Corotation Resonence as Low T/W Dynamical Instability in Differentially Rotating Stars

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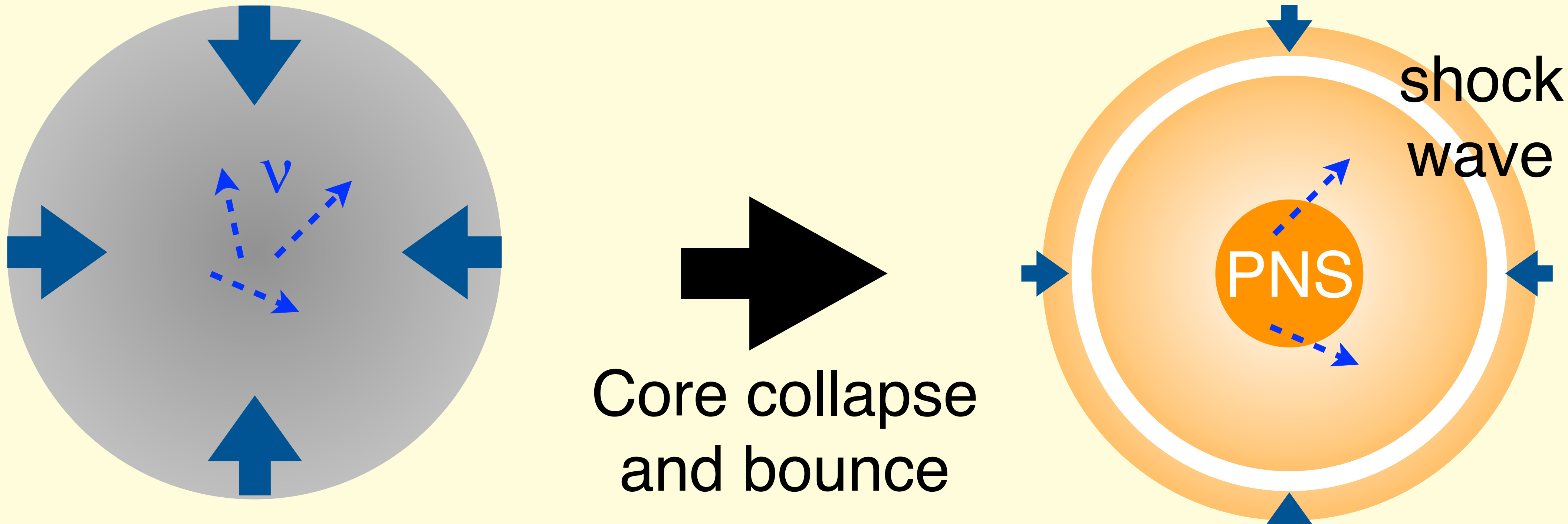
CONTENTS

1. Introduction

Proto-Neutron Star (PNS) Formation and Hydrodynamical Instabilities



Iron core

Post bounce configuration

Fluid instabilities

- Convectional instabilities
- Standing Accretion Shock Instability (SASI)
- g-mode oscillations of PNS

Rotational instabilities

- Low T/W dynamical instability (spiral, bar, …)

Magnetic Fields

Brief history of low T/W dynamical instabilities

m=1 dynamical instability

(e.g. Picketet al. 96; Centrella et al. 01; MS et al. 03)

- Strong concentration of the angular momentum in the envelope region of the star (n=1.5,T/W~0.20; n=3.33, T/W~0.14; n=3, T/W~0.15)
- Even appears in n=1 polytropic star (Ou & Tohline 05)

m=2 dynamical instability

(e.g. Shibata et al. 02, 03)

- Require high degree of differential rotation (Ω_c/Ω_e=10; T/W~0.01)
- Regarded as f-mode (unstablised f-mode; Passamonti & Andersson 14)

Role of corotation

(e.g. Watts et al. 05; MS & Yoshida 06)

- Flow and the turbulence of the fluid has a resonance interaction at the corotation radius inside the star $\omega = m\Omega$
- Diagnose both numerically and perturbatively using canonical angular momentum

Magnetic fields

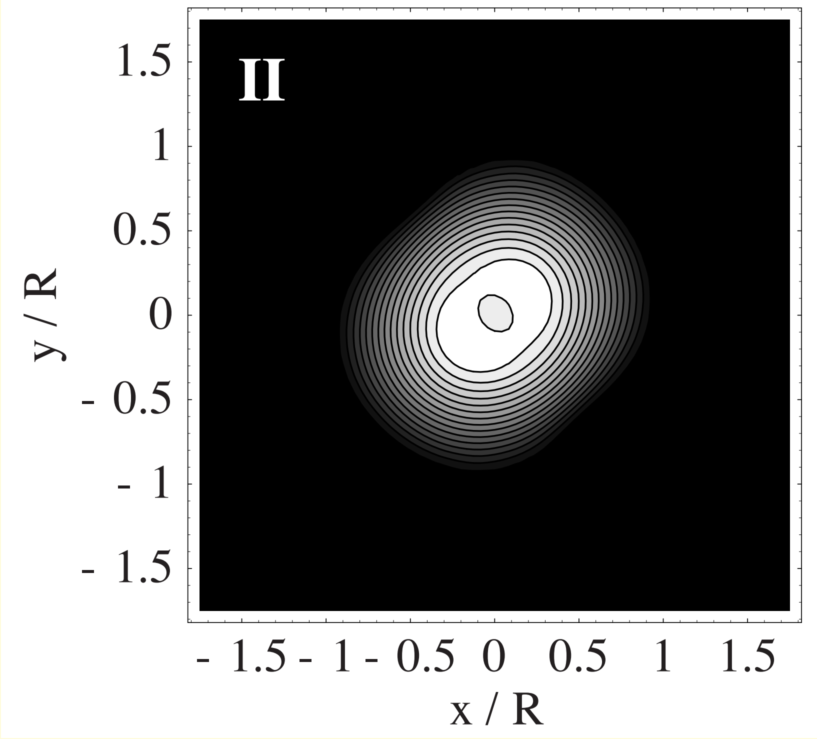
(Fu & Lai 11; Muhlberger et al. 14)

