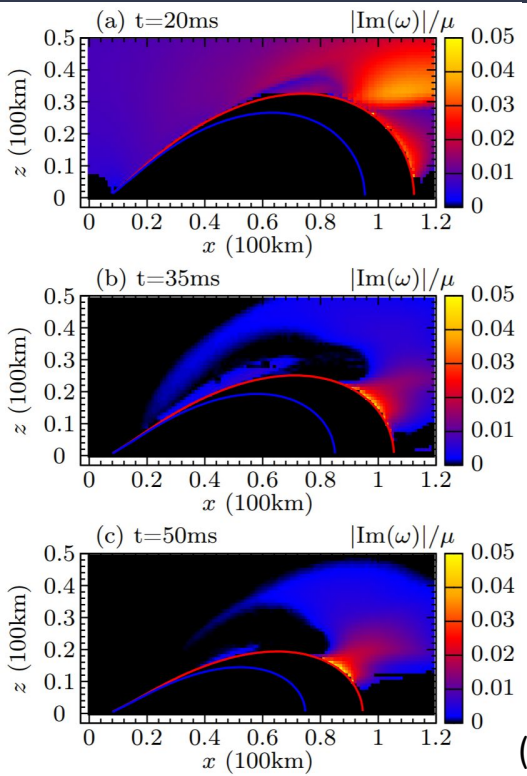




Conclusions

	Supernovae	Neutron Star Mergers
Standard Model	**Moments** work well in 1D (worse in multi-D) **Proof of concept** MC closure does little in 1D	**Analytic closures** are inadequate for merger simulations
Quantum Kinetics	Can use existing neutrino interaction physics to build **quantum collision terms**	

Supernovae and Mergers are Unstable! (since 2015)

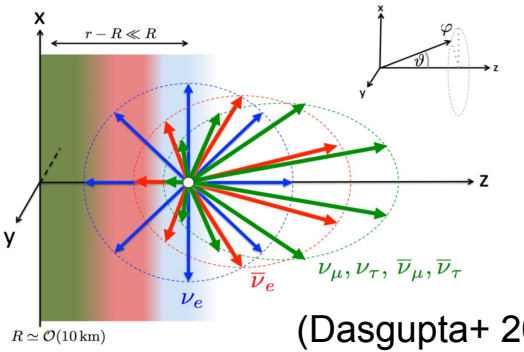


(Wu+ 2017)

