International conference “The Evolution of Massive Stars and Formation of Compact Stars: from the Cradle to the Grave” @ Waseda University, February 26–28, 2020

Collaborators:
Hiroki Nagakura (Princeton University)
Chinami Kato (Tohoku University)
Shoichi Yamada (Waseda University)

Taiki Morinaga (Waseda University)

COLLECTIVE NEUTRINO-FLAVOR CONVERSION IN THE PRE-SHOCK REGION OF CORE-COLLAPSE SUPERNOVA

Contents

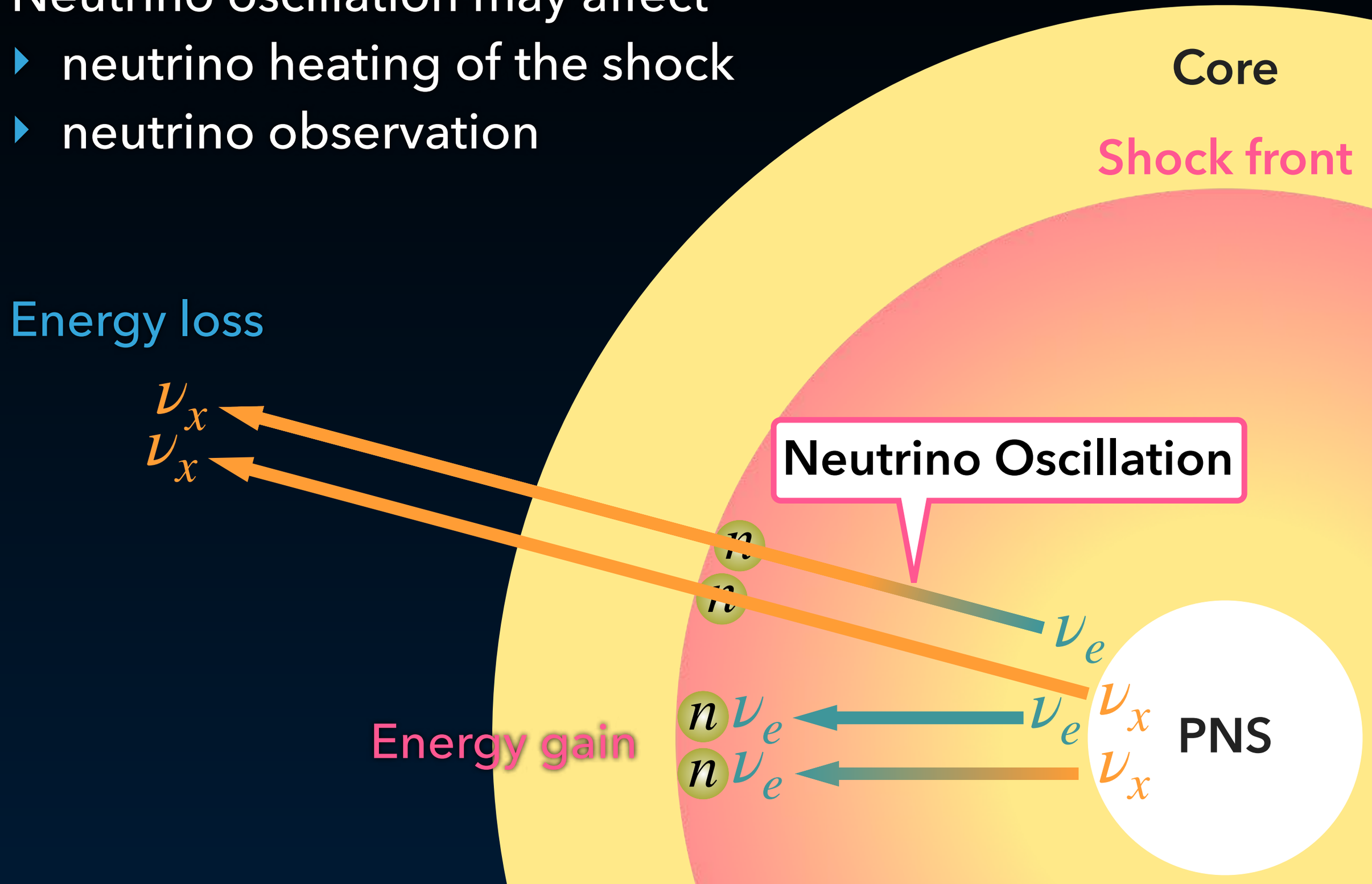
1. Collective neutrino oscillation in core-collapse supernova
2. Instability condition for collective neutrino oscillation
3. Flavor conversion in the pre-shock region of core-collapse supernova
4. Summary

Contents

1. Collective neutrino oscillation in core-collapse supernova
2. Instability condition for collective neutrino oscillation
3. Flavor conversion in the pre-shock region of core-collapse supernova
4. Summary

