The Evolution of Massive Stars and Formation of Compact Stars: from the Cradle to the Grave

The Core-Collapse Supernova Simulations with the Full Boltzmann Neutrino Transport in Three-Dimensional Space

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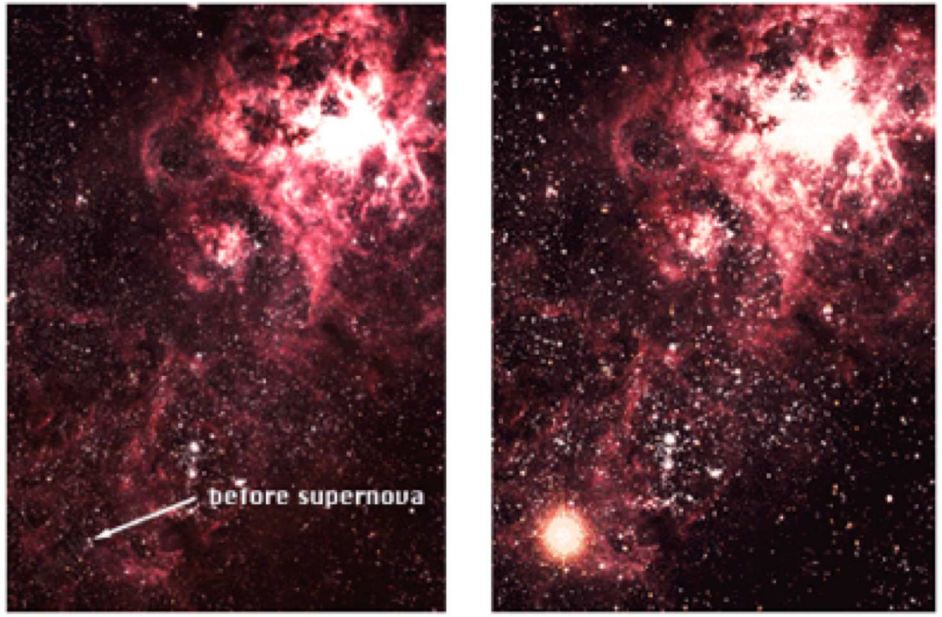