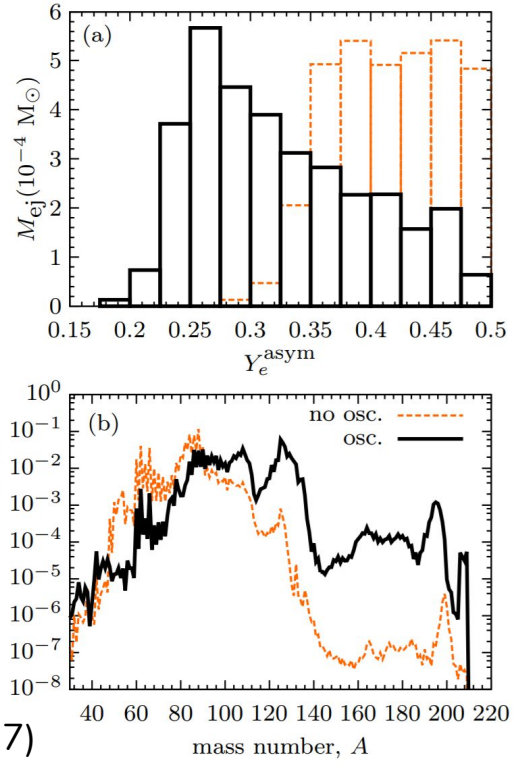
Oscillations

⬇

Less r-process

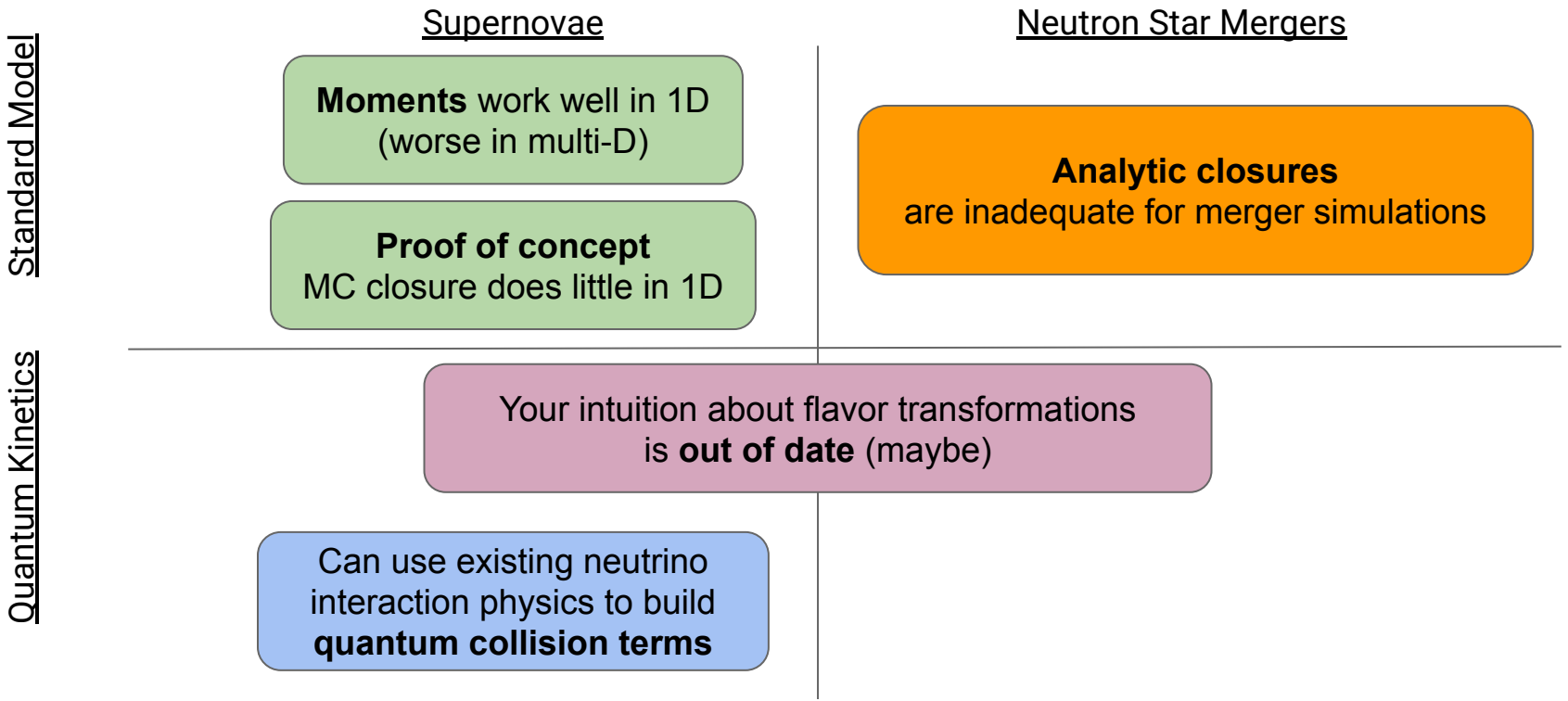


(Dasgupta+ 2017)



(Wu+ 2017)

Outline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Proof of concept
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Analytic closures
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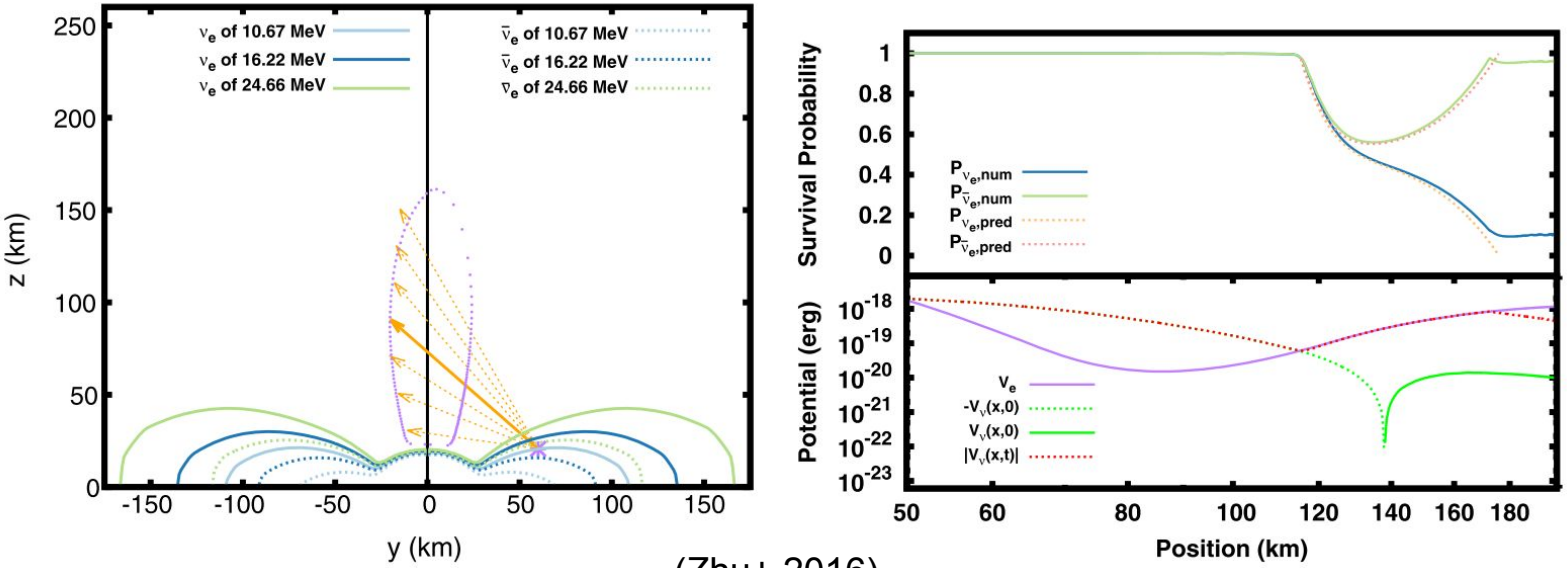
Your intuition about flavor transformations
is **out of date** (maybe)
Can use existing neutrino
interaction physics to build
quantum collision terms