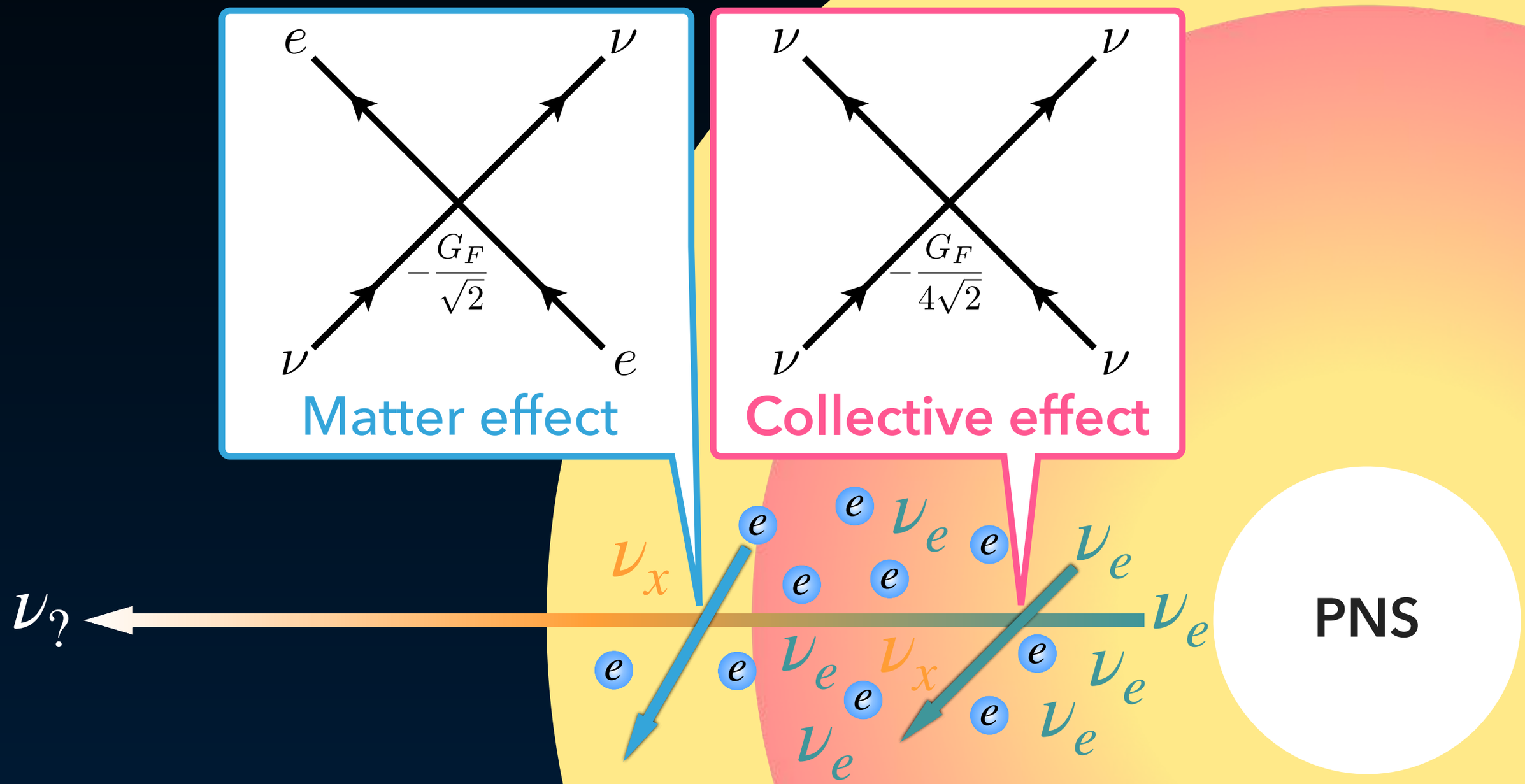
Core-collapse supernova and neutrino oscillation

- Neutrino oscillation may affect
  - neutrino heating of the shock
  - neutrino observation



Neutrino oscillation in medium

- Dense neutrinos inside the core modify the neutrino oscillations to nonlinear phenomena



Neutrino oscillation in medium

- The kinetic equation governs the time evolution of "distribution function matrix" f
  - The diagonal component $f_{\alpha\alpha}$ is the distribution function of ν_α

$$v \cdot \partial \mathsf{f}(x,\Gamma) = -i[\mathsf{H}(x,\Gamma), \mathsf{f}(x,\Gamma)] + \underline{\mathcal{C}[\mathsf{f}]}$$

Collision term

$$\mathsf{H}(x,\Gamma) \equiv \underline{\frac{\mathsf{M}^2}{2E}} + v \cdot \Lambda(x)$$

Vacuum mass

$$\Lambda^\mu(x) \equiv \sqrt{2} G_F \left[\underline{\begin{pmatrix} j_e^\mu(x) & 0 & 0 \\ 0 & j_\mu^\mu(x) & 0 \\ 0 & 0 & j_\tau^\mu(x) \end{pmatrix}} + \underline{\int d\Gamma \mathsf{f}(x,\Gamma) v^\mu} \right]$$

Matter effect　　Collective effect

Contents

1. Collective neutrino oscillation in core-collapse supernova
2. Instability condition for collective neutrino oscillation
3. Flavor conversion in the pre-shock region of core-collapse supernova
4. Summary

Linear stability analysis

I. Izaguirre, G. Raffelt, and I. Tamborra, Phys. Rev. Lett. **118**, 021101 (2017)
M. Chakraborty and S. Chakraborty, J. Cosmol. Astropart. Phys. 01 (2020) 005

- We work in the framework of 2-flavor mixing
  - Even if we consider 3-flavor mixing, the linearized equation is decomposed into 3 independent equations for 2-flavor mixing