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CORE-COLLAPSE SUPERNOVAE

Explosion of Massive Star

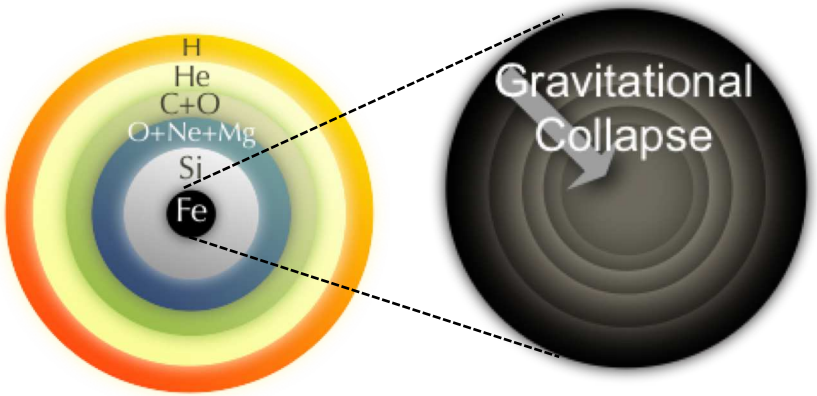
OBSERVATION

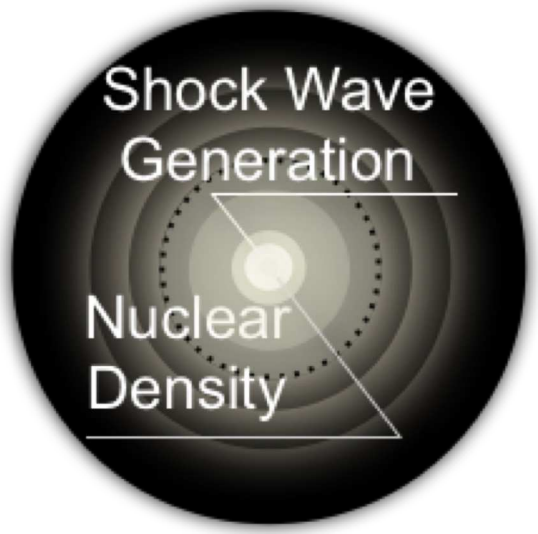
Neutrino

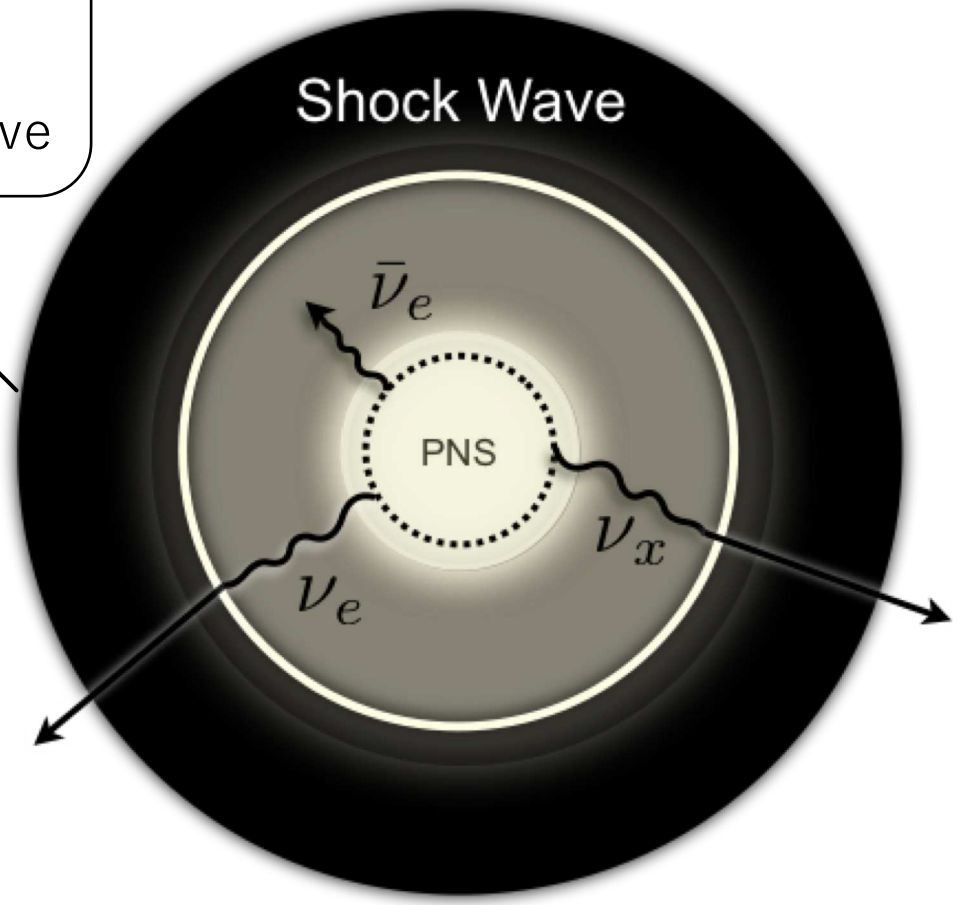
Gravitational Wave

EXPLOSION MECHANISM

Neutrino Heating Mechanism





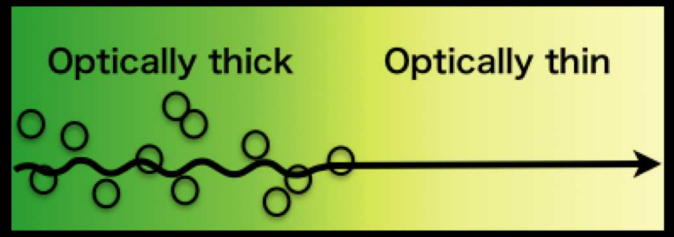


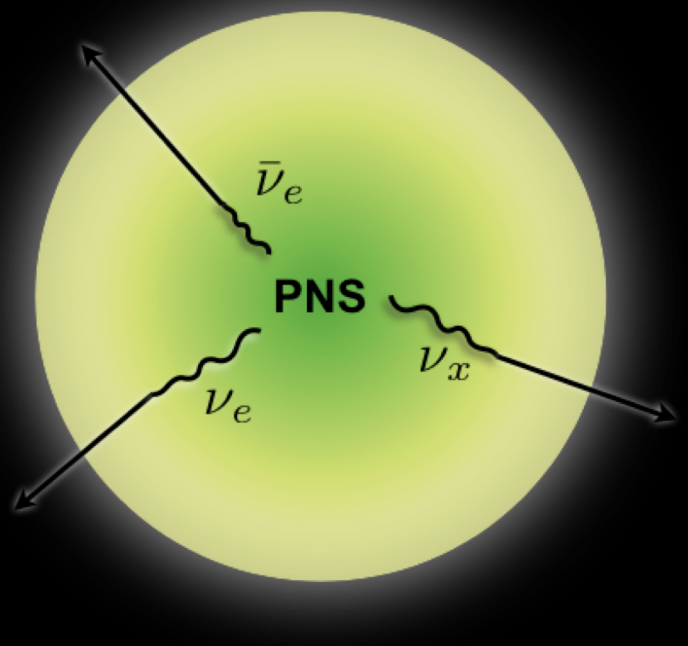
Approximation Method for Neutrino Transport

Boltzmann equation

Boltzmann Equation

$$\frac{dx^\mu}{d\lambda}\frac{\partial f}{\partial x^\mu}+\frac{dp^i}{d\lambda}\frac{\partial f}{\partial p^i}=\left(\frac{\delta f}{\delta \lambda}\right)_{\text{collision}}$$





- Leakage Scheme
- Light Bulb Approximation
- Ray-by-Ray Approach
- IDSA (Isotropic Diffusion Source Approximation)
- Moment method
- MGFLD (Multi-Group Flux Limited Diffusion) method

OUR OBJECTIVE

- Analysis of neutrino transport in the intermediate region between optically thick and thin
- Clarification of explosion mechanism
- Validation and improvement of approximation method

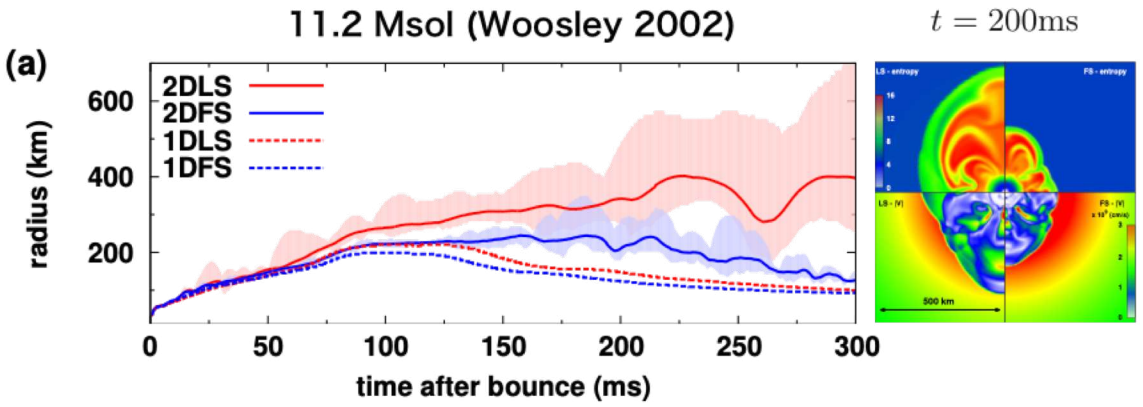
Results for 2D Boltzmann Radiation Hydrodynamic Simulation

EOS Dependence by Nagakura

Six-Dimensional Simulations of Core-Collapse Supernovae with Full Boltzmann Neutrino Transport

Hiroki Nagakura[1], Wakana Iwakami[2,3], Shun Furusawa[4], Hirotada Okawa[2,3], Akira Harada[5], Kohsuke Sumiyoshi[6], Shoichi Yamada[3,7], Hideo Matsufuru[8] and Akira Imakura[9]

arXiv:1702.01752v1



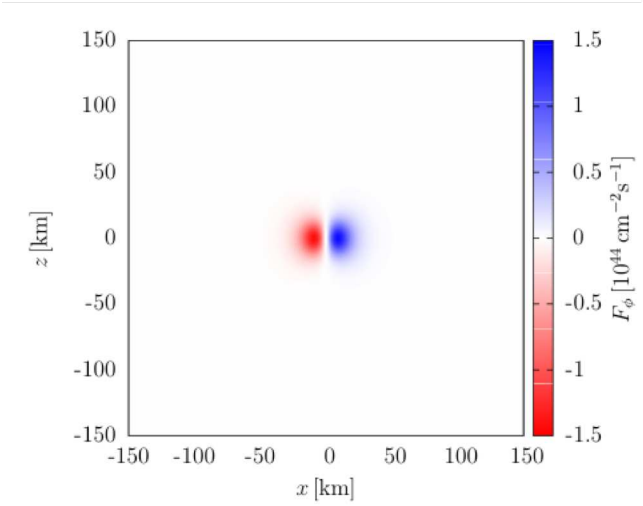
LS: Lattimer & Swesty (K=220MeV) EOS FS: Furusawa EOS

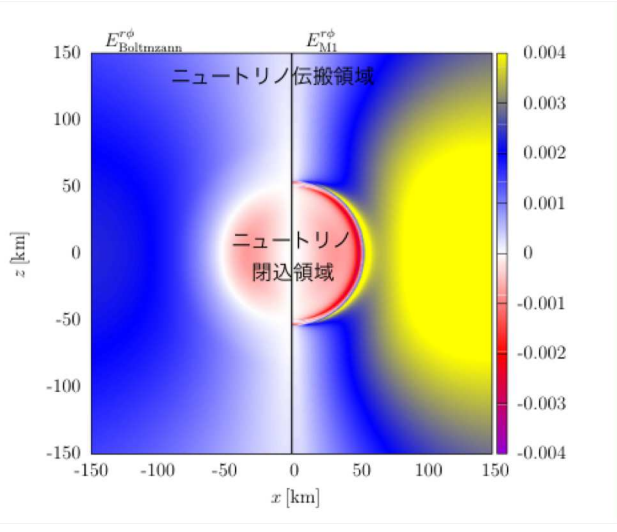
Rotational Effect by Harada

ON THE NEUTRINO DISTRIBUTIONS IN PHASE SPACE FOR THE ROTATING CORE-COLLAPSE SUPERNOVA SIMULATED WITH A BOLTZMANN-NEUTRINO-RADIATION-HYDRODYNAMICS CODE