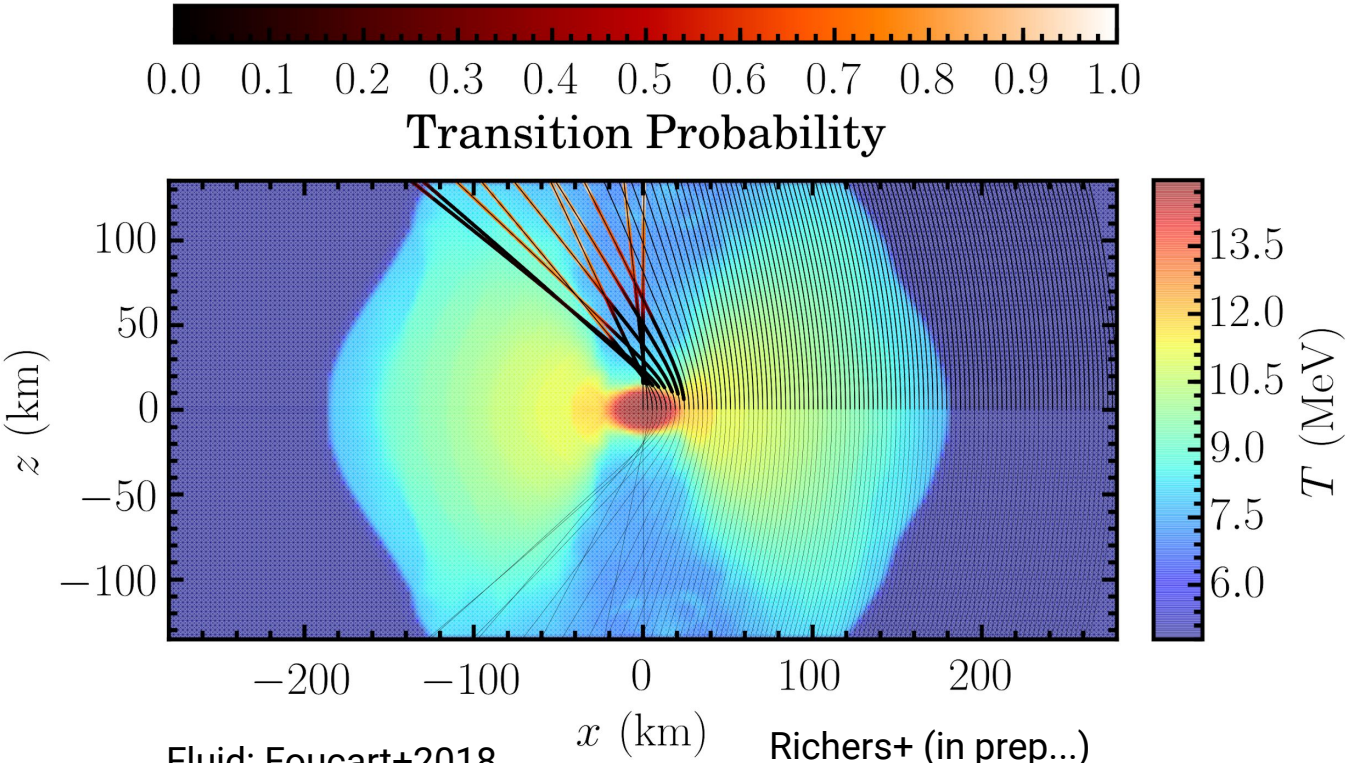
Pure oscillations in neutron star mergers



(Zhu+ 2016)

Neutrino-matter resonance efficiently transforms neutrinos.