Kinetic equation: $v \cdot \partial \mathsf{f}(x,\Gamma) = -i[\mathsf{H}(x,\Gamma), \mathsf{f}(x,\Gamma)] + \mathcal{C}[\mathsf{f}]$

$$\mathsf{H}(x,\Gamma) \equiv \frac{\mathsf{M}^2}{2E} + v \cdot \Lambda(x)$$

2-flavor

$$\mathsf{M}^2 = \mathsf{U} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \mathsf{U}^\dagger,\ \mathsf{U} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} e^{i\delta\sigma_3}$$

$$\Lambda^\mu = \sqrt{2} G_F \left[\begin{pmatrix} j_e^\mu & 0 \\ 0 & j_x^\mu \end{pmatrix} + \int d\Gamma \rho(x,\Gamma) v^\mu \right]$$

Linear stability analysis

- We neglect the off-diagonal components of M^2 and the collision term $\mathcal{C}[\mathrm{f}]$
  - Under this assumption, the flavor eigenstate

$$\mathrm{f}(x,\Gamma) = \begin{pmatrix} f_{\nu_e}(\Gamma) & 0 \\ 0 & f_{\nu_x}(\Gamma) \end{pmatrix}$$

    is a solution of the kinetic equation
- We consider a small perturbation from the flavor eigenstate

$$\mathrm{f}(x,\Gamma) = \begin{pmatrix} f_{\nu_e}(\Gamma) & 0 \\ 0 & f_{\nu_x}(\Gamma) \end{pmatrix} + \frac{f_c(\Gamma)}{2} \begin{pmatrix} 0 & \bar{S}(x,\Gamma) \\ S(x,\Gamma) & 0 \end{pmatrix}$$

Perturbation

$$f_c(\Gamma) \equiv f_{\nu_e}(\Gamma) - f_{\nu_x}(\Gamma)$$

Linear stability analysis

- Linearized equation for the perturbation S

$$[v \cdot \{i\partial - \Lambda_c(x)\} - \omega_E]\, S(x,\Gamma) + \sqrt{2}G_F \int d\Gamma' f_c(\Gamma') v \cdot v' S(x,\Gamma') = 0$$

$$\left(\begin{array}{r} \omega_E \equiv \frac{m_1^2 - m_2^2}{2E} \cos 2\theta \\ \Lambda_c^\mu(x) \equiv \sqrt{2}G_F \left[j_e^\mu(x) - j_x^\mu(x) + \int d\Gamma\, f_c(\Gamma) v^\mu \right] \end{array} \right)$$

Plane wave ansatz $S(x,\Gamma) = Q(k,\Gamma) e^{i(k+\Lambda_c)\cdot x}$

- Non-trivial solution exists if k satisfies the following dispersion relation

$$\det \Pi(k) = 0$$

$$\Pi^{\mu\nu}(k) \equiv \eta^{\mu\nu} + \sqrt{2}G_F \int d\Gamma\, f_c(\Gamma) \frac{v^\mu v^\nu}{v \cdot k - \omega_E}$$

Fast flavor conversion

- In a sufficiently dense neutrino gas, ω_E can be neglected
  - Flavor conversion under this assumption is called fast flavor conversion

$$\Pi^{\mu\nu}(k) = \eta^{\mu\nu} + \sqrt{2}G_F \int d\Gamma f_c(\Gamma) \frac{v^\mu v^\nu}{v\cdot k - \omega_E}$$

$$\simeq \eta^{\mu\nu} + \int \frac{d^2\boldsymbol{v}}{4\pi} \underline{\mathscr{G}(\boldsymbol{v})} \frac{v^\mu v^\nu}{v\cdot k}$$

- Dispersion relation depends on $\mathscr{G}(\boldsymbol{v})$, which can be regarded as an electron lepton number (ELN) angular distribution carried by neutrinos

$$\mathscr{G}(\boldsymbol{v}) \equiv \sqrt{2}G_F \int_{-\infty}^{\infty} \frac{dE E^2}{2\pi^2} f_c(\Gamma)$$

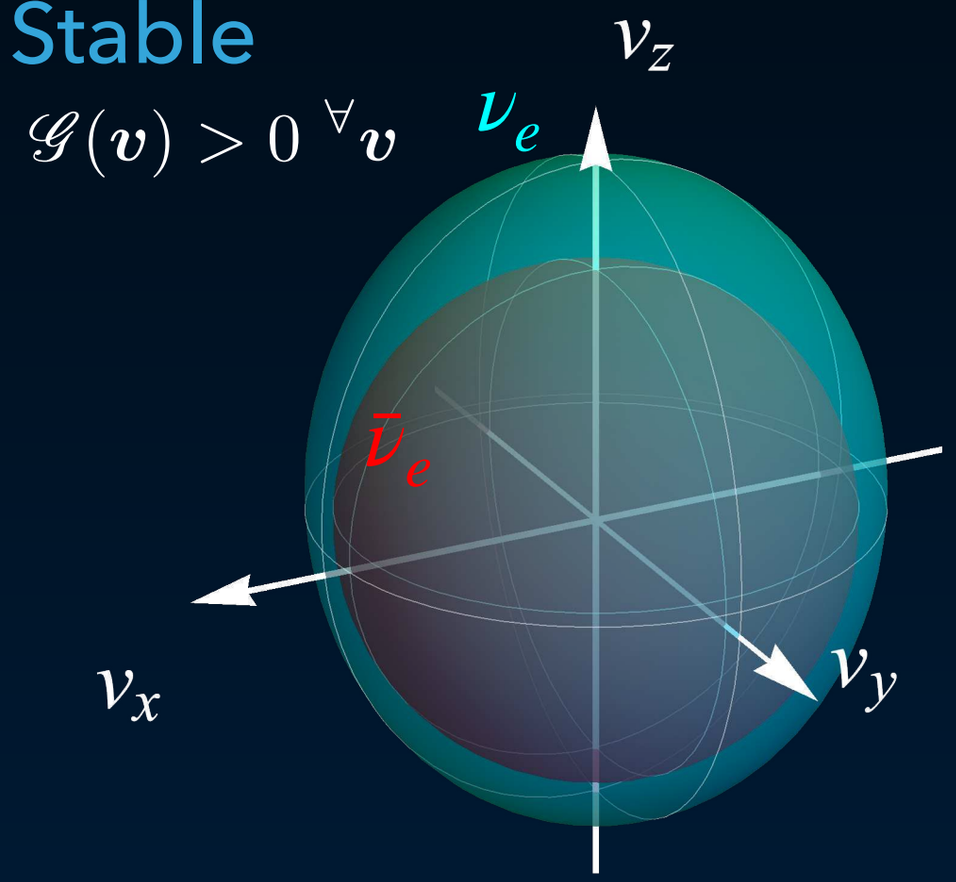
$$\simeq \sqrt{2}G_F \int_{0}^{\infty} \frac{dE E^2}{2\pi^2} \left[f_{\nu_e}(E,\boldsymbol{v}) - f_{\bar{\nu}_e}(E,\boldsymbol{v})\right]$$

