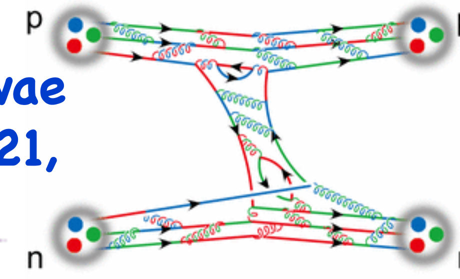
Workshop "Physics of core-collapse supernovae and compact star formations", Mar. 19-Mar.21, 2018, Waseda University, Japan



The properties of neutron star in the relativistic central variational method

Jinniu Hu (胡 金牛)

School of Physics, Nankai University

Outline

- Introduction
- Relativistic variational method
- Neutron star with variational method
- Summary

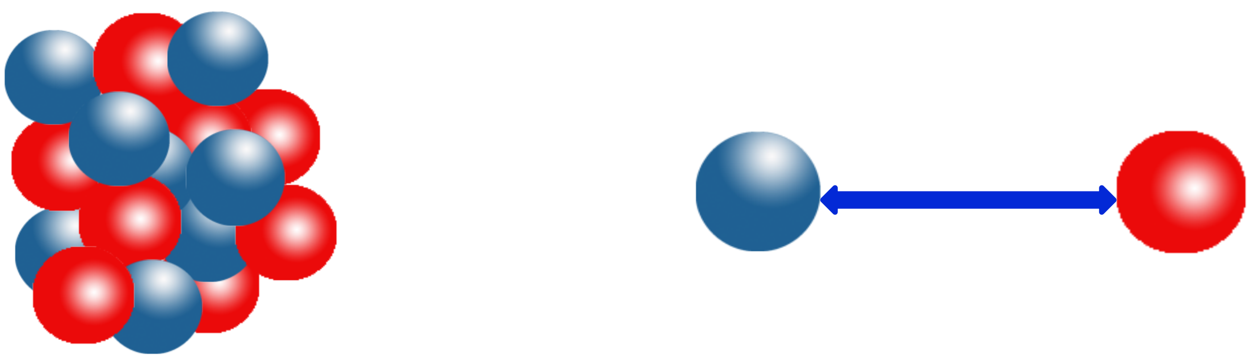
The nucleon-nucleon interaction

- **Realistic nucleon-nucleon (NN) interaction:**

  NN interaction in free space

- **Effective nucleon-nucleon interaction**

  NN interaction in nuclear medium



The nucleon-nucleon interaction

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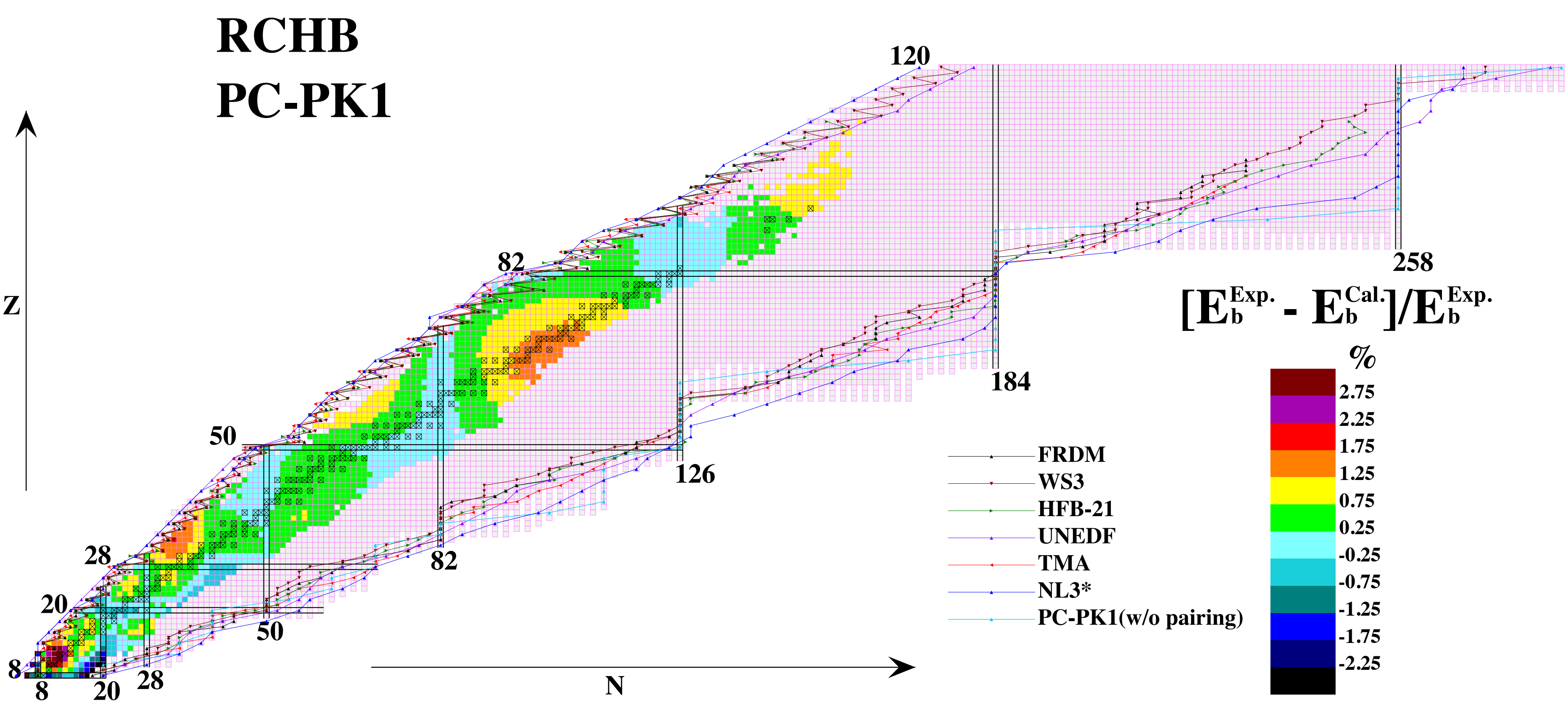

X. Xia et al., Atomic Data and Nuclear Data Tables, 121(2018)1

The nucleon-nucleon interaction

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- **Effective nucleon-nucleon interaction**