Matter-Neutrino Resonance (3D GR)

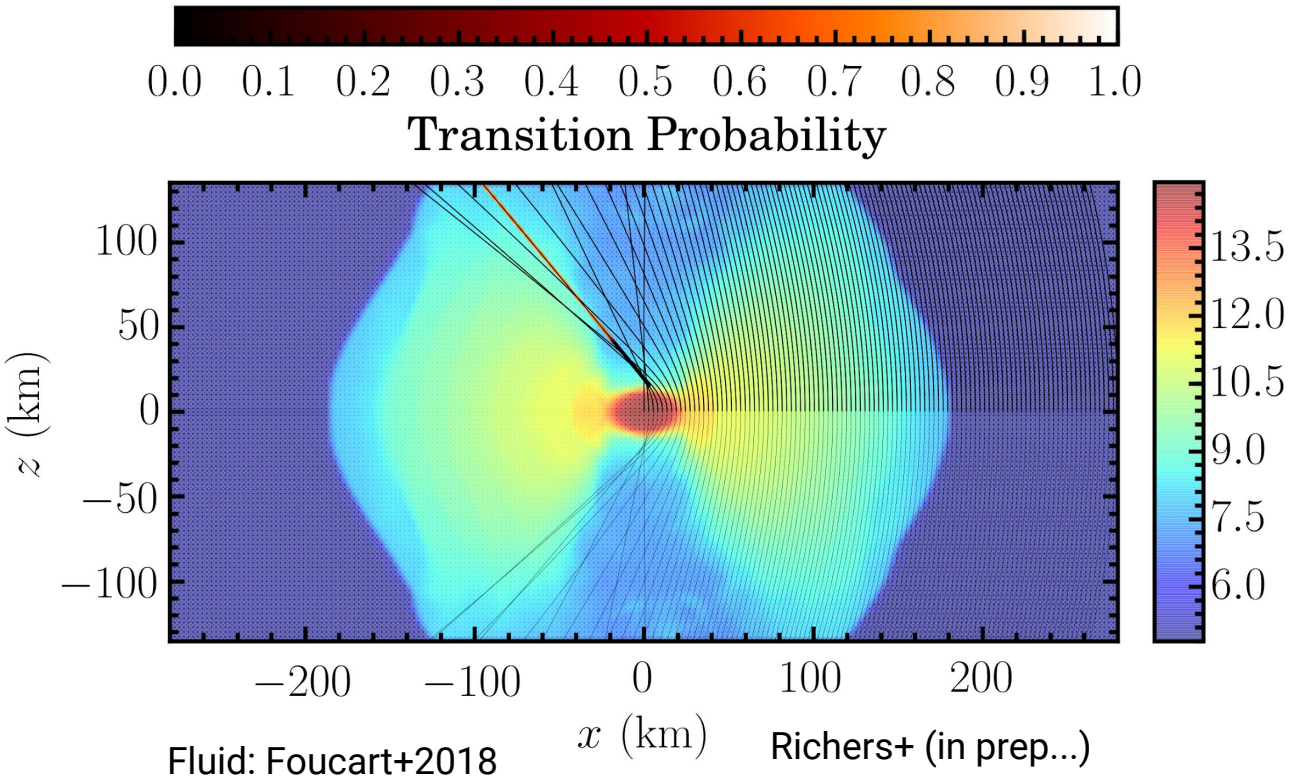


Fluid: Foucart+2018

Richers+ (in prep...)

The same process occurs in 3D and GR!