Instability condition

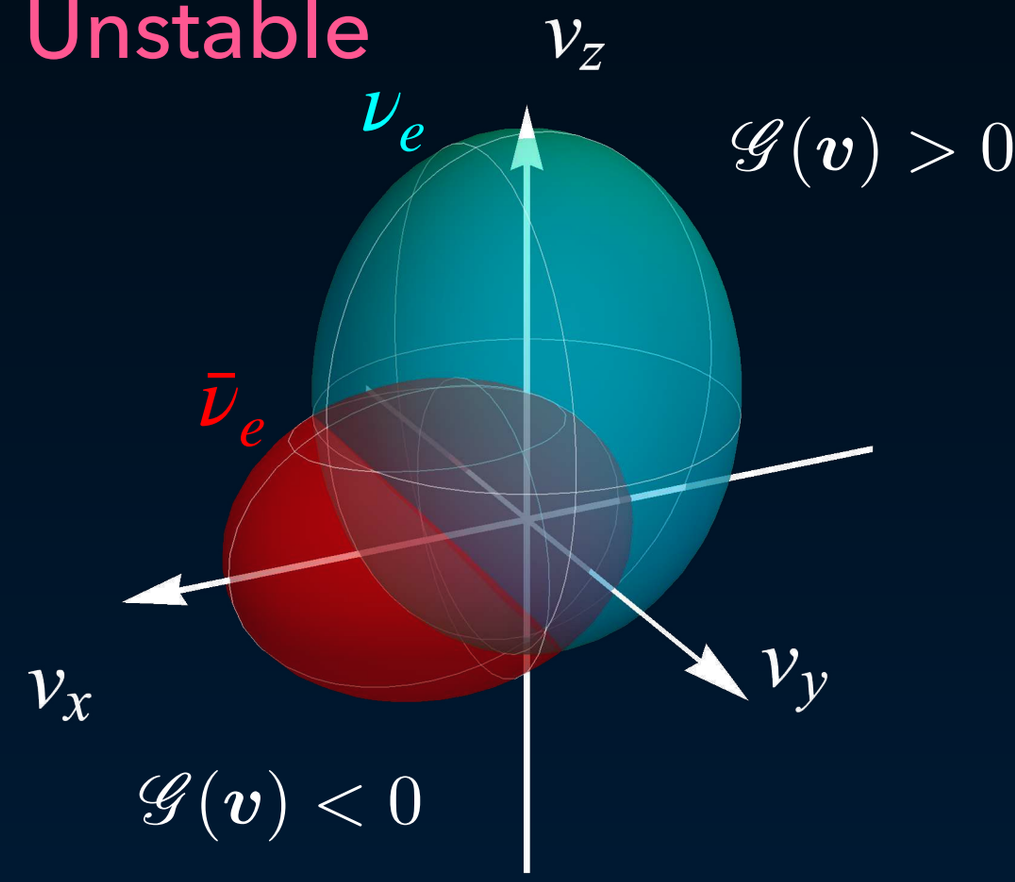
$$S(x,\Gamma) = Q(k,\Gamma)e^{i(k+\Lambda_c)\cdot x}$$

- If there exists a mode with $\mathrm{Im}\,\omega(\boldsymbol{k}) > 0$ and $\boldsymbol{k} \in \mathbb{R}^3$, the flavor eigenstate is unstable, implying flavor conversions can occur
- It is empirically known that a flavor eigenstate is unstable if the ELN angular distribution crosses with 0

