The numerical construction of axisymmetric equilibria of rotating stars on Lagrangian coordinates

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Abstract

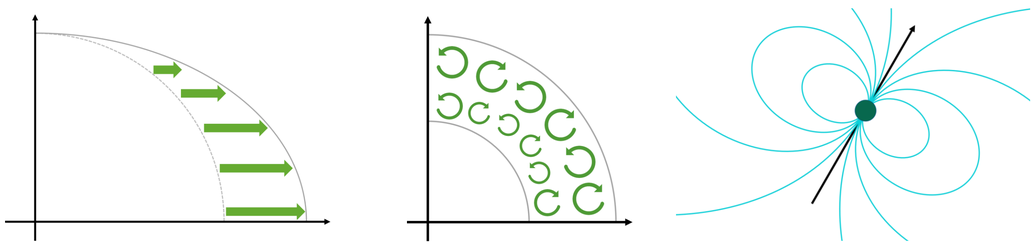
The stellar rotation affects the configuration and evolution: it is particularly important to consider the effects of rotation on core-collapse supernovae and on merger remnants of the coalescence of two stars.

To construct axisymmetric equilibria of rotating stars numerically and then calculate their evolutions, we solve the hydrostatic equilibrium equation by using Lagrangian coordinates. Moreover, we employ a new numerical method, W4 method. We calculate the pseudo-evolution to the actual non-spherical configuration with rotation. We are able to obtain the rotating star that has a rotation speed which is very close to this critical case and the obtained configuration is flattened.

Introduction

◆ **Rotational effects on stellar evolution**

- ✓ Shape of stars
- ✓ Internal mixing
- ✓ Effects of magnetic field



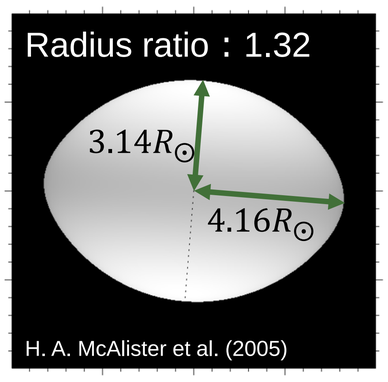
◆ **Theoretical studies**

Equilibrium state

→ Lagrangian coordinate → Stellar evolution calculation

1D calculations with rotational effects (ex. MESA)
2D calculations are nothing.

→ Euler coordinate



◆ **Observations**

1. Rotational velocity (Regulus)
   $v_{\rm rot} = 317 \pm 3\ {\rm km/s}\ (0.86 v_{\rm crit})$
2. Gravitational wave detections
   Merger remnants = rapidly rotating stars

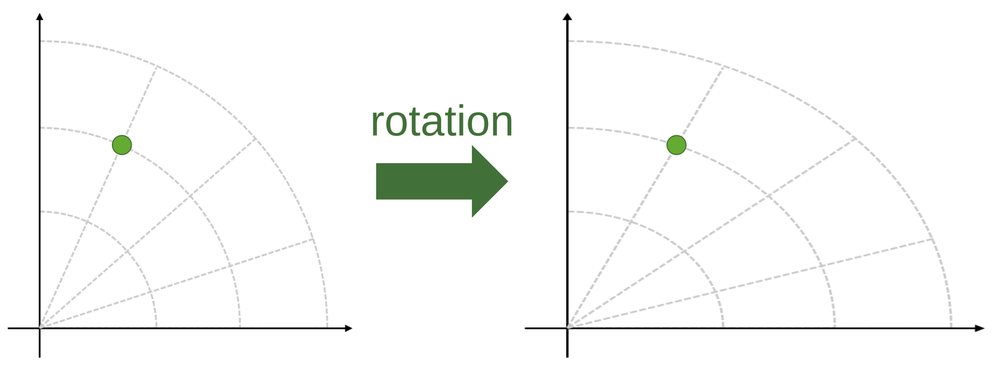
Conventional		This study	Advantage
Spherical (1D)	→	Axisymmetry (2D)	Multi-dimensional
Euler coordinate	→	Lagrange coordinate	Conserved quantity
Newton method	→	W4 method	Wide convergence

Method

$$\frac{1}{\rho}\boldsymbol{\nabla} P - \boldsymbol{\nabla}\phi + \omega^2 (r\sin\theta)\boldsymbol{e_\varpi} = \boldsymbol{0}$$

$$\rightarrow \begin{cases} F_{\rm r} = \dfrac{1}{\rho}\dfrac{\partial {\rm P}}{\partial r} - \dfrac{\partial \phi}{\partial r} + \omega^2 (r\sin\Theta)\sin\Theta = 0 \\ F_\Theta = \dfrac{1}{\rho}\dfrac{\partial {\rm P}}{{\rm r}\partial\Theta} - \dfrac{\partial \phi}{{\rm r}\partial\Theta} - \omega^2 (r\sin\Theta)\cos\Theta = 0 \end{cases}$$

Solve on Lagrangian coordinate
= Solve the position of each mesh (r, θ)