Akira Harada,[1] Hiroki Nagakura,[2,3] Wakana Iwakami,[4,5] Hirotada Okawa,[4,5] Shun Furusawa,[6] Hideo Matsufuru,[7] Kohsuke Sumiyoshi,[8] and Shoichi Yamada[5]

arXiv:1810.12316v2

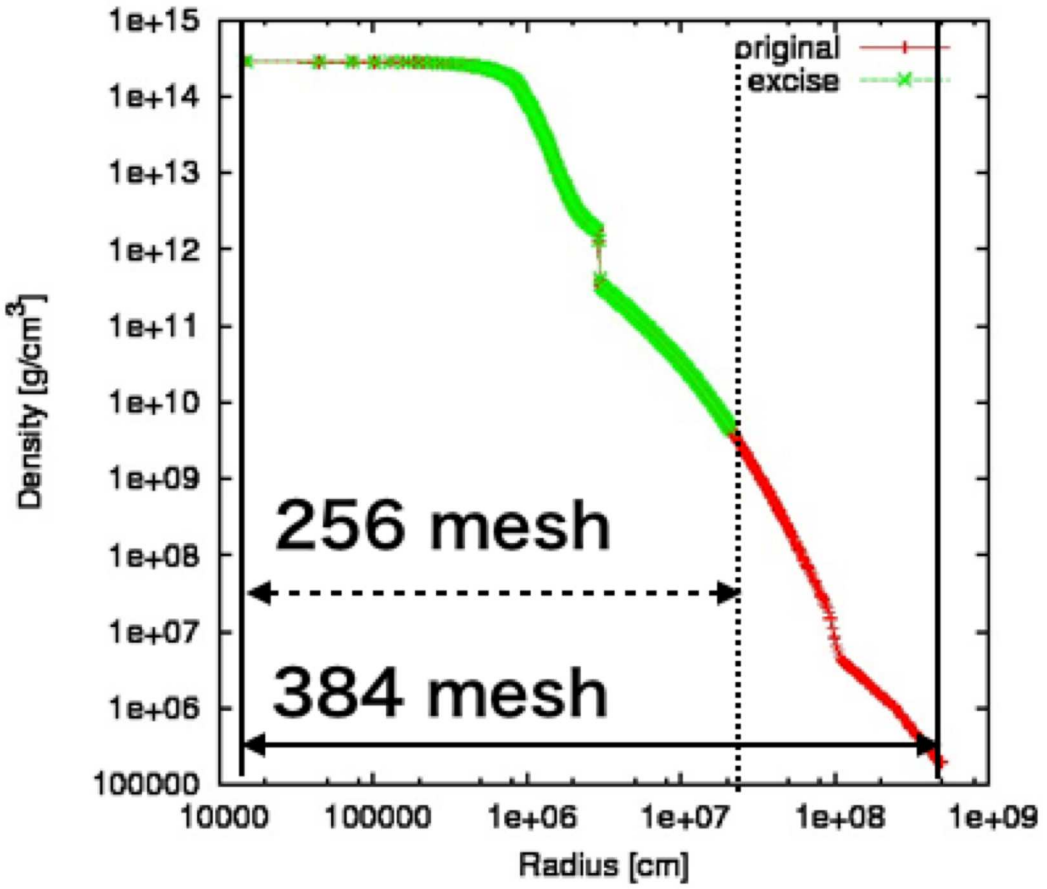




Numerical Setup for 3D Boltzmann Radiation Hydrodynamic Simulation

$$Nr \times N\theta \times N\phi \times N\nu e \times N\nu\theta \times N\nu\phi = 256 \times 48 \times 96 \times 16 \times 6 \times 6$$

京: ～10% of peak performance　　FX100: ～10%



1D simulation has been done from the onset of core-collapse to shock stagnation.

The random perturbation is imposed on the 1D data meeting the instability condition for convection

The boundary data extracted from the 1D data is added to the outer boundary each time step.

1D average has been done within 8km, and the origin of the space coordinate is fixed in this study.

The quantities are averaged over some Φ meshes for relaxation of numerical instability under 12km.

Euler Equations

Hydrodynamics

Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x^j}\left(\rho v^j\right) = 0$$

Equations of Motion:

$$\frac{\partial}{\partial t}\left(\rho v_i\right) + \frac{\partial}{\partial x^j}\left(\rho v_i v^j + P\delta_i^j\right) = -\rho\frac{\partial \psi}{\partial x^j} - G^i$$

Energy Equation:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + e\right) + \frac{\partial}{\partial x^j}\left[\left(\frac{1}{2}\rho v^2 + e + P\right)v^j\right] = -\rho v^j\frac{\partial \psi}{\partial x^j} - G^0$$

Time-Evolution Equation of Electron Number:

$$\frac{\partial}{\partial t}\left(\frac{\rho Y_e}{m_A}\right) + \frac{\partial}{\partial x^j}\left(\frac{\rho Y_e}{m_A}v^j\right) = -\Gamma$$

1D Poisson's equation for gravity:

$$\Delta\psi = 4\pi G\rho$$

EOS table of Nuclear Matter:

$$P = P(\rho, T, Y_e)$$

ρ : density, v: velocity, P: pressure, e: internal energy, ψ: the gravitational potential,
G : the gravitational constant ($=6.67\times10^{-8}[\mathrm{cm^3 g^{-1} s^2}]$), Y_e: electron fraction, T : temperature,
m_A: the atomic mass unit, G^0: neutrino energy density, G^i: neutrino flux,
Γ: deleptonization rate ($\equiv \Gamma_{\nu_e} - \Gamma_{\bar{\nu}_e}$), Γ_s : neutrino reaction rate

Collision Terms

Coupling term between hydrodynamics and neutrino radiation transport

Neutrino Number Density

s : species ($s = \nu_e,\ \bar{\nu}_e,\ \nu_x$)

$$\Gamma_s \equiv \int \left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} d^3\mathbf{p} \qquad \Gamma \equiv \Gamma_{\nu_e} - \Gamma_{\bar{\nu}_e}$$

Neutrino Energy Density (μ = 0)

Neutrino Flux (μ = 1, 2, 3)

$$G_s^\mu \equiv \int p_s^\mu \left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} d^3\mathbf{p} \qquad G^\mu \equiv \sum_s G_s^\mu$$

$$\left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} = \left[\frac{\delta f}{\delta \tau}\right]_{\text{emis-abs (s)}} + \left[\frac{\delta f}{\delta \tau}\right]_{\text{scat (s)}} + \left[\frac{\delta f}{\delta \tau}\right]_{\text{pair (s)}}$$

Emission/Absorption

Electron Capture

$e^- + p \longleftrightarrow \nu_e + n$ [ecp]

Positron Capture

$e^+ + n \longleftrightarrow \bar{\nu}_e + p$ [aecp]

Electron Capture on nuclei

$e^- + A \longleftrightarrow \nu_e + A'$ [eca]