Stable



Unstable



Contents

1. Collective neutrino oscillation in core-collapse supernova

2. Instability condition for collective neutrino oscillation

3. Flavor conversion in the pre-shock region of core-collapse supernova

4. Summary

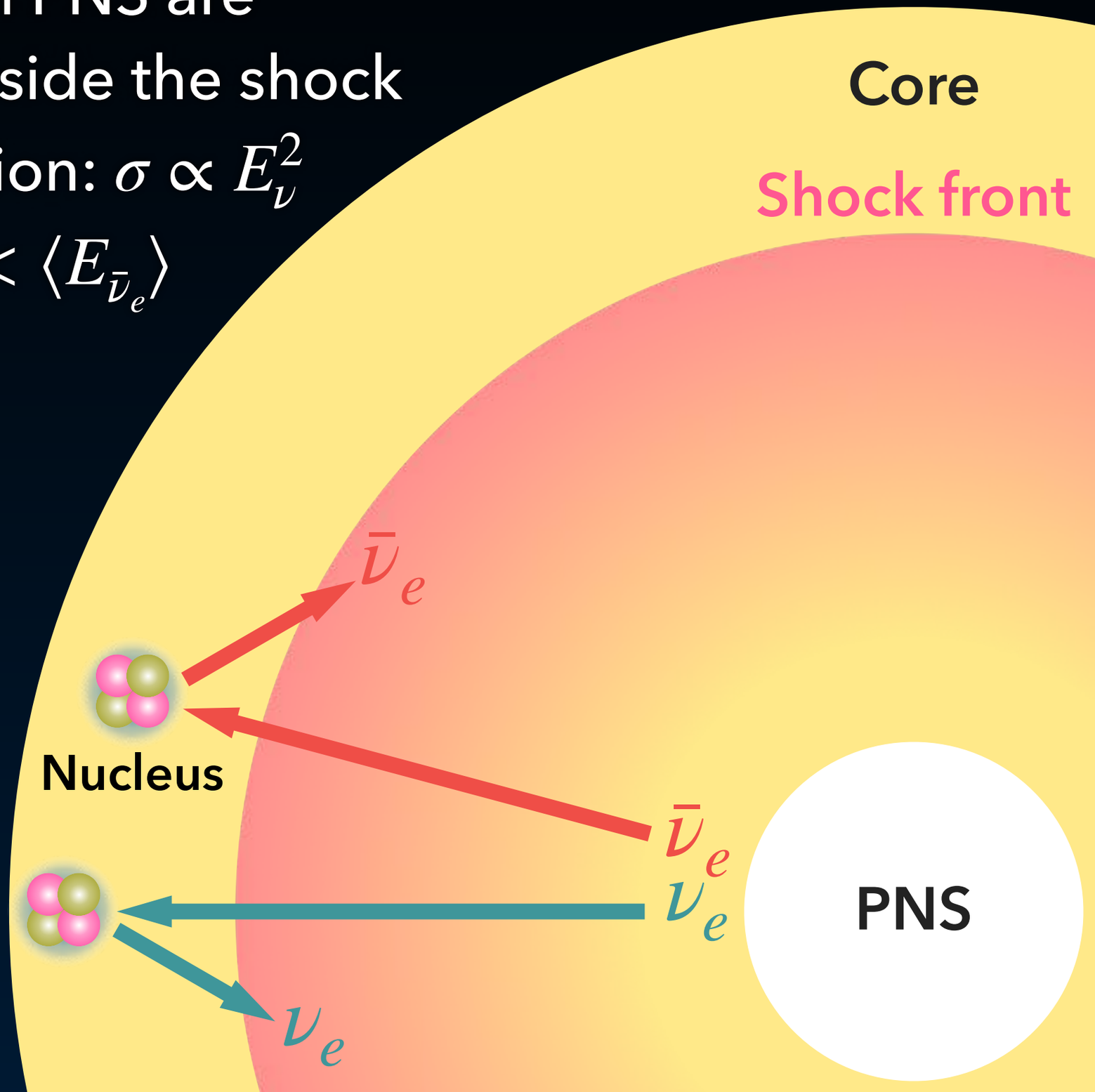
Coherent neutrino–nucleus scattering

T. Morinaga, H. Nagakura, C. Kato, and S. Yamada, Phys. Rev. Research **2**, 012046(R) (2020).

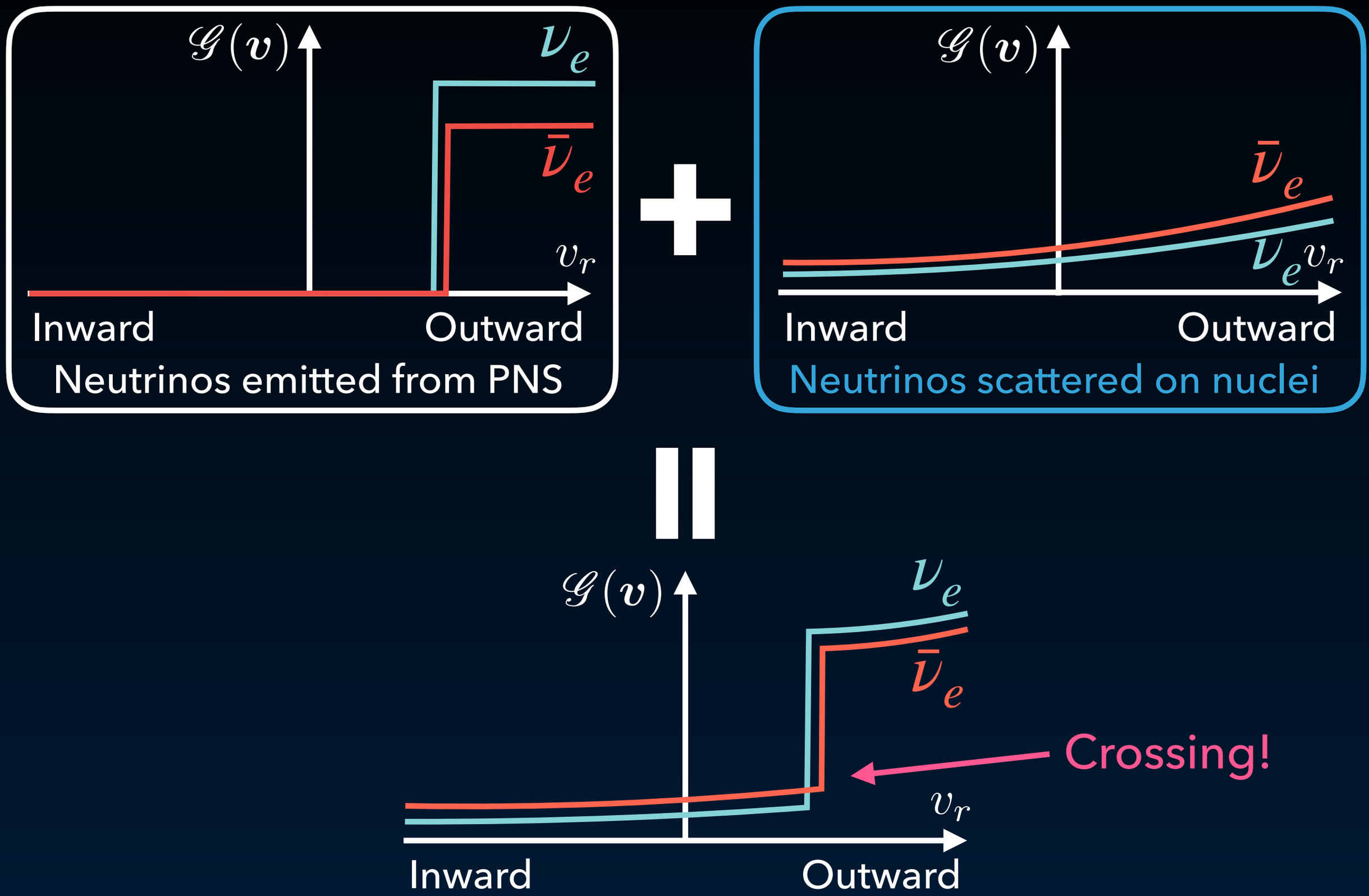