Simultaneous Oscillations and Collisions



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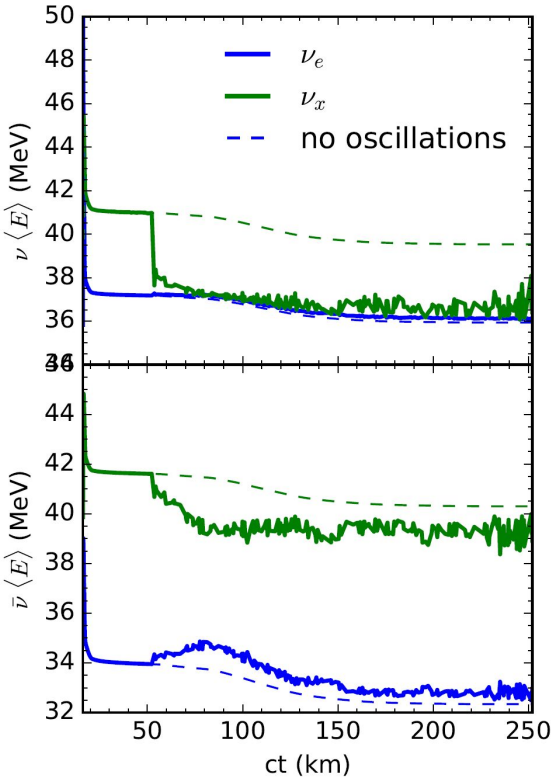
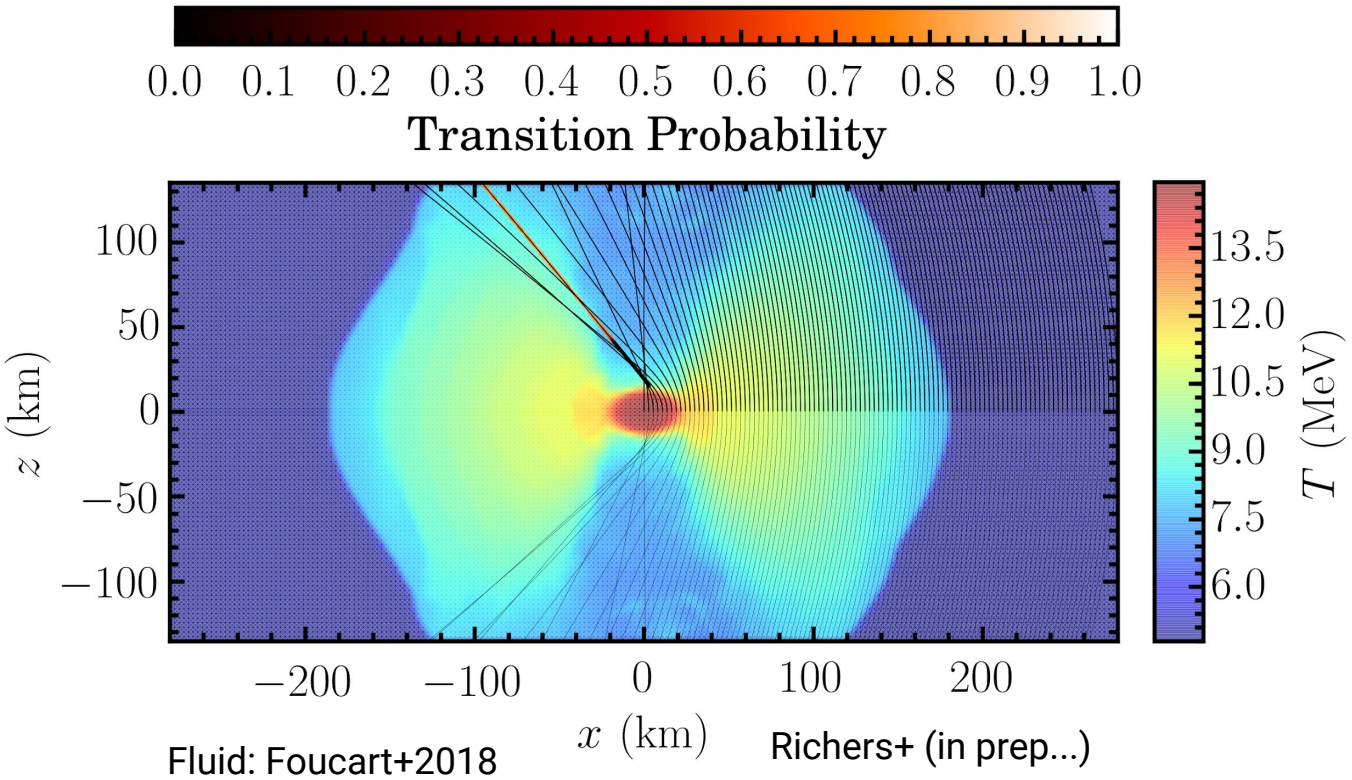
Richers+ (in prep...)



Simultaneous Oscillations and Collisions



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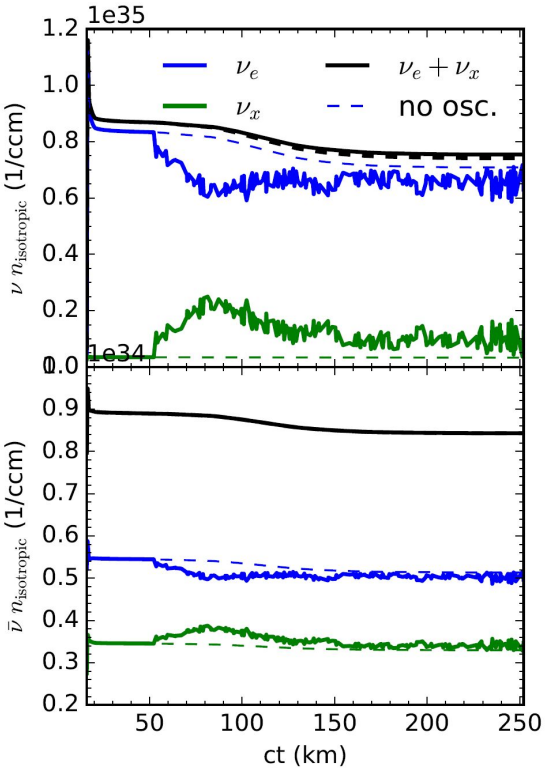