Scattering

Neutrino-Nucleon scattering

$\nu + N \longleftrightarrow \nu + N$ [nsc]

Neutrino-Nuclei scattering

$\nu + A \longleftrightarrow \nu + A$ [csc]

Neutrino-Electron scattering

$\nu + e \longleftrightarrow \nu + e$ [esc]

Pair Process

Electron-positron pair process

$e^- + e^+ \longleftrightarrow \nu_i + \bar{\nu}_i$ [pap]

Nucleon-nucleon bremsstrahlung

$N + N \longleftrightarrow N + N + \nu_i + \bar{\nu}_i$ [nbr]

Collision Terms

Coupling term between hydrodynamics and neutrino radiation fields

Neutrino Number Density

s : species ($s = \nu_e,\ \bar{\nu}_e,\ \nu_x$)

$$\Gamma_s \equiv \int \left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} d^3\mathbf{p} \qquad \boxed{\Gamma} \equiv \Gamma_{\nu_e} - \Gamma_{\bar{\nu}_e}$$

Neutrino Energy Density (μ = 0)

Neutrino Flux (μ = 1, 2, 3)

$$G_s^\mu \equiv \int p_s^\mu \left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} d^3\mathbf{p} \qquad \boxed{G^\mu} \equiv \sum_s G_s^\mu$$

$$\left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} = \left[\frac{\delta f}{\delta \tau}\right]_{\text{emis-abs (s)}} + \left[\frac{\delta f}{\delta \tau}\right]_{\text{scat (s)}} + \left[\frac{\delta f}{\delta \tau}\right]_{\text{pair (s)}}$$

$$\left[\frac{\delta f}{\delta \tau}\right]_{\text{emis-abs}} = -R_{\text{abs}}(\varepsilon, \Omega) f(\varepsilon, \Omega) + R_{\text{emis}}(\varepsilon, \Omega)[1 - f(\varepsilon, \Omega)].$$

$$\left[\frac{\delta f}{\delta \tau}\right]_{\text{pair}} = -\int \frac{d\varepsilon' \varepsilon'^2}{(2\pi)^3} \int d\Omega' R_{\text{pair-anni}}(\varepsilon, \Omega; \varepsilon', \Omega') \times f(\varepsilon, \Omega)\bar{f}(\varepsilon', \Omega') + \int \frac{d\varepsilon' \varepsilon'^2}{(2\pi)^3} \int d\Omega' R_{\text{pair-emis}}(\varepsilon, \Omega; \varepsilon', \Omega') \times [1 - f(\varepsilon, \Omega)][1 - \bar{f}(\varepsilon', \Omega')],$$

$$\left[\frac{\delta f}{\delta \tau}\right]_{\text{scat}} = -\int \frac{d\varepsilon' \varepsilon'^2}{(2\pi)^3} \int d\Omega' R_{\text{scat}}(\varepsilon, \Omega; \varepsilon', \Omega') f(\varepsilon, \Omega) \times [1 - f(\varepsilon', \Omega')] + \int \frac{d\varepsilon' \varepsilon'^2}{(2\pi)^3} \int d\Omega' R_{\text{scat}}(\varepsilon', \Omega'; \varepsilon, \Omega) \times f(\varepsilon', \Omega')[1 - f(\varepsilon, \Omega)],$$

Ω' denotes the angle variables after/before the scattering

$\bar{f}(\varepsilon', \Omega')$ denotes the distribution of anti-neutrinos, which is the angle-averaged distribution in the previous time step.

Boltzmann Equation

Neutrino Radiation

Boltzmann Equation

$$\frac{dx^{\mu}}{d\lambda}\frac{\partial f}{\partial x^{\mu}}+\frac{dp^{i}}{d\lambda}\frac{\partial f}{\partial p^{i}}=\left(\frac{\delta f}{\delta\lambda}\right)_{\text{collision}}$$

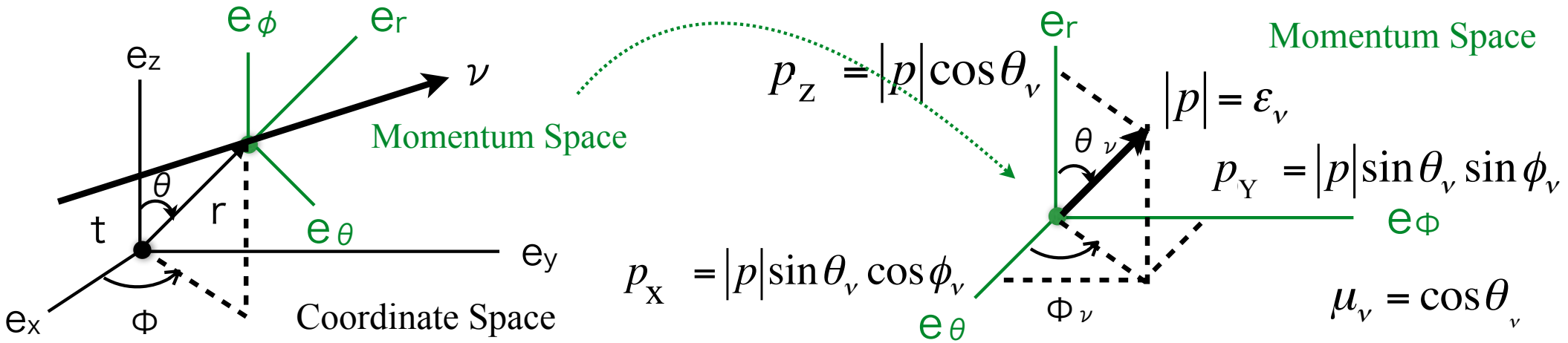
Neutrino distribution function

$$f(t,r,\theta,\phi;\varepsilon_{\nu},\mu_{\nu},\phi_{\nu})$$

Boltzmann Equation in the spherical coordinate

$$\frac{\partial f}{\partial t}+\cos\theta_{\nu}\frac{\partial f}{\partial r}+\frac{\sin\theta_{\nu}\cos\theta_{\nu}}{r}\frac{\partial f}{\partial\theta}+\frac{\sin\theta_{\nu}\sin\phi_{\nu}}{r\sin\theta}\frac{\partial f}{\partial\phi}$$

$$-\frac{\sin\theta_{\nu}}{r}\frac{\partial f}{\partial\theta_{\nu}}-\frac{\cos\theta}{\sin\theta}\frac{\sin\theta_{\nu}\sin\phi_{\nu}}{r}\frac{\partial f}{\partial\phi_{\nu}}=\left(\frac{\delta f}{\delta t}\right)_{\text{collision}}$$