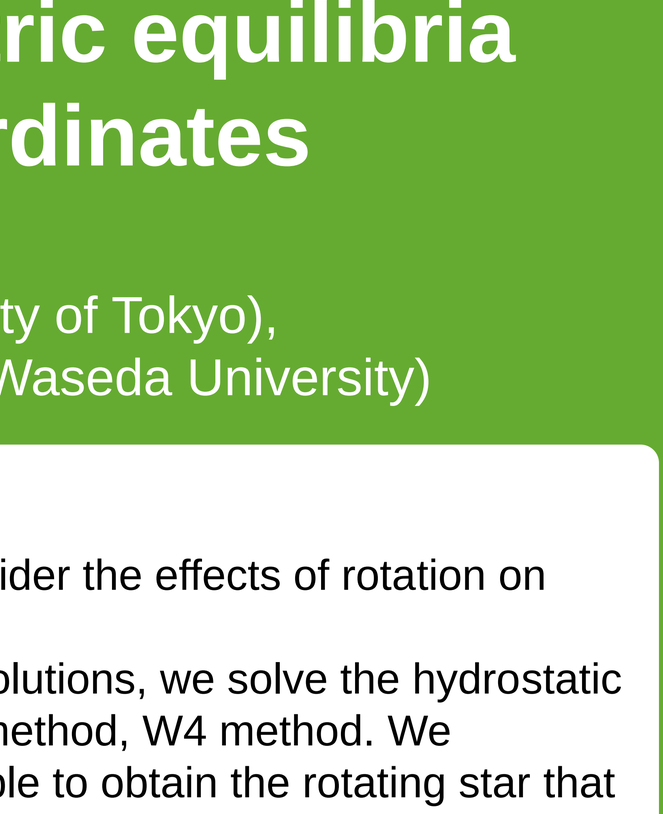


Results

◆ **Roche model**

- Point-mass gravity
  $$\frac{{\rm d}\phi}{{\rm dr}} = \frac{GM}{{\rm r}^2}$$
- Rigid rotation
  $$\Omega = {\rm const.}$$
- Polytrope (n=1)
  $${\rm P} = \kappa\rho^{1+1/n}$$

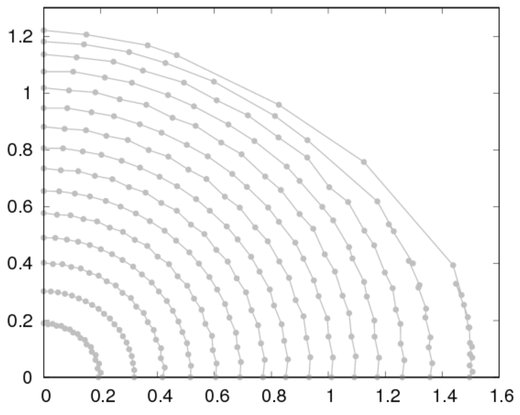
Radius ratio : 1.3525



◆ **Self-gravity**

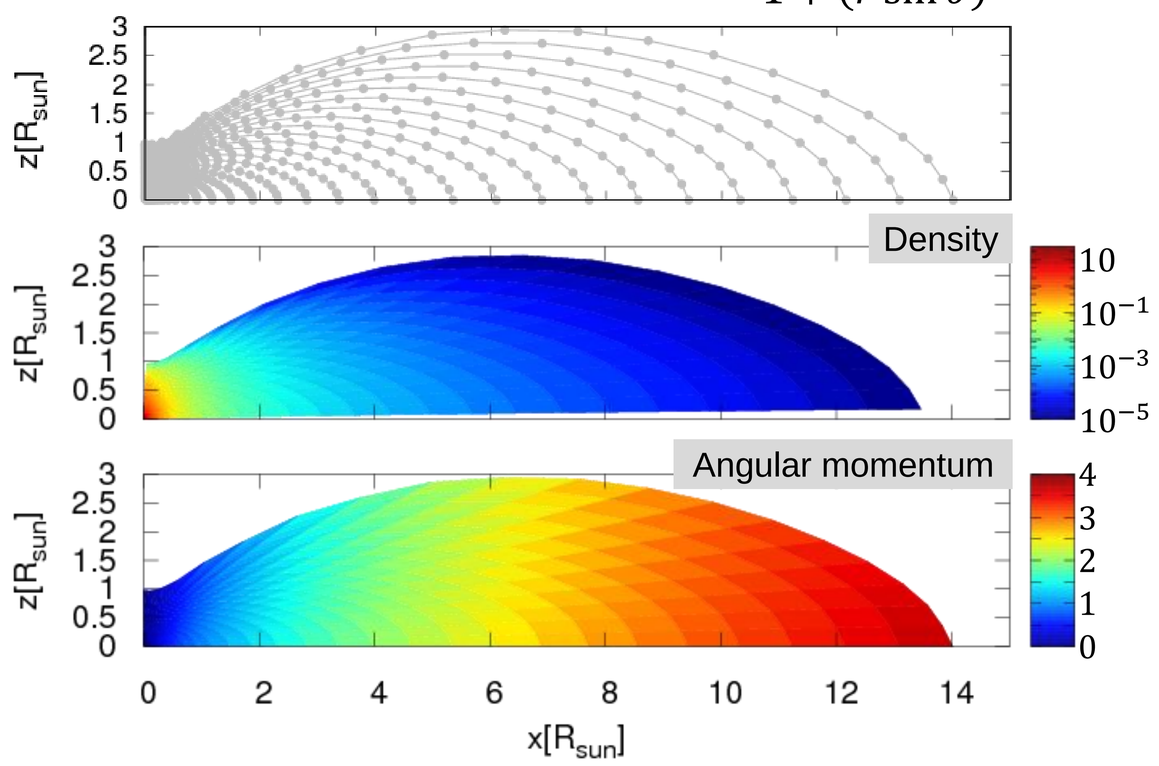
- Spectral method
  (θ-direction is ignored)
  $$\Delta\phi = 4\pi G\rho(r)$$

Pole : 1.2209
Equator : 1.4964



◆ **Differential rotation**

$$J = \frac{j_0 (r\sin\theta)^2}{1 + (r\sin\theta)^2}$$



Conclusion

We calculate equilibrium structures of many physical setting. In some models, the rotational speed is close to the critical velocity and their shape is distorted.

→The axisymmetric equilibria of rotating stars on Lagrange coordinates are calculated!

Future work

More realistic star cases : nuclear reaction, energy transport