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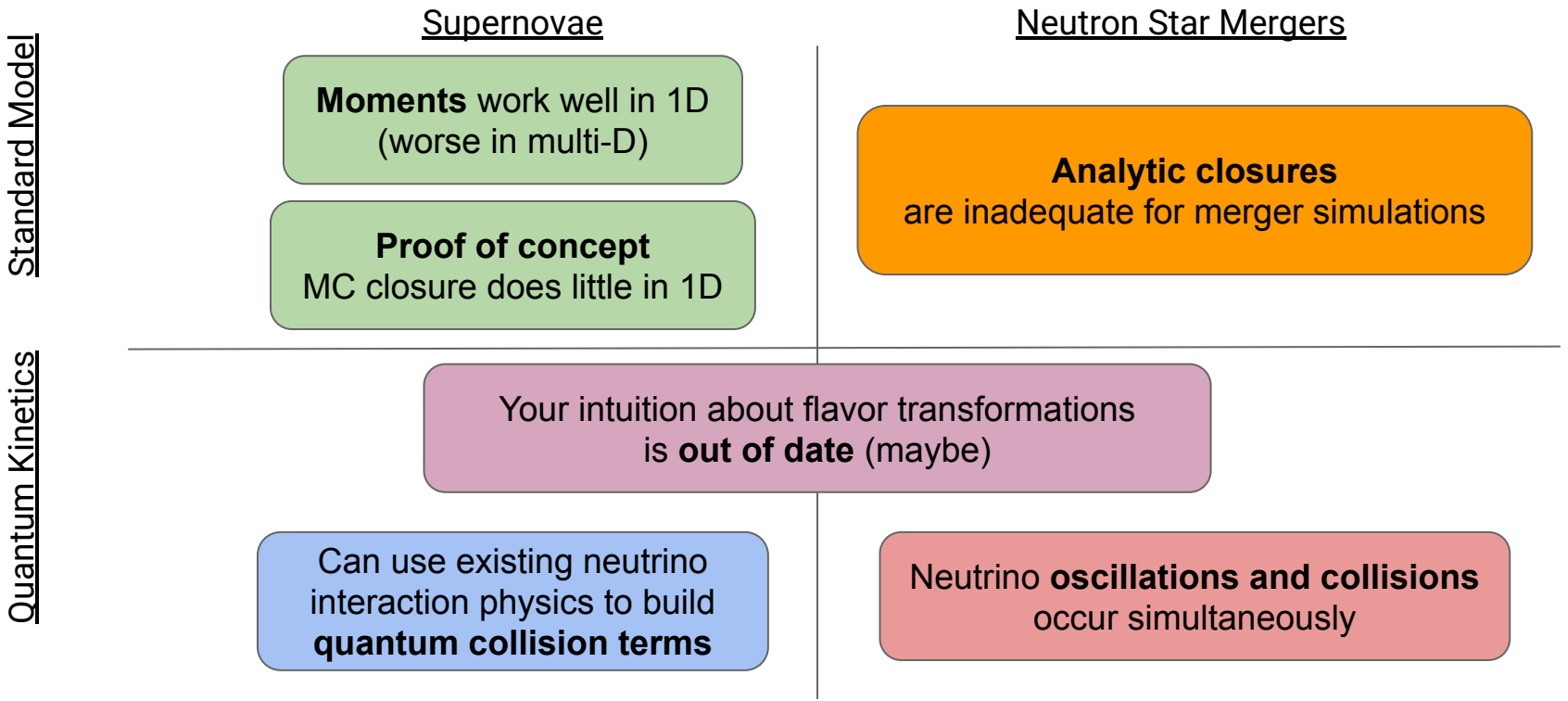