- m=1 dynamical instability is excited, and the frequency is split into two frequencies
- Toroidal magnetic fields may stabilise low T/W dynamical instabilities

Features of Low T/W Dynamical Instabilities

Common features

- Bar structure appears throughout time evolution when there is certain amount of rotation
- Dynamically unstable to bar mode (m=2)
- Generates quasi-periodic gravitational waves
- Considered as an outcome of binary neutron star merger



T/W=0.119, n=1,
$\Omega_c / \Omega_{eq} = 26.0$

Significant Difference

- Instability occurs significantly lower T/W from the standard dynamical and secular (bar) instability
- Weak bar formation
- Existence of spiral instability
- Corotation may play a role
- May occur in the realistic T/W range of supernova explosion

Require some degree of differential rotation to trigger instability

Applications

Binary neutron star merger

(e.g. Paschalidis et al. 15)

- Form differentially rotating objects and trigger a spiral type of low T/W dynamical instabilities

Supernova explosion

(e.g. Ott et al. 05, Kuroda et al. 14, Takiwaki et al. 16)

- Post-bounce cores are subject to a so-called m=1 low T/W nonaxisymmetric instability
- May play an essential role in supernova explosion for efficient energy and angular momentum transport after the core bounce

Purpose

- Understand low T/W dynamical instability in terms of mode analysis
- Towards understanding the physical mechanism of low T/W dynamical instability
- Implication to gravitational waves and equation of state

2. Dynamics of Low T/W Dynamical Instabilities

- 3D Newtonian hydrodynamics in HLLE scheme (no symmetry assumed)
- Γ-law equation of state