  Density functional theory from the properties of nuclear matter or finite nuclei

- **Skyrme interaction (zero range)**

  J. W. Negele and D. Vautherin, Phys. Rev. C 5(1972)1472

- **Gogny interaction (Finite range)**

  J. Decharge and D. Gogny, Phys. Rev. C 21(1980)1568

- **Relativistic mean field (RMF) (meson exchange force)**

  H. P. Duerr, Phys. Rev. 103(1956)469, NL3,TM1,DD-M2……

- **Relativistic Hartree Fock (RHF)**

  A. Bouyssy, J. -F Mathiot, and N. Van Giai, Phys. Rev. C 36(1987)380, PKO, PKA,...

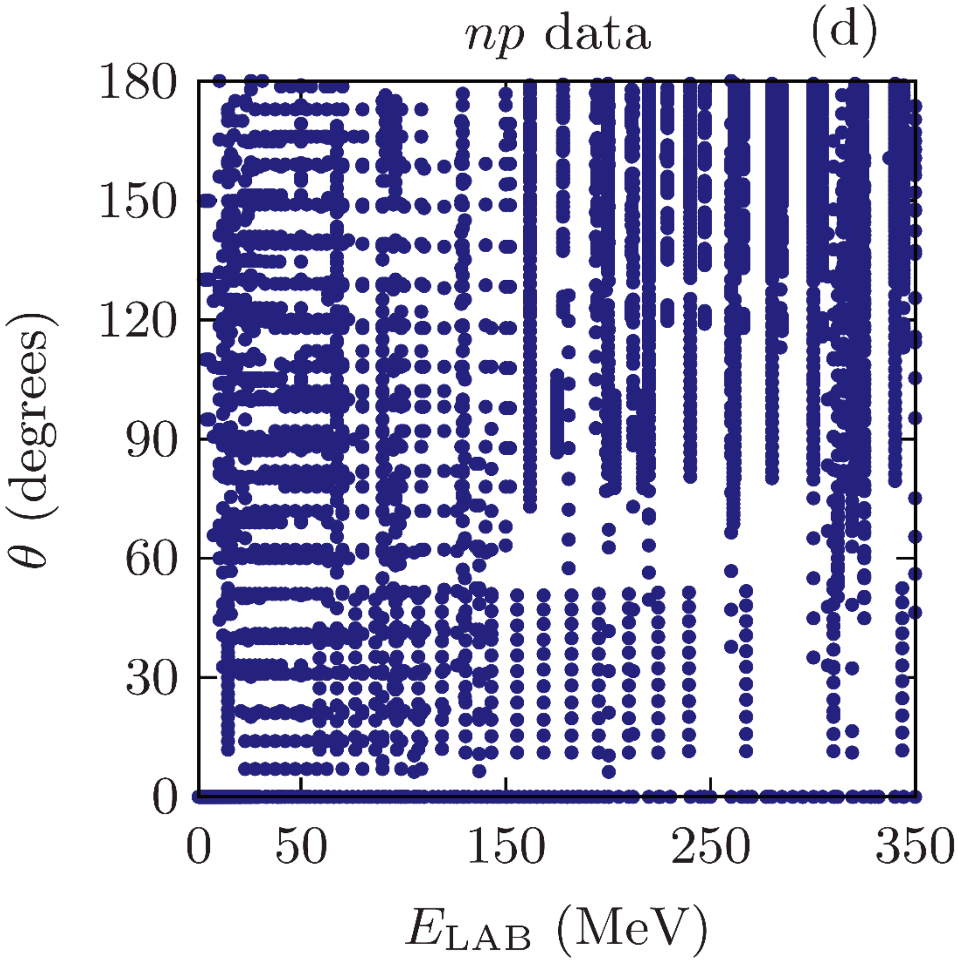
- **Point coupling interaction (zero range)**

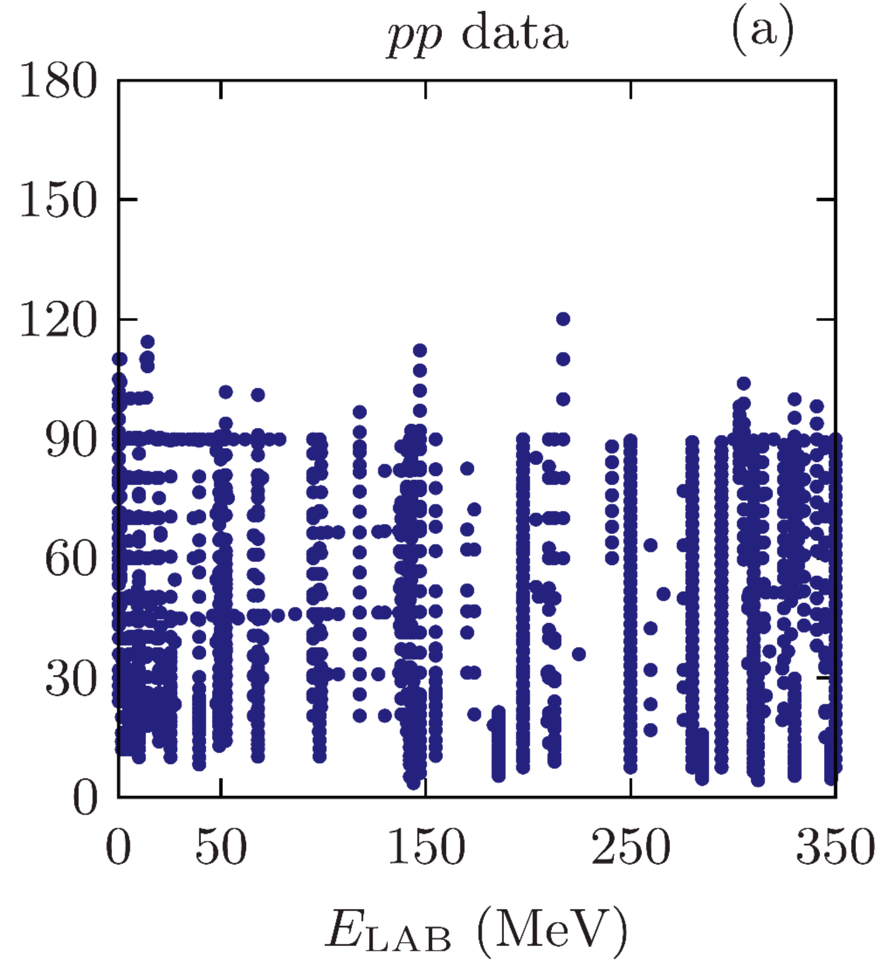
  B.A. Nikolaus, T. Hoch, and D.G. Madland, Phys. Rev. C 46(1992)1757, PC-PK1, DD-PC……

