General Relativistic Boltzmann Solver for Core-collapse of Massive Stars

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1. Introduction

Neutrino plays very important role in core-collapse of massive stars, carrying almost $\sim 99\%$ of energy released by gravitational collapse.

Since neutrinos are NOT in thermodynamic equilibrium, accurate treatment of neutrinos should be based on direct numerical calculation of Boltzmann equation. Unfortunately, many groups have employed approximated schemes because of expensive computational resources required to do so. The Boltzmann group, however, have developed Boltzmann solver for core-collapse simulations, and many simulations of core-collapse supernovae (CCSNe) have been performed.

In addition to accurate neutrino treatment, general relativistic (GR) effects are important due to the extremely high density. Especially, full GR simulation based on numerical relativity is required to follow black hole formation.

In this work, the Boltzmann solver is extended to be able to treat full GR effects, and verification tests are performed. Energy shift test and momentum advection tests are done in 1D.

2. Methods

1. Boltzmann Equation

The GR Boltzmann equation solved is written in conservative form with tetrad components:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^a}\Big|_{q_{(i)}}\left[\left(e^a_{(0)}+\sum_{i=1}^{3}\ell_{(i)}e^a_{(i)}\right)\sqrt{-g}f\right] - \frac{1}{\nu^2}\frac{\partial}{\partial\nu}(\nu^3 f\omega_{(0)}) + \frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}(\sin\bar{\theta}f\omega_{(\bar{\theta})}) + \frac{1}{\sin^2\bar{\theta}}\frac{\partial}{\partial\bar{\varphi}}(f\omega_{(\bar{\varphi})}) = S_{\rm rad}$$

In this work, the source terms in the right hand side is set to zero, because the main purpose is to show GR performance.

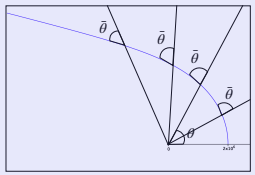
2. Test Calculations

Energy Shift Test

Because of the spacetime curvature, neutrino energy is changed. As a test case, a single neutrino beam with purely radial velocity is set as the input in the Schwarzschild spacetime.

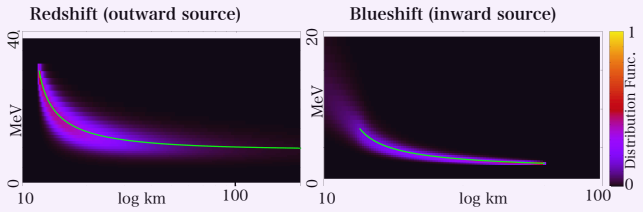
Momentum Advection Test

Momentum advection occurs by the nonzero Christoffel symbols, and behaves differently by different spacetime curvature. Confirmation of momentum advection is an indirect test in 1D to "light-deflection". As a test case, a single neutrino beam with purely angular velocity $\bar{\theta}$ is set as the input in the Schwarzschild spacetime.



3. Results

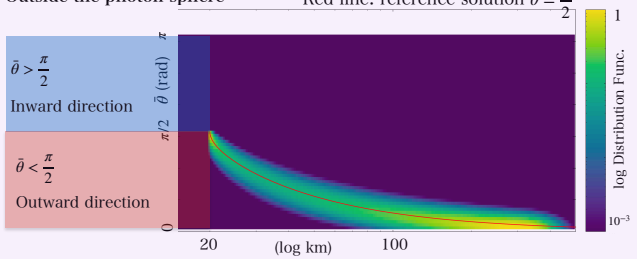
Energy Shift Test



Momentum Advection Test

Outside the photon sphere

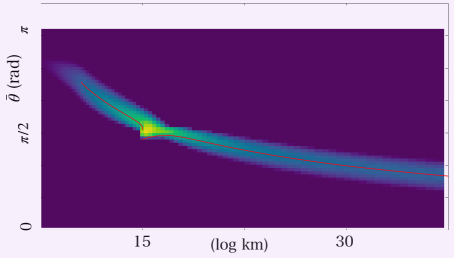
Red line: reference solution $\bar{\theta}=\frac{\pi}{2}$



Inside the photon sphere

Red line: Reference solution with initial condition $\bar{\theta}=1.44$ rad and $\bar{\theta}=1.54$ rad

Because the trajectory is very sensitive to the initial angle, the neutrino flux split into two.



4. Summary

Although only 1D calculations were performed, the code has a capability to simulate in 3D. Further code tests in multi-D, and interplay with hydrodynamics and numerical relativity will enable us to calculate core-collapse and compact object formation in the unprecedented accuracy.