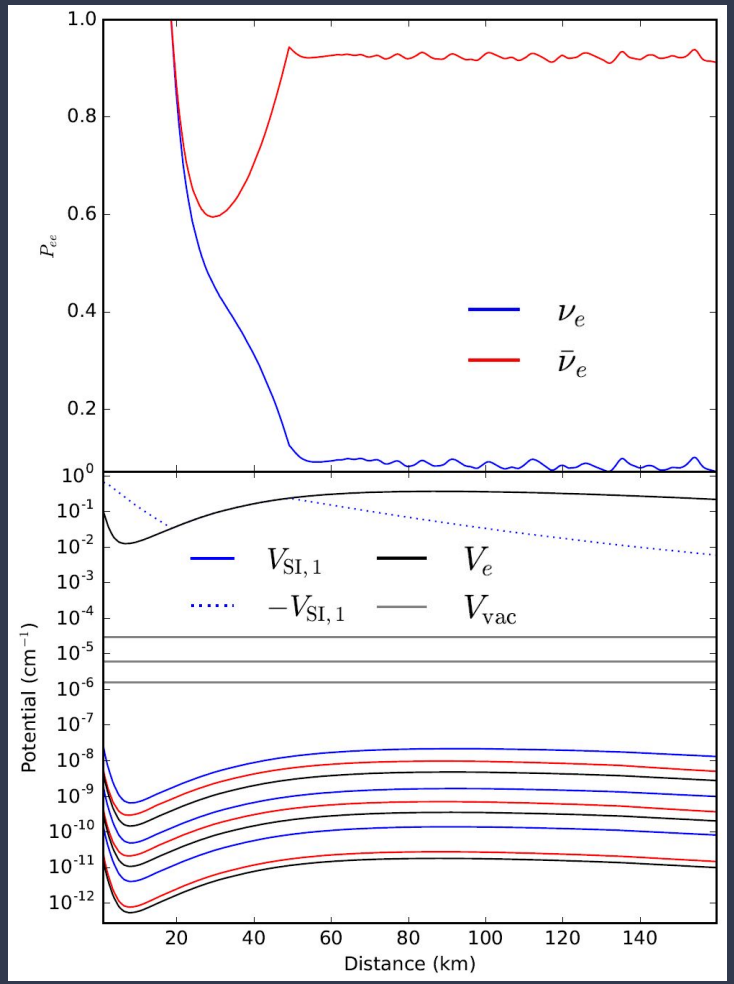
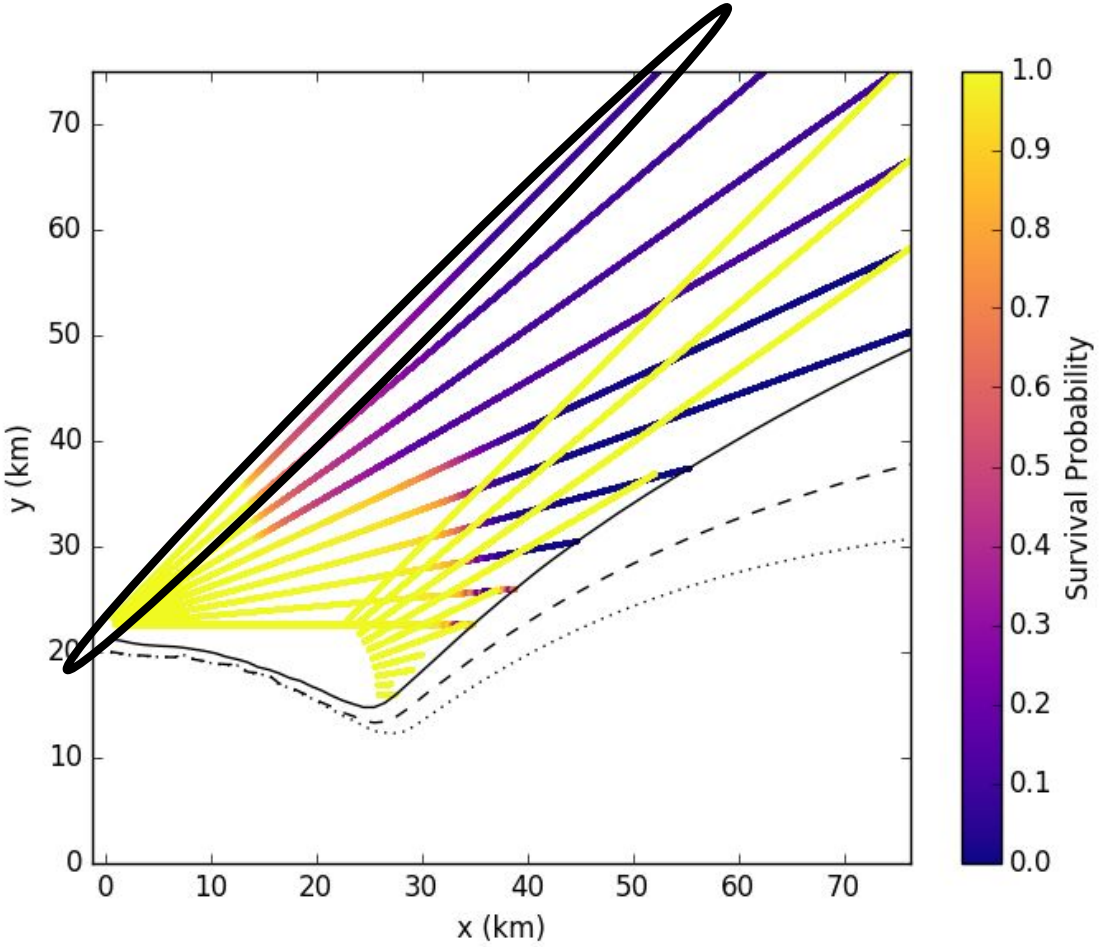
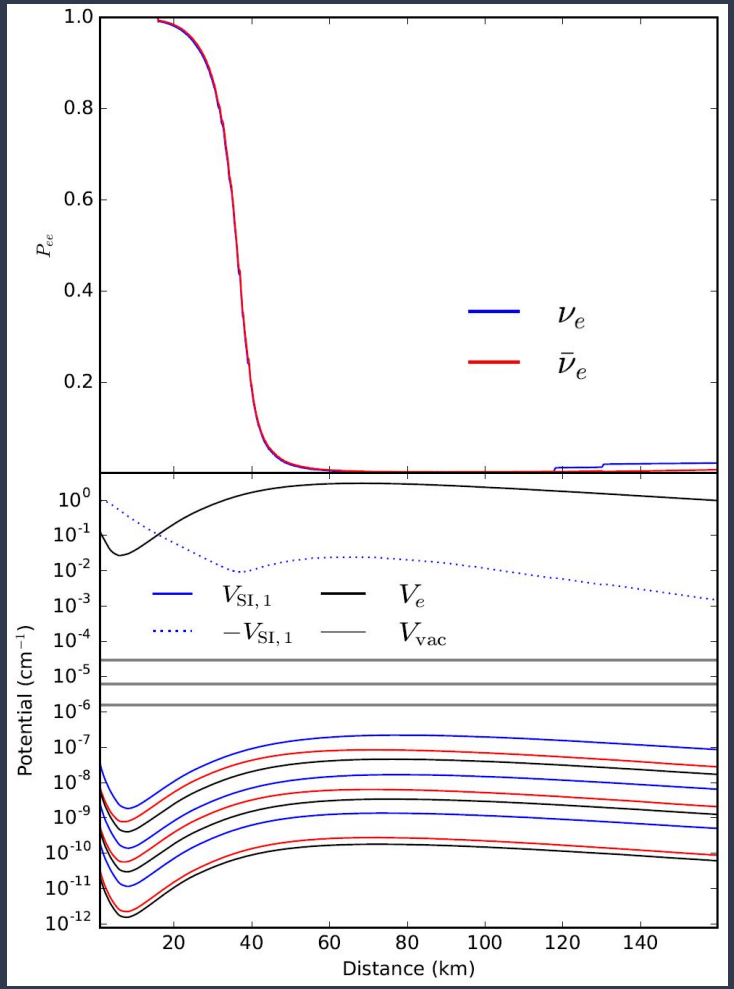