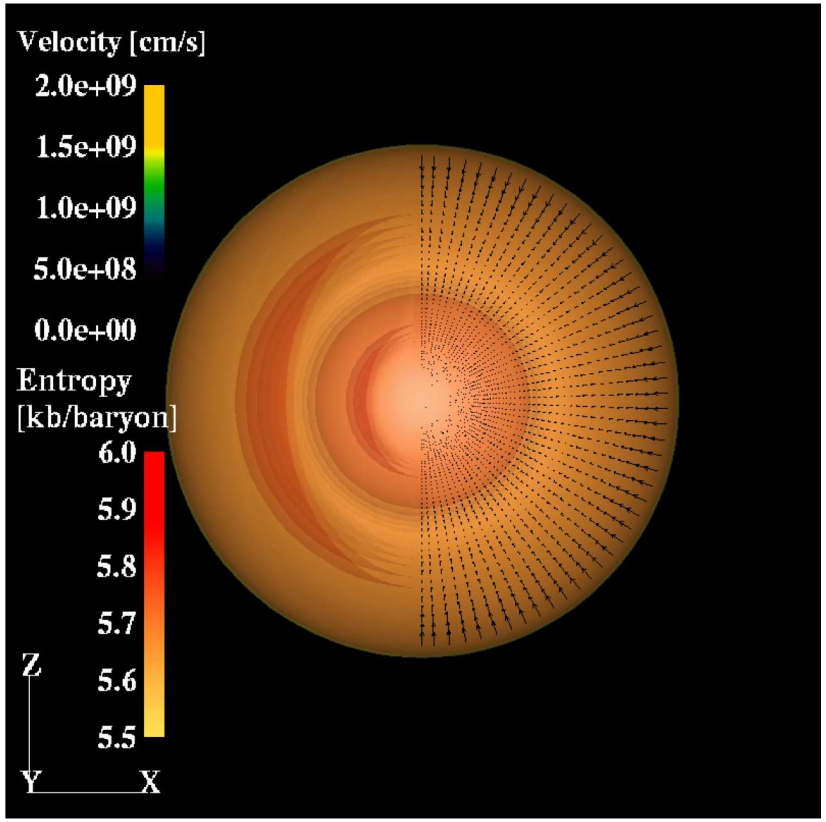


Hydrodynamics

Iso-Surfaces of Entropy and Vectors of Velocity

Growth and attenuation of the prompt convection inside proto-neutron star just after shock generation（~20ms）

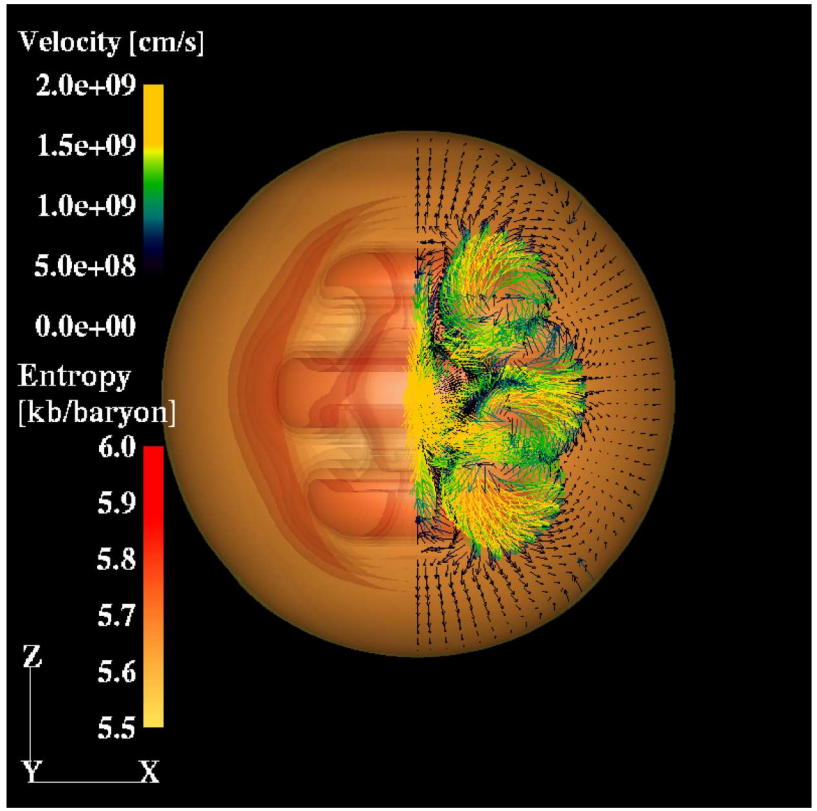
1D



No convection

~100km

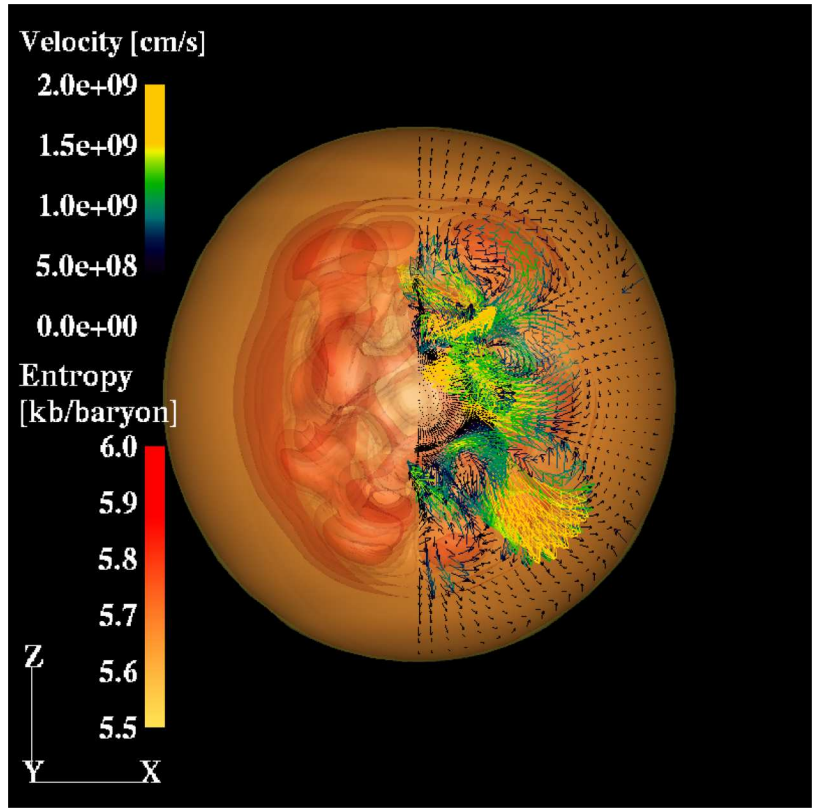
2D



Vortex rings

~100km

3D



Cellular vortex

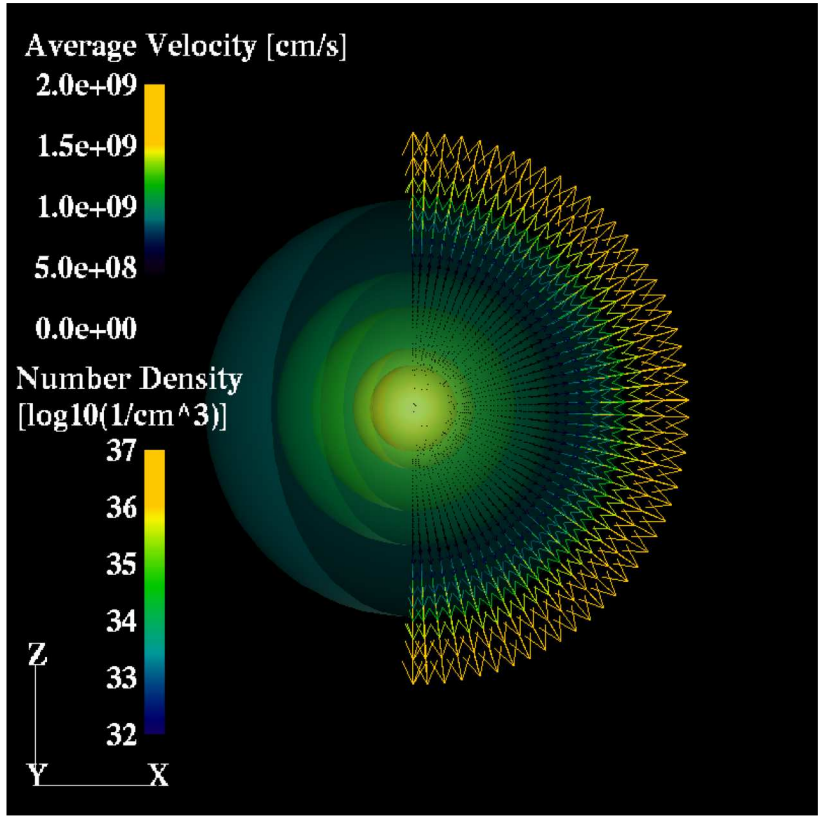
~100km

Neutrino Radiation Fields

Iso-Surfaces of Number Density and Vectors of Flux Factor for Electron Neutrino

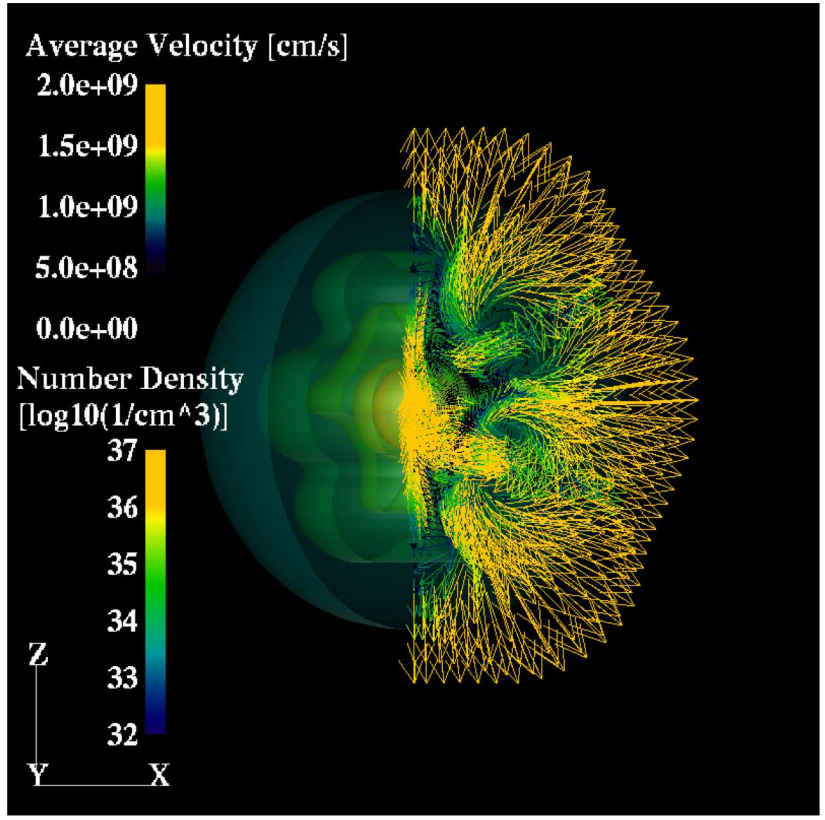
Neutrino move with the matter in the optically thick region, begin to deviate from the matter in the intermediate region between optically thick and thin limit, and propagate outward in the optically thin region

1D



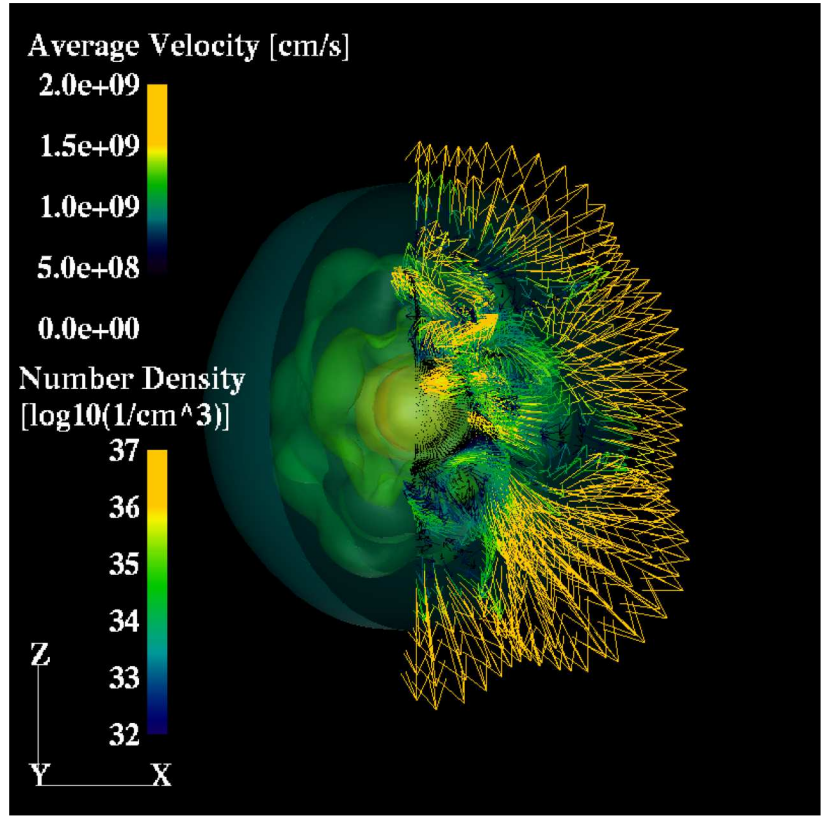
~100km

2D



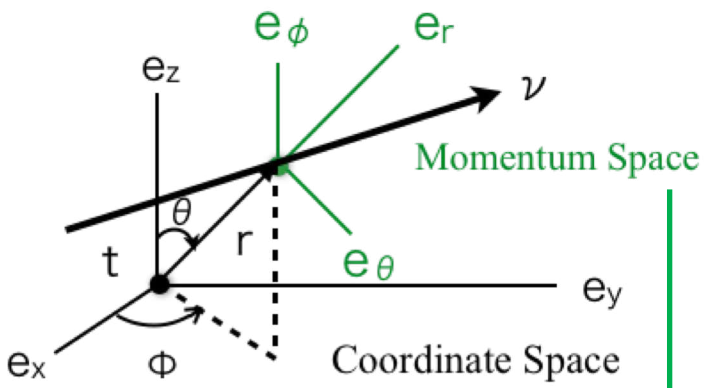
~100km

3D

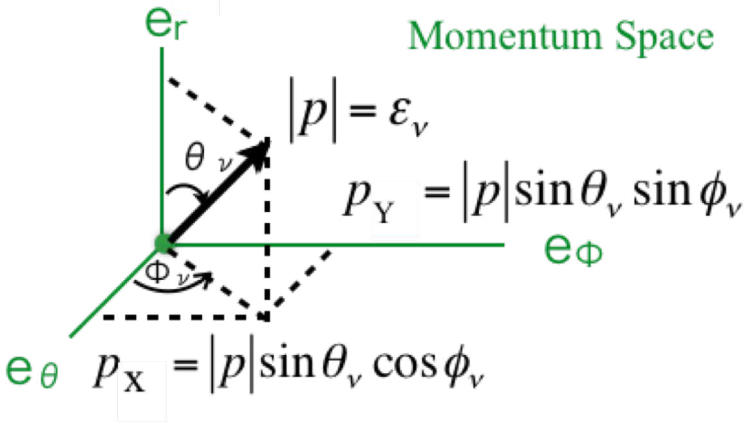


~100km

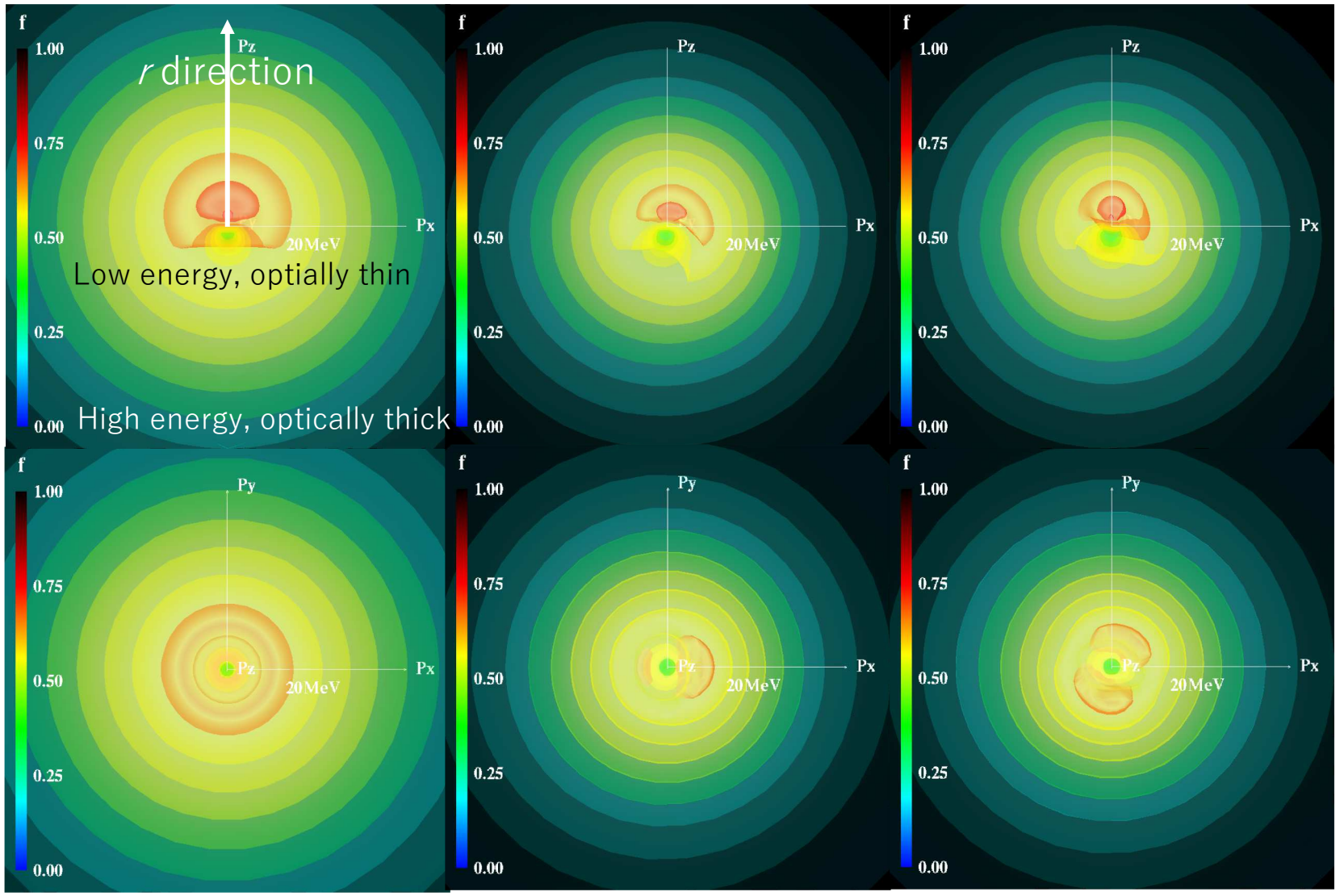
Distribution function f (ε , θ_ν, ϕ_ν) of ν_e t = 10s, r=30km



$$p_Z = |p|\cos\theta_\nu$$



1D 2D 3D



Axisymmetric to Pz axis　　Plane symmetry on Pz-Px　　No symmetry

Eddington Tensor for Boltzmann and M1 Closure Approximation

Energy Density

$$E_\nu = \int d\Omega \varepsilon f(\varepsilon, \Omega).$$

Energy Flux

$$F_\nu^i = \int d\Omega \varepsilon \, n_i f(\varepsilon, \Omega)$$

Radiation Pressure Tensor

$$P_\nu^{ij} = \int d\Omega \varepsilon n_i n_j f(\varepsilon, \Omega)$$

Eddington Tensor

$$k_{(\nu)}^{ij} = \frac{P_{(\nu)}^{ij}}{E_{(\nu)}} = \begin{pmatrix} k^{11} & k^{12} & k^{13} \\ k^{21} & k^{22} & k^{23} \\ k^{31} & k^{32} & k^{33} \end{pmatrix}$$

M1 closure approximation

$$P_{(\nu)}^{ij} = \frac{3\chi - 1}{2}(P_{(\nu)}^{ij})_{\text{thin}} + \frac{3(1-\chi)}{2}(P_{(\nu)}^{ij})_{\text{thick}}$$

optically thin $\quad P_{(\nu)\ \text{thin}}^{\alpha\beta} = E_{(\nu)} \dfrac{F_{(\nu)}^{\alpha} F_{(\nu)}^{\beta}}{\gamma_{ij} F_{(\nu)}^{i} F_{(\nu)}^{j}}$

optically thick $\quad P_{(\nu)\ \text{thick}}^{ij} = J_{(\nu)} \left[\dfrac{\gamma^{ij} + 4V^i V^j}{3} \right] + H_{(\nu)}^{i} V^j + H_{(\nu)}^{j} V^i$

Eddington Factor

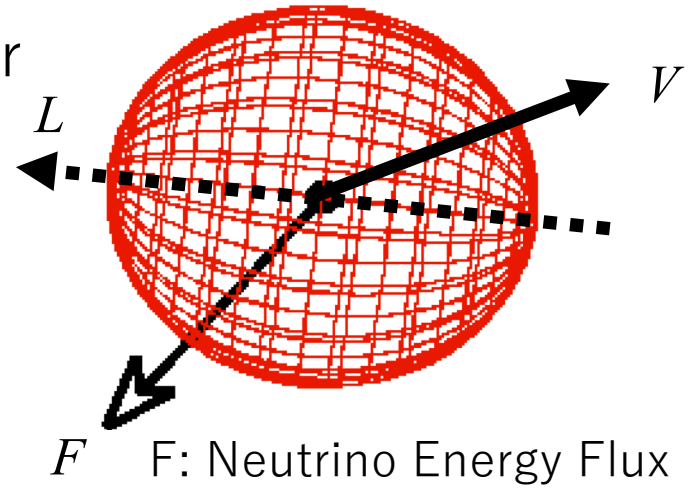
$$\chi = \frac{3 + 4\bar{F}^2}{5 + 2\sqrt{4 - 3\bar{F}^2}}.$$

Flux factor

$$\bar{F} := \left(\frac{h_{\alpha\beta} H_{(\nu)}^{\alpha} H_{(\nu)}^{\beta}}{J_{(\nu)}^2} \right)^{1/2}$$

Analysis of eigen-values and -vectors of Eddington Tensor

L: longest axis V: Matter Velocity



F: Neutrino Energy Flux

Eddington Tensor $$k^{ij}_{(\nu)} = \frac{P^{ij}_{(\nu)}}{E_{(\nu)}} = \begin{pmatrix} k^{11} & k^{12} & k^{13} \\ k^{21} & k^{22} & k^{23} \\ k^{31} & k^{32} & k^{33} \end{pmatrix}$$

Neutrino Energy $E_\nu = \int d\Omega \varepsilon f(\varepsilon, \Omega)$.

Neutrino Strain Tensor $P^{ij}_\nu = \int d\Omega \varepsilon n_i n_j f(\varepsilon, \Omega)$

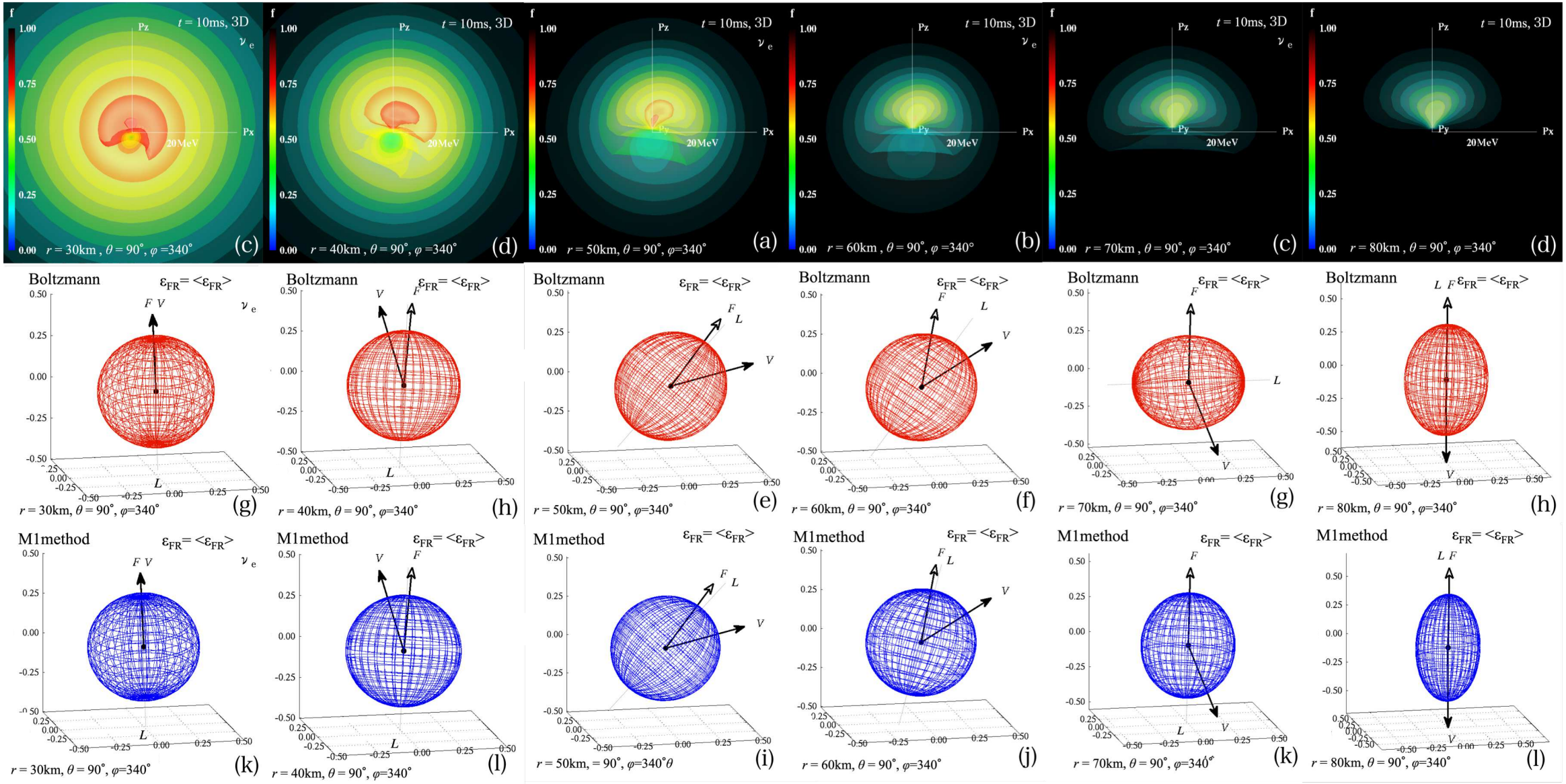
Ellipsoid $$\begin{pmatrix} P_X \\ P_Y \\ P_Z \end{pmatrix} = \begin{pmatrix} V^{\theta 1} & V^{\theta 2} & V^{\theta 3} \\ V^{\phi 1} & V^{\phi 2} & V^{\phi 3} \\ V^{r1} & V^{r2} & V^{r3} \end{pmatrix} \begin{pmatrix} \lambda^1 \sin u \cos v \\ \lambda^2 \sin u \sin v \\ \lambda^3 \cos u \end{pmatrix}$$

1) Obtain eigenvalue λ_i and eigenvectors (V^{ri}, $V^{\theta i}$, $V^{\phi i}$) of Eddington Tensor by Jacobi method

2) Visualize with the ellipsoid whose axes are parallel to eigen vectors with the coefficients of eigen values

3) Comparison between the ellipsoids given by Boltzmann and M1 closure relation

Neutrino Distribution in the Momentum Space and Ellipsoid of Eddington Tensor



isotoropic、
F and V are overlapped.　F deviate from V.　F is almost parallel to L.　F deviate from L.　Ellipsoids are different between Boltzmann and M1.

SUMMARY

1) Boltzmann radiation hydrodynamic simulation in 3D space has been done until 20ms.

2) Growth and attenuation of the prompt convection inside proto-neutron star has been confirmed.

3) Neutrinos move with the matter in the optically thick region, begin to deviate from the matter in the intermediate region between optically thick and thin limit, and propagate outward in the optically thin region.

4) The neutrino distribution function is axisymmetric to Pz axis, plane-symmetric, and no symmetric in 1D, 2D, and 3D, respectively.

5) M1 closure relation cannot reproduce the horizontally long ellipsoid which made by the eigenvalues and eigenvectors of Eddington tensor obtained from Boltzmann