...

The NN scattering data

R. Navarro Pérez, J. E. Amaro, and E. Ruiz Arriola, Phys. Rev. C 89(2014)064006





The realistic NN interaction

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➢ Realistic nucleon-nucleon interaction

Meson exchange potential models from the NN scattering data

➢ Reid potential

R.V. Reid, Ann. Phys. (N.Y.) 50(1968)411

➢ Argonne V18 potential

R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C 51(1995)38

➢ CD Bonn potential

R. Machleidt, Phys. Rev. C 63(2001)024001

➢ N^4LO chiral potential

D. R. Entem, N. Kaiser, R. Machleidt, and Y. Nosyk, Phys. Rev. C 91 (2015) 014002

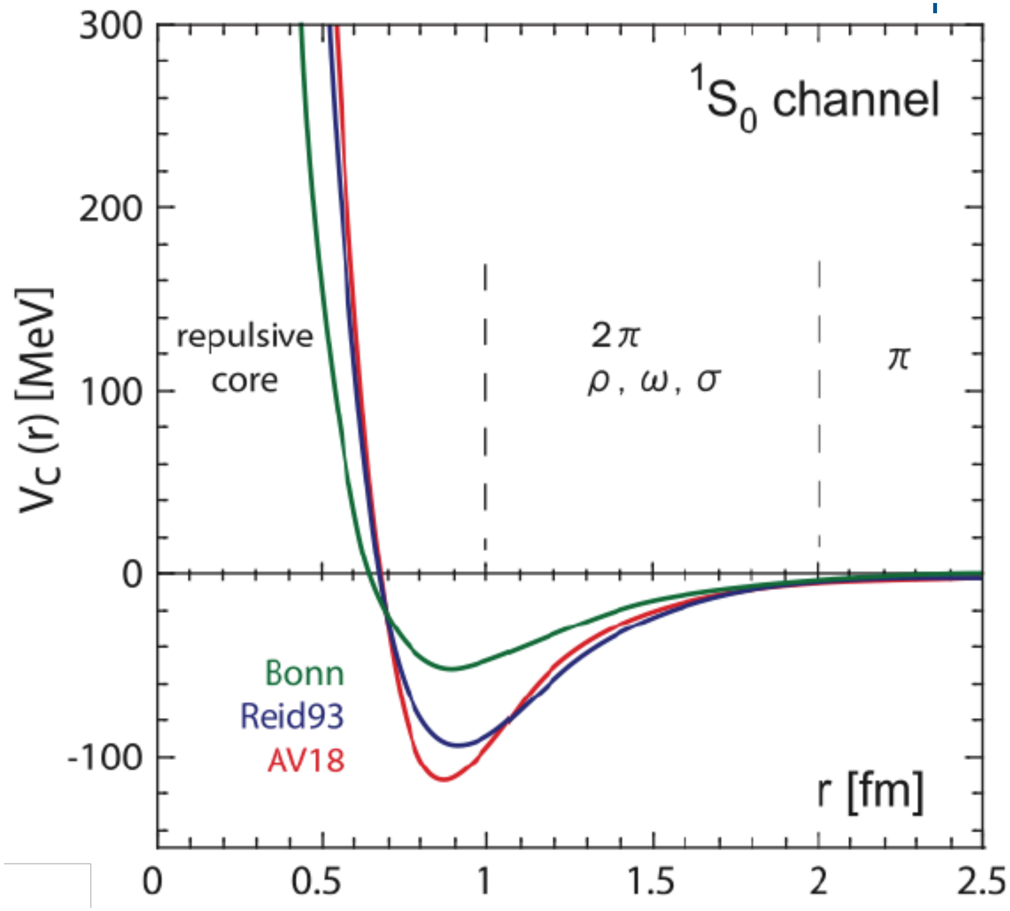
E. Epelbaum, H. Krebs, U.-G. Meissner, Phys. Rev. Lett. 115 (2015) 122301