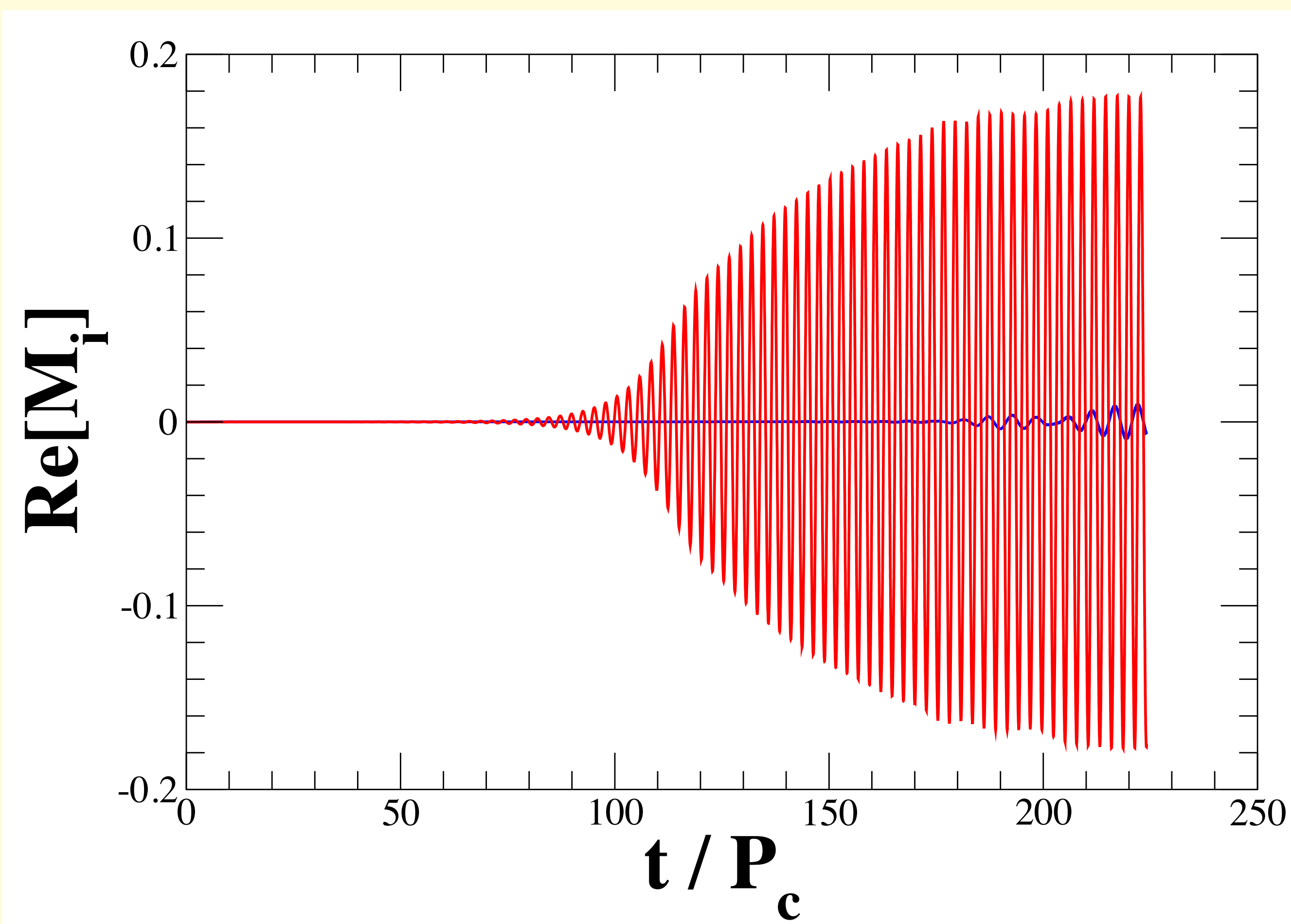
Equilibrium configuration

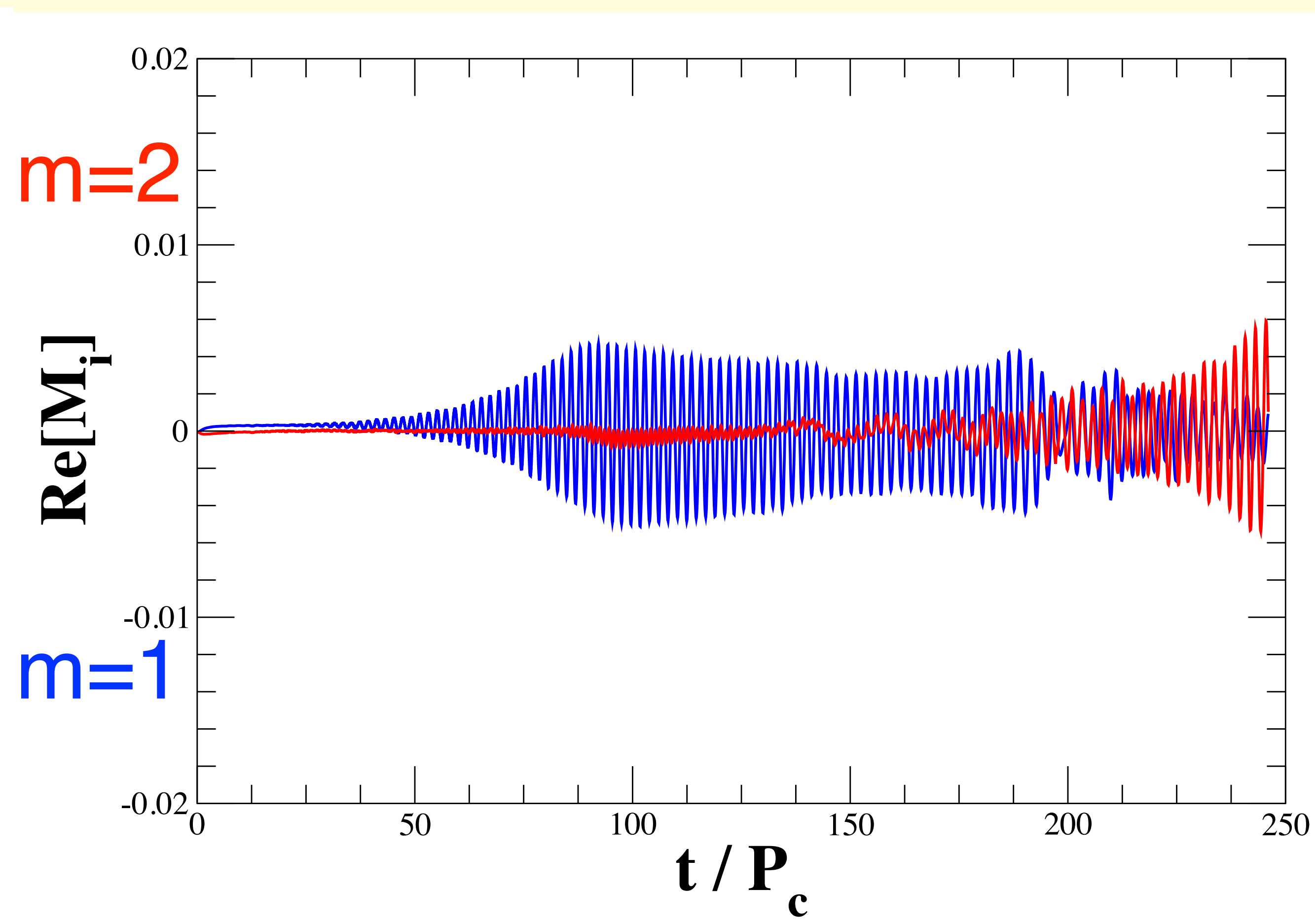
n	T/W
1	0.06
3	0.07

$$\frac{\Omega_c}{\Omega_e} = 26$$

Diagnostics $M_m = \langle e^{im\varphi} \rangle$ density weighted average

n=1 polytropic star | **n=3 polytropic star**





m=2 dominance

m=1 dominance

3. Perturbative Approaches

Non-axisymmetric Perturbations in Rapidly Rotating Stars

δh : perturbed enthalpy

$\delta\Phi$: perturbed gravitational potential

$$\delta U \equiv \delta h + \delta\Phi$$ (e.g. Ipser & Lindblom 90)

$$\delta q = \delta q(\varpi, z) e^{-i(\omega t - m\varphi)}$$

Basic Equations (axisymmetric background)

$$\begin{cases} \mathcal{L}_U(\delta U) = S_U(\delta\Phi) \\ \mathcal{L}_\Phi(\delta\Phi) = S_\Phi(\delta U) \end{cases}$$

⬅

- perturbed continuity equation
- perturbed Euler's equation
- perturbed Poisson's equation

Model

Assumption

Only equatorial motion of the perturbed quantities is taken into account

⬅ Characteristic wave propagation lies in the equatorial plane

- Cylindrical model … no stellar structure in rotational axis direction
- Spheroidal model … assume Legendre polynomial function for perturbed quantities in rotational axis direction

Normal Mode Analysis

(MS & Yoshida 16)