Supernovae
Neutron Star Mergers
Standard Model
Quantum Kinetics
Moments work well in 1D (worse in multi-D)
Proof of concept
MC closure does little in 1D
Analytic closures
are inadequate for merger simulations
Your intuition about flavor transformations is out of date (maybe)
Can use existing neutrino interaction physics to build quantum collision terms
Neutrino oscillations and collisions occur simultaneously

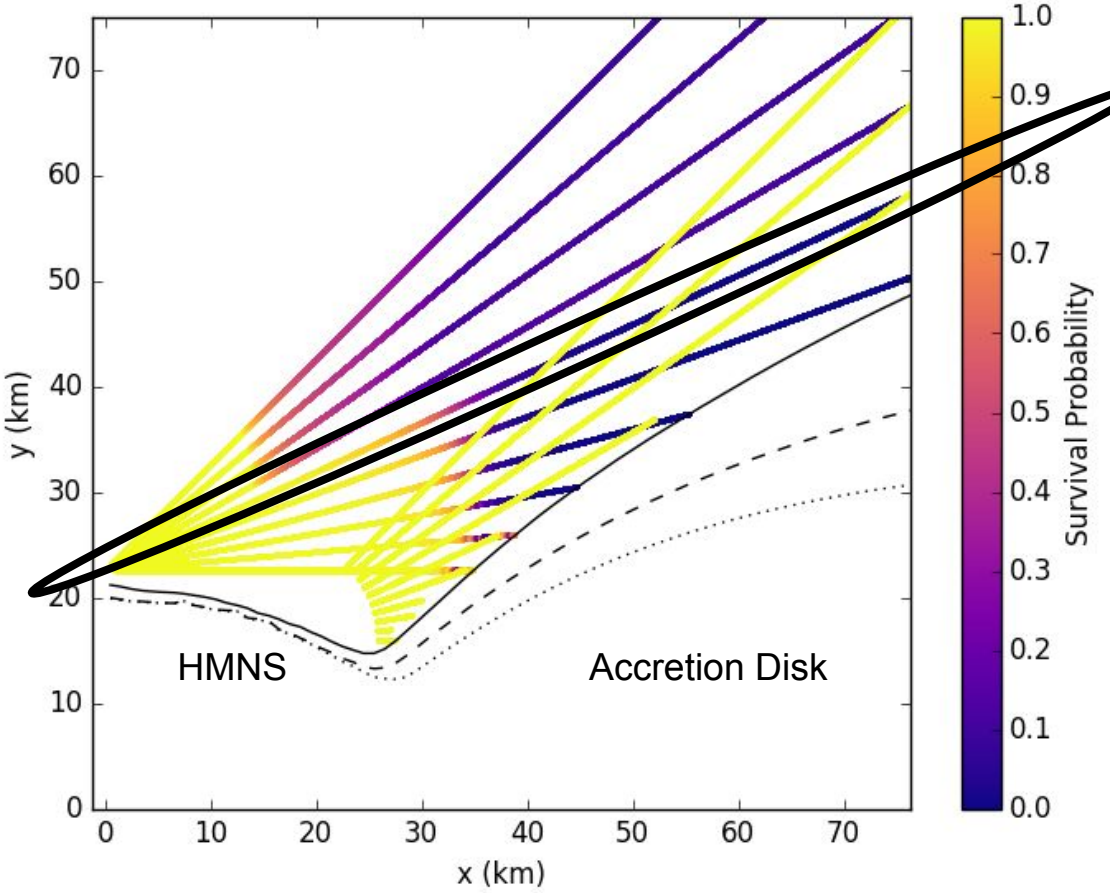


Matter-Neutrino Resonance!





Synchronized MSW



Monte Carlo Random Walk Approximation