D. R. Entem, R. Machleidt, and Y. Nosyk, Phys. Rev. C 96 (2017) 024004

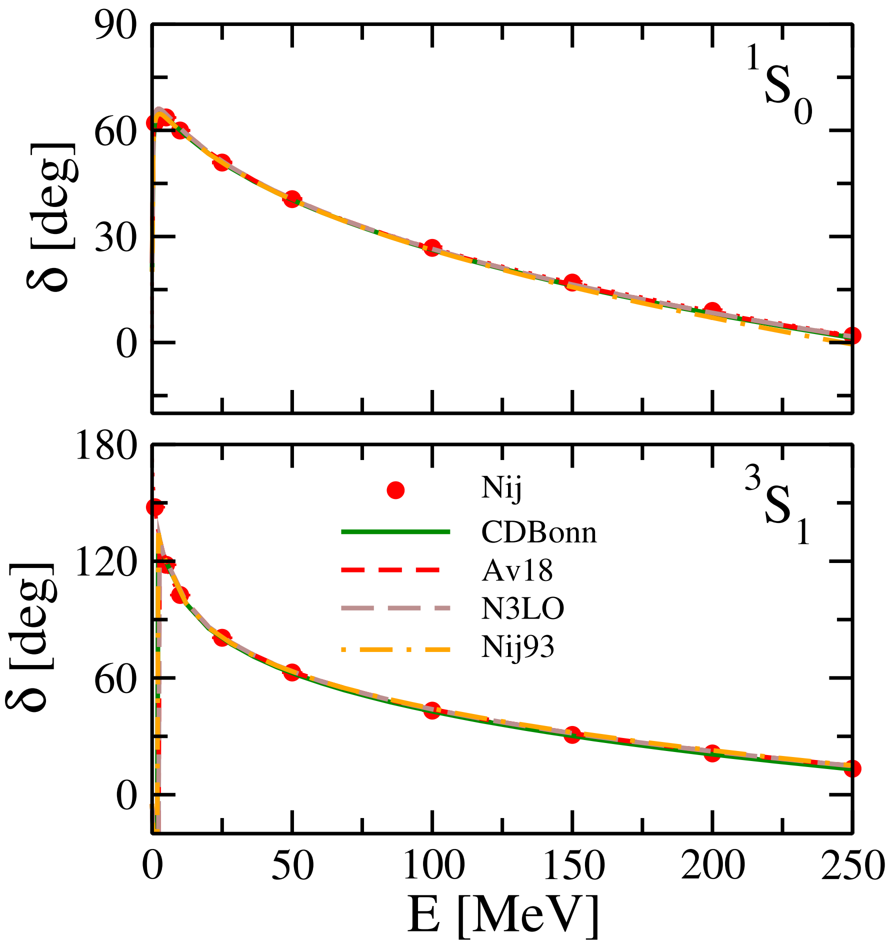
➢ Lattice QCD potential

N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. 99(2007)022001

➢**on-shell behavior**



➢**Phase shifts**



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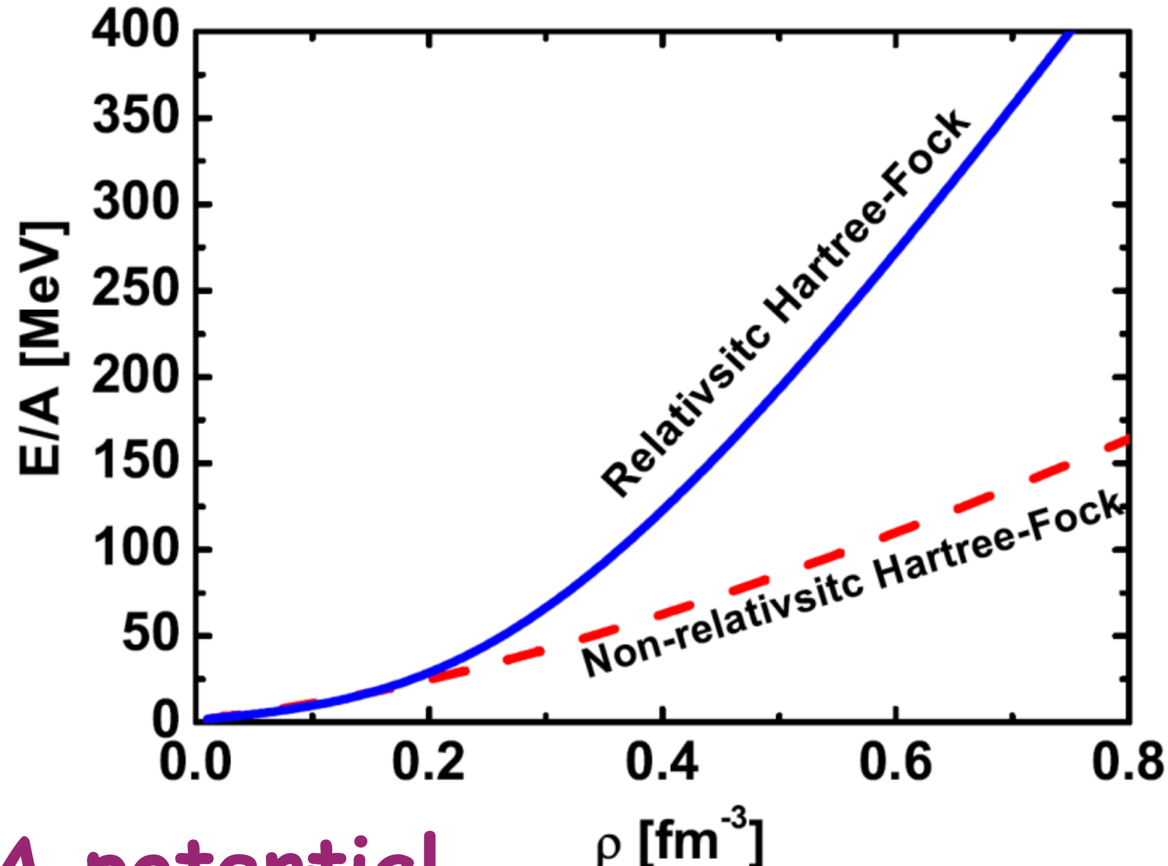
Realistic NN interaction for symmetric nuclear matter with mean-field theory



Bonn A potential

Ab initio calculation

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- **Variational methods**

A.Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58 (1998)180
H. Togashi and M. Takano, Nucl. Phys. A 902(2013)53

- **Brueckner-Hartree-Fock (BHF) method**

Z. H. Li, et al., Phys. Rev. C 74 (2006)047304

- **Self-consistent Green's function method**

A. Carbone, A. Polls, A. Rios, Phys. Rev. C 88 (2013)044302

- **Coupled-cluster theory**

G. Hagen, at al., Phys. Rev. C 89 (2014) 014319

- **Many-body perturbation theory**

C. Drischler, V. Soma, A. Schwenk, Phys. Rev. C 89 (2014)025806.

- **Relativistic Brueckner-Hartree-Fock (RBHF) method**

R.Brockmann and R. Machleidt, Phys. Rev. C 42 (1990)1965.

Cluster expansion

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Different ways to treat cluster expansion

Lowest order constrained variational (LOCV) method

M. Modarres, A. Tafrihi, A. Hatami, Nucl. Phys. A 879 (2012) 1.

Fermi hypernetted chain (FHNC) method