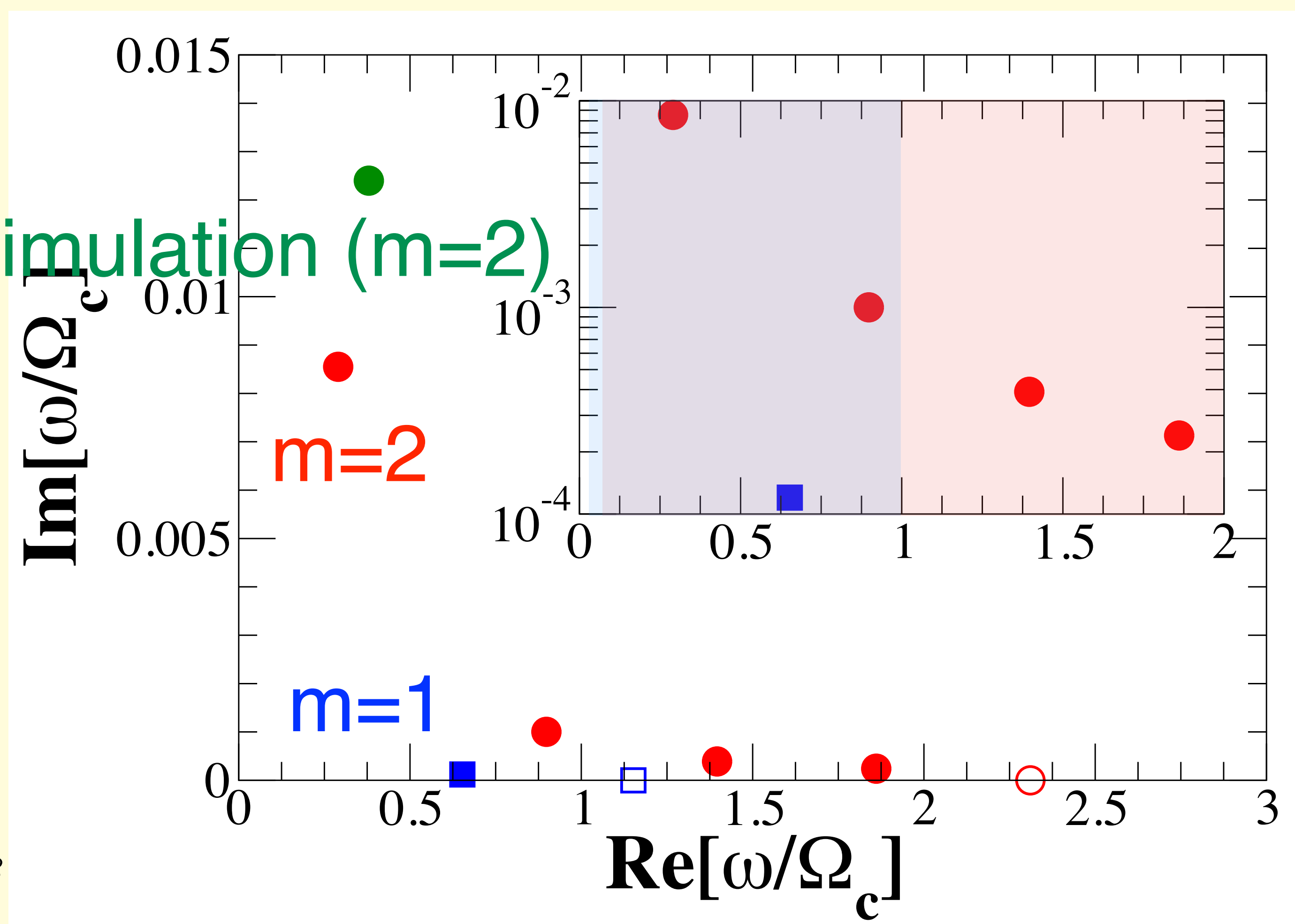
Eigenvalue Problem

$$\begin{cases} \mathcal{L}_U(\delta U) = S_U(\delta\Phi) \\ \mathcal{L}_\Phi(\delta\Phi) = S_\Phi(\delta U) \end{cases}$$

Overall scaling

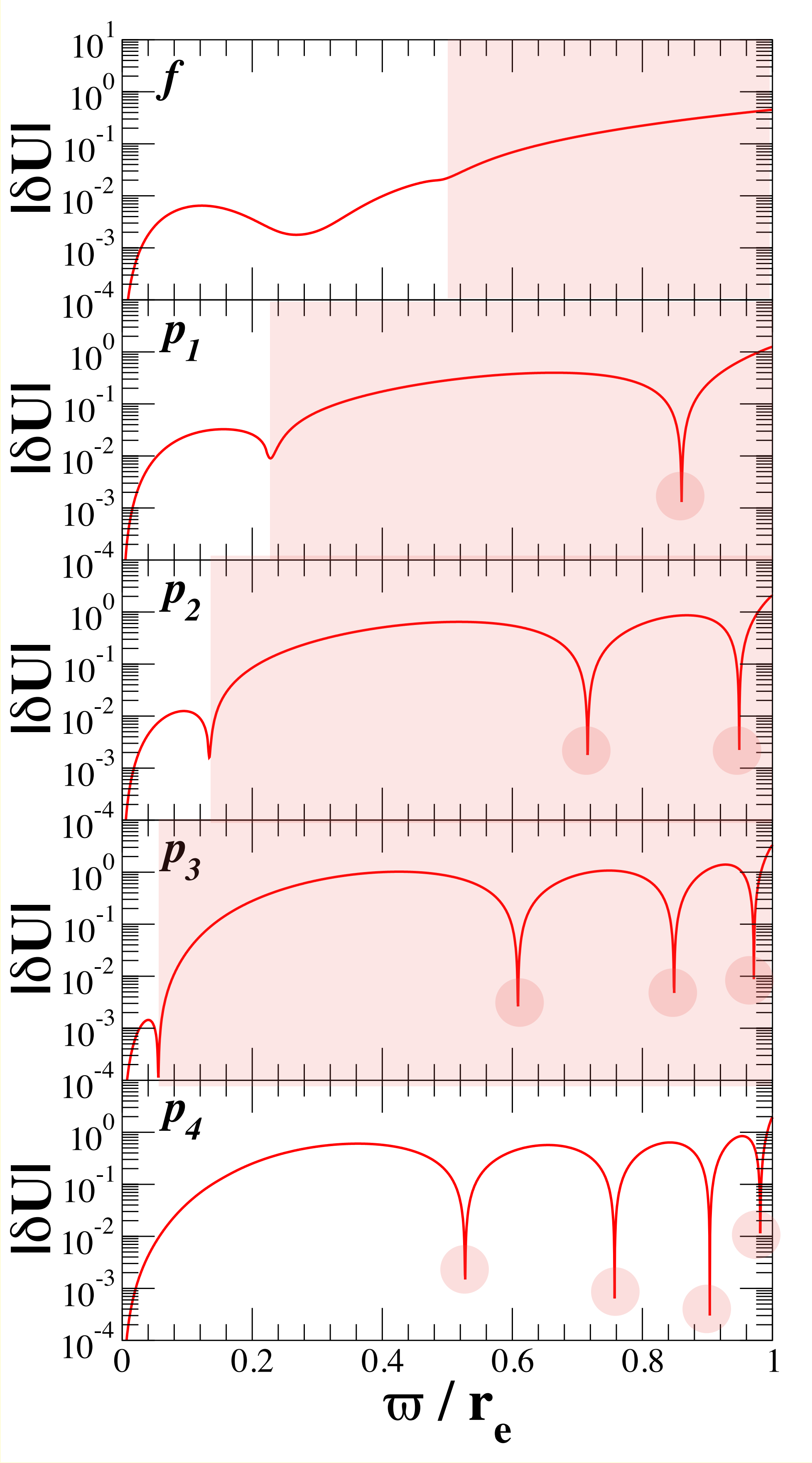
Regularity condition		Vanishing enthalpy	Regularity condition
$\delta U \quad \delta\Phi$	$\times$	$\delta h + \xi^j \nabla_j h = 0$	$\delta\Phi$

Eigenfrequencies

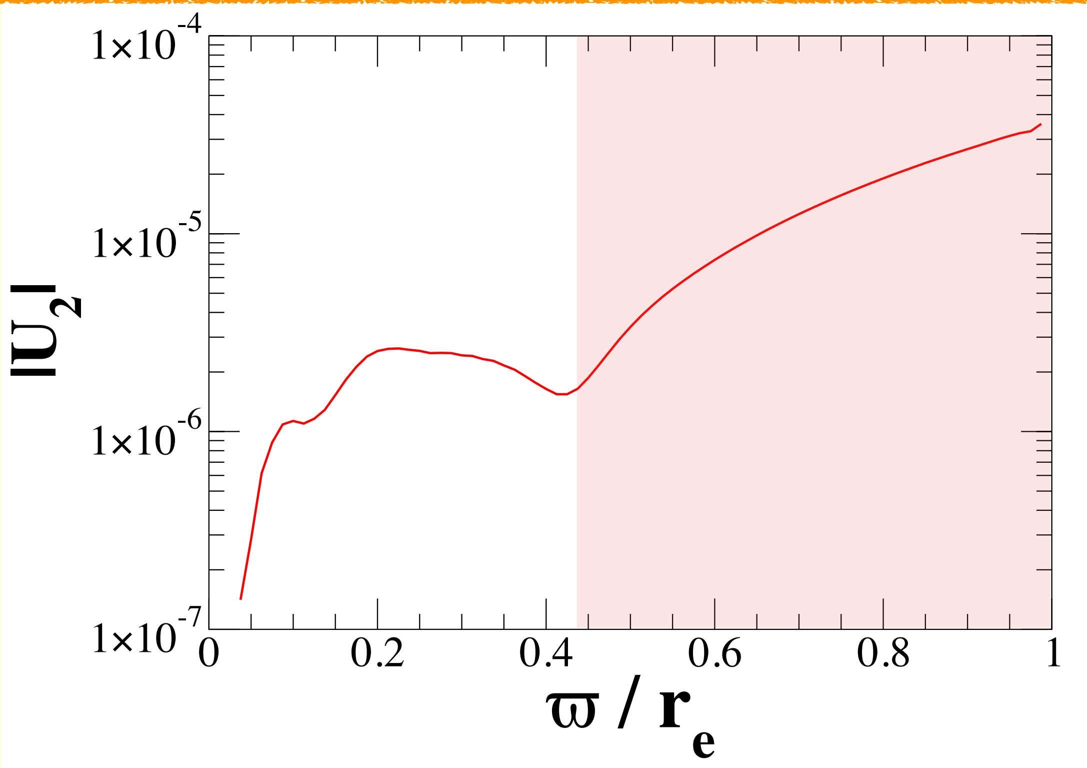


- Discrete Eigenfrequencies are found in the system
- Unstable modes are found only when corotation exists

Eigenfunctions



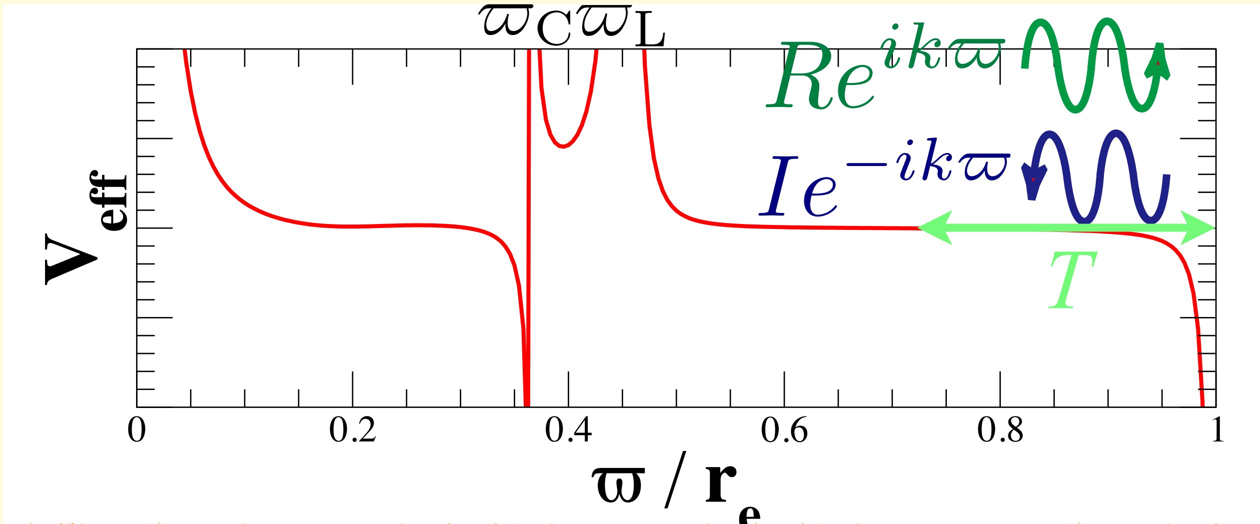
- Continuous node number is found in each mode (interpret these modes as f- and p-modes)
- Eigenfunctions oscillate between corotation and surface



Simulation results have the same feature!

Scattering Problem

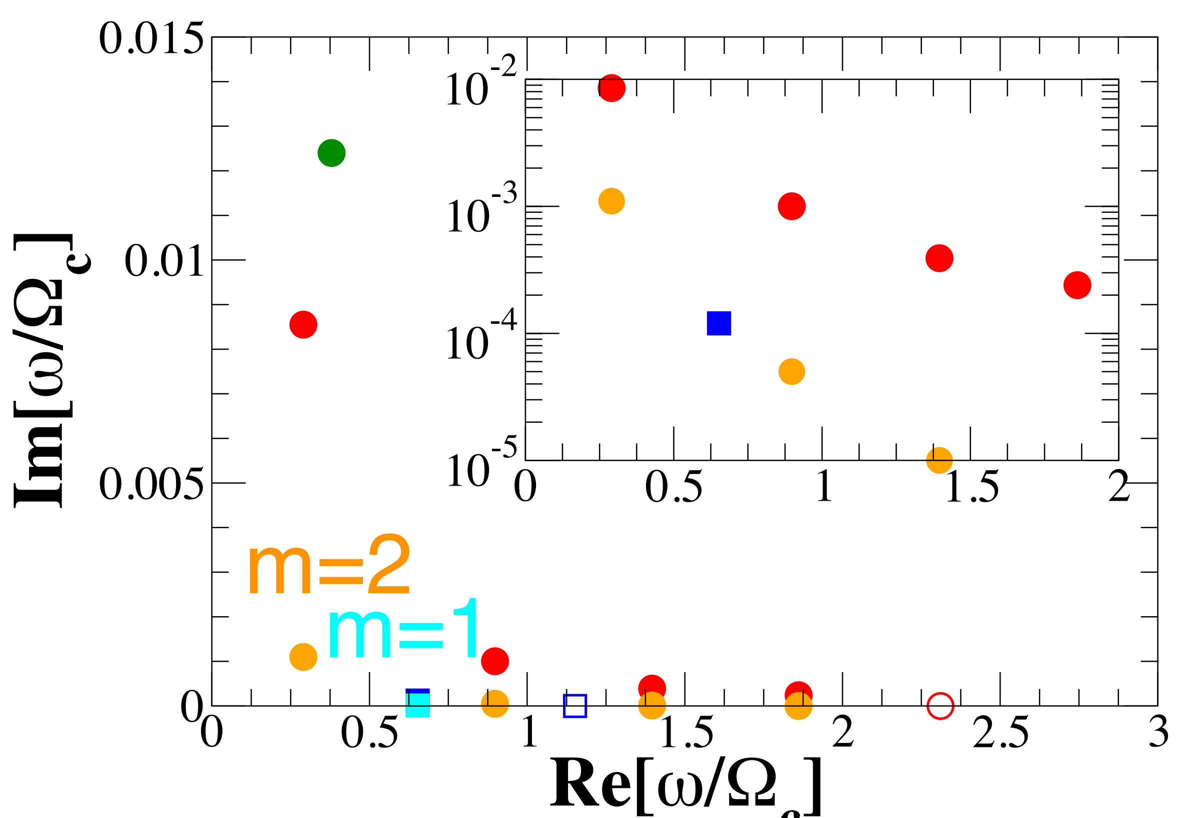
(Yoshida & MS 17; MS 18)



(only cylindrical model)

$$\begin{cases} \left[\dfrac{d^2}{d\varpi^2} - V_{\rm eff}(\varpi)\right]\delta\eta(\delta U) = S_\eta(\delta\Phi) \\ \mathcal{L}_\Phi \delta\Phi = S_\Phi(\delta U) \end{cases}$$

- Insert “incoming wave” with real part of unstable eigenfrequency from the surface along the equatorial plane
- Corotation acts as a potential barrier
- Investigate the amplification rate of the “reflection wave”



Traveling time $T = \dfrac{2}{\Re[\omega_{\rm ref}]}\displaystyle\int_{\varpi_{V_{\rm min}}}^{\varpi_{V_{\rm max}}} k\, d\varpi$

Amplification rate $\dfrac{|R|}{|I|} = \exp[T/\tau]$

Growth Timescale $\tau = \dfrac{T}{\ln|R| - \ln|I|}$

Picture of low T/W dynamical instabilities

1. Suppose a mode contains corotation inside the star
2. The mode grows exponentially in a nonaxisymmetric sense because of amplification mechanism
3. After angular momentum transport sets in, the amplification condition may no longer be satisfied
4. The mode saturates (or damps)

If this picture is correct,

All modes (f, p, r, w, …) are the potential candidates for gravitational wave sources

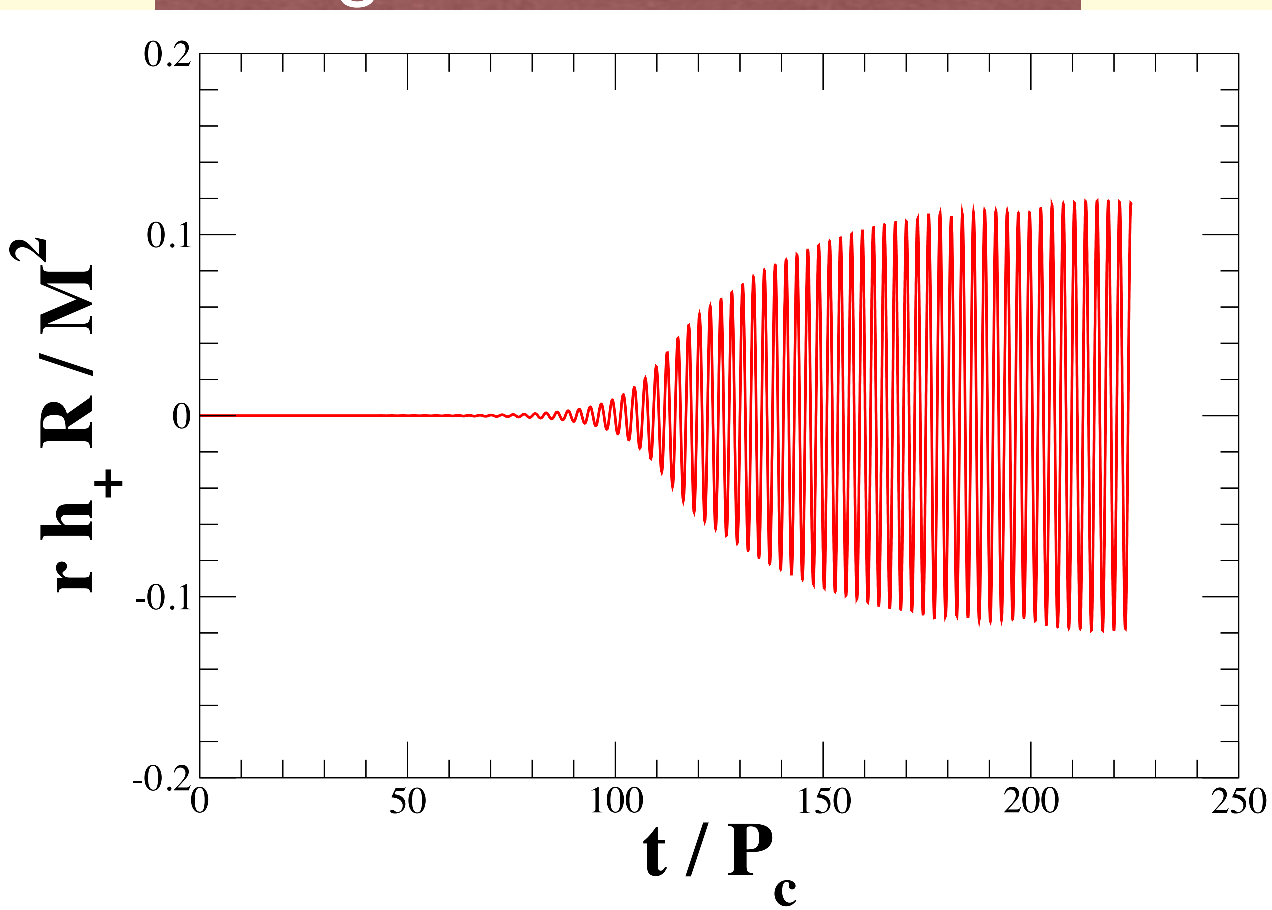
However,

1. Existence of corotation inside the star is quite limited, and requires differential rotation
2. Growth timescale depends on the configuration of effective potential (normally powerful for f-mode)
3. Saturation amplitude depends on the efficiency of angular momentum transport

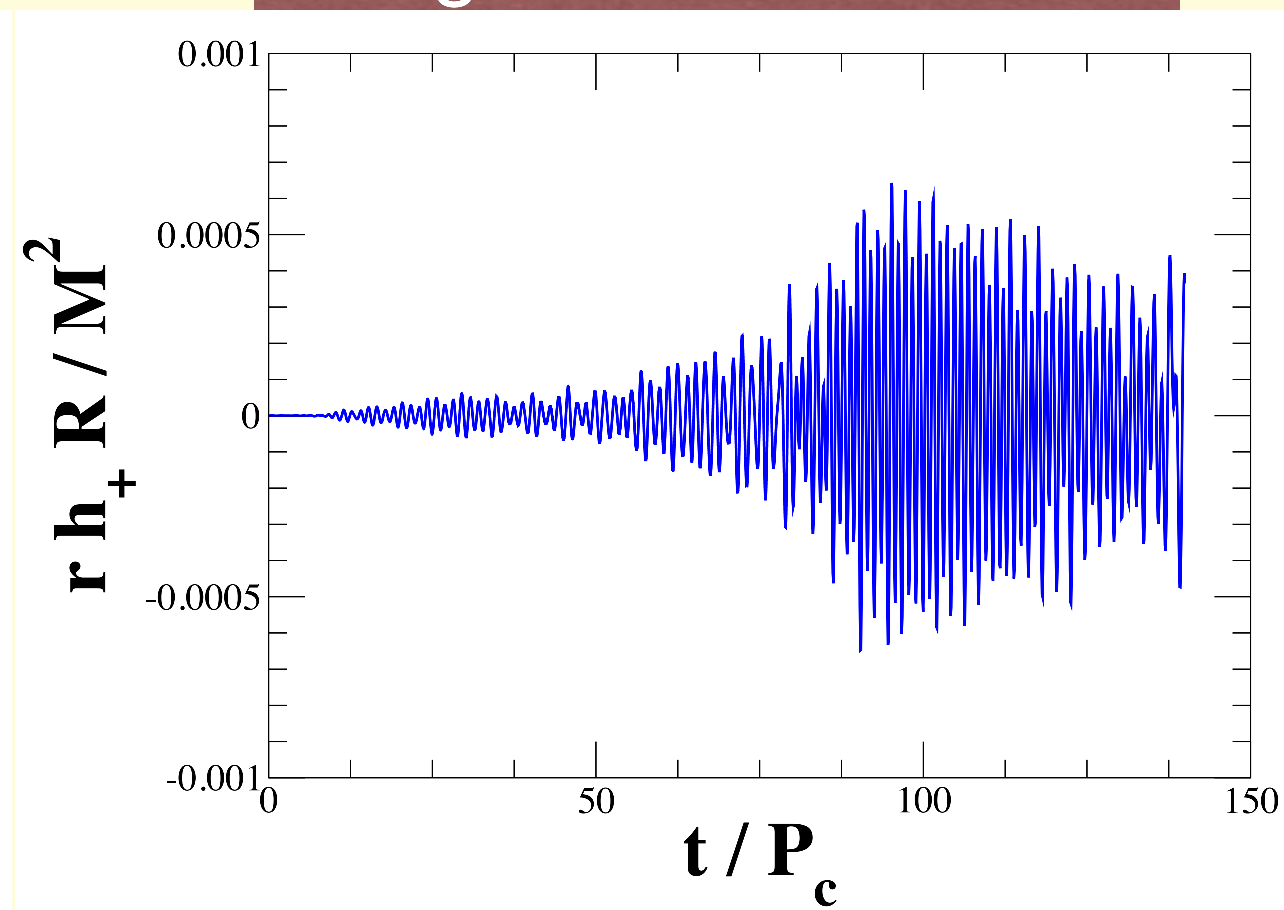
4. Gravitational Waves and Stiffness of Equation of State

Extract gravitational waves from rotational axis using quadrupole formula

e.g. m=2 dominant



e.g. m=1 dominant

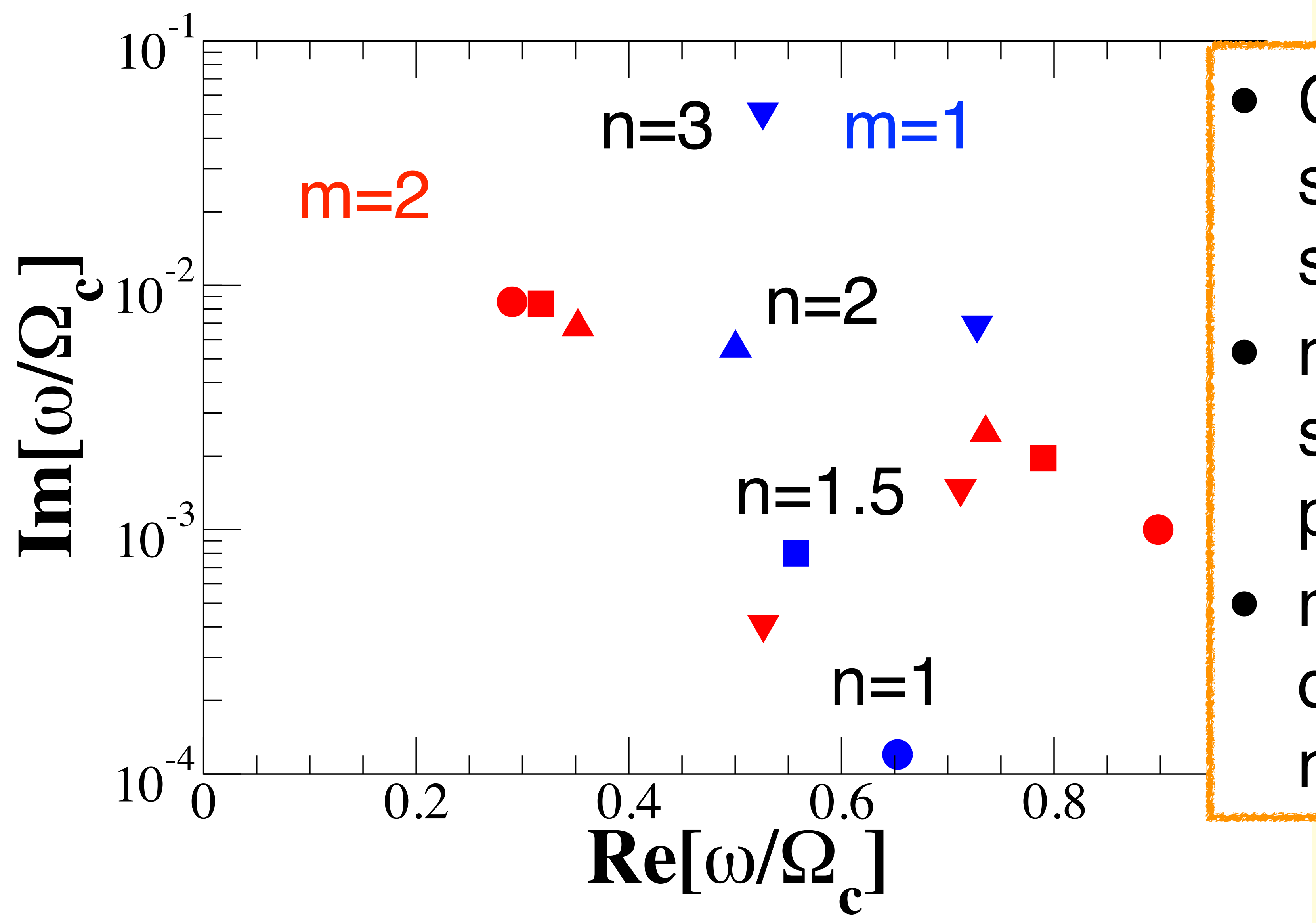


If we have succeeded in decomposing m=1 and m=2 waveform (depends on the observation angle to the source) …

- Enable to constrain stiffness of EoS from the decomposed waveform (azimuthal mode dominancy)

Stiffness of equation of state

(MS 18)



- Growth rate of m=1 strongly depends on stiffness of EoS
- m=1 and m=2 have almost same contribution in n=2 polytropic star
- m=1 mode takes a dominant contribution in n=3 polytropic star

→ Azimuthal mode dominancy depends on stiffness of EoS, which may be a useful tool in principle

5. Summary and Future directions

We have investigated perturbative approaches of low T/W dynamical instability in differential rotating stars in the equatorial plane, and compare the results with those of numerical simulations

- No new mode is created, but existing (f- and p-)modes become unstable in the corotation band
- Eigenfunction oscillates between corotation and surface, which requires reinterpretation of pulsation in differentially rotating stars
- Amplification of the reflection sound waves because of corotation can be understood as a physical mechanism
- Enable to constrain stiffness of EoS by the direct observation of mode decomposed gravitational waves

- Perturbative approaches in axisymmetric background are necessary for complete understanding
- Plays some role in realistic supernovae explosion (requires 3D simulation)?