Neutrinos emitted from PNS are scattered on nuclei outside the shock

- Scattering cross section: $\sigma \propto E_\nu^2$
- Mean energy: $\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle$

More $\bar{\nu}_e$ are scattered than ν_e

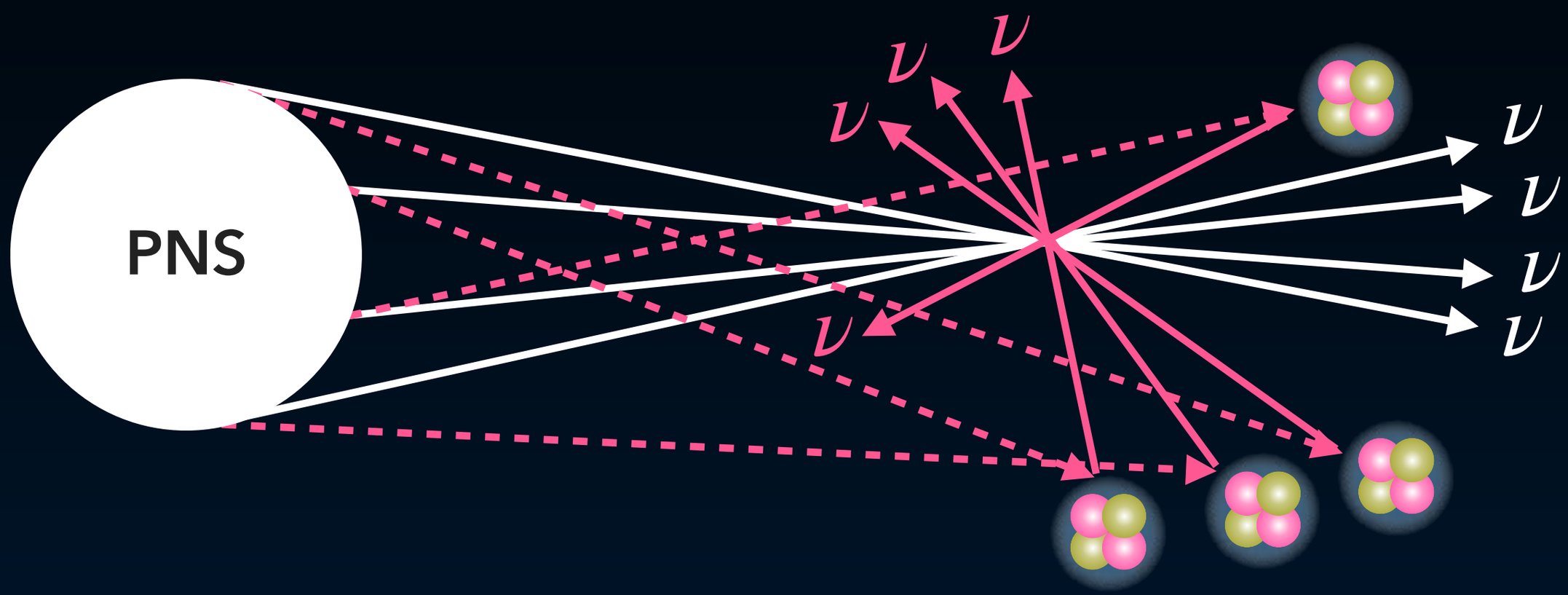


ELN crossing



Bulb model with the coherent scattering

- We consider the neutrino bulb model with the coherent scattering to evaluate the growth rate concretely



Bulb model + Scatterings

Bulb model with the coherent scattering

- ELN angular distributions of direct neutrinos

$$\mathscr{G}^{\rm bulb}(v_r) \sim \underline{10^{-1}\,{\rm cm}^{-1}}\,\Theta\left(v_r - \sqrt{1-(R_\nu/r)^2}\right)$$

- ELN angular distributions of scattered neutrinos

$$\mathscr{G}^{\rm scat}(v_r) \sim \underline{-10^{-6}\,{\rm cm}^{-1}}(v_r+1) + O\left((v_r+1)^2\right)$$

Empirical law

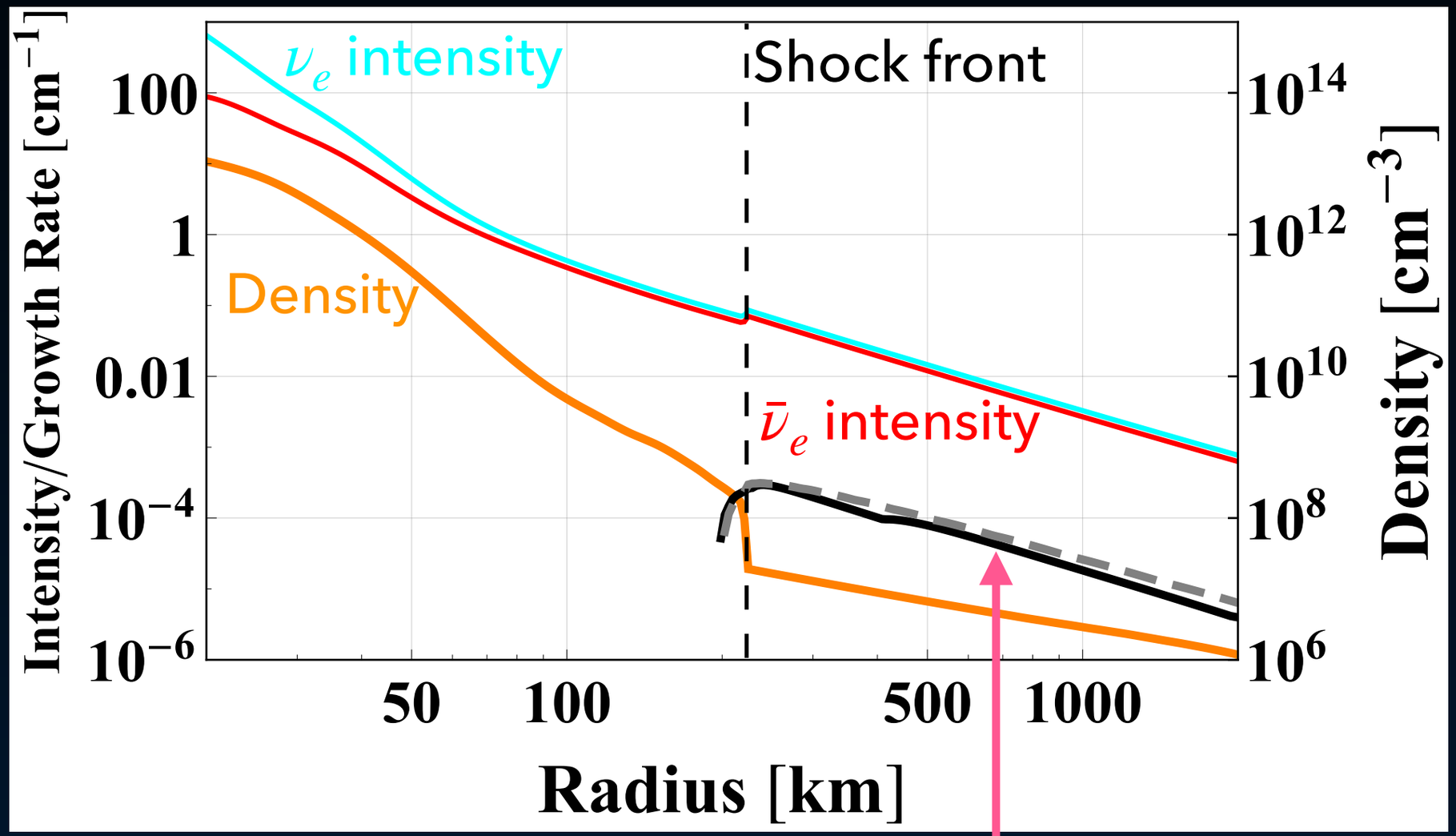
Growth rate: $\sigma \simeq \sqrt{-\left(\int_{\mathscr{G}(\boldsymbol{v})>0}\frac{d^2\boldsymbol{v}}{4\pi}\mathscr{G}(\boldsymbol{v})\right)\left(\int_{\mathscr{G}(\boldsymbol{v})<0}\frac{d^2\boldsymbol{v}}{4\pi}\mathscr{G}(\boldsymbol{v})\right)}$

$$\gtrsim \sqrt{-(10^{-1}\,{\rm cm}^{-1})(-10^{-6}\,{\rm cm}^{-1})} \gtrsim \underline{\frac{1}{100\,{\rm m}}}$$

Large enough for perturbations to develop in supernovae

Realistic model

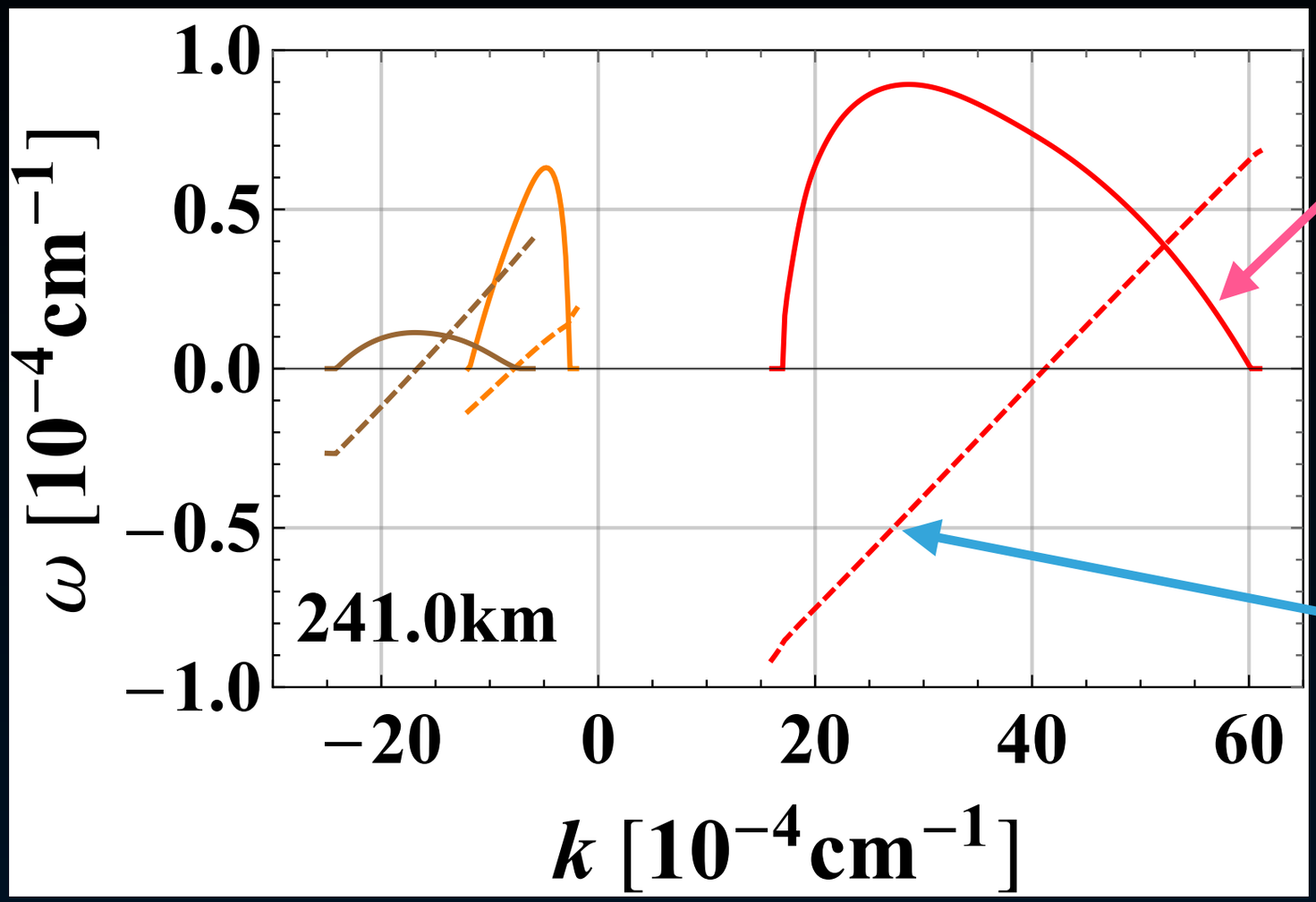
- We also performed a 1D supernova simulation with full Boltzmann neutrino transport without neutrino oscillation



(100ms after bounce)

Flavor conversion occurs outside the shock

Dispersion relation



Dispersion relation at 241km

Imaginary part

Growth rate

$$\sigma \sim 10^{-4}\,\mathrm{cm}^{-1} = \frac{1}{100\,\mathrm{m}}$$

Real part ($\times 0.05$)

Group velocity

$$v_g = \frac{d\,\mathrm{Re}\,\omega}{dk} \sim 0.7c > 0$$

Flavor conversions occur only in the outward direction

→ This conversion can affect { ✗ ~~neutrino heating~~ ; ✓ neutrino observation }

Contents

1. Collective neutrino oscillation in core-collapse supernova
2. Instability condition for collective neutrino oscillation
3. Flavor conversion in the pre-shock region of core-collapse supernova
4. Summary

Summary

- Linear analysis shows that the collective neutrino oscillations can be induced by the neutrino-nucleus scatterings in the pre-shock regions of core-collapse supernovae
- This instability grows towards the radially-outward direction and may affect neutrino observations

How are the neutrino signals from supernovae changed?



Nonlinear evolutions should be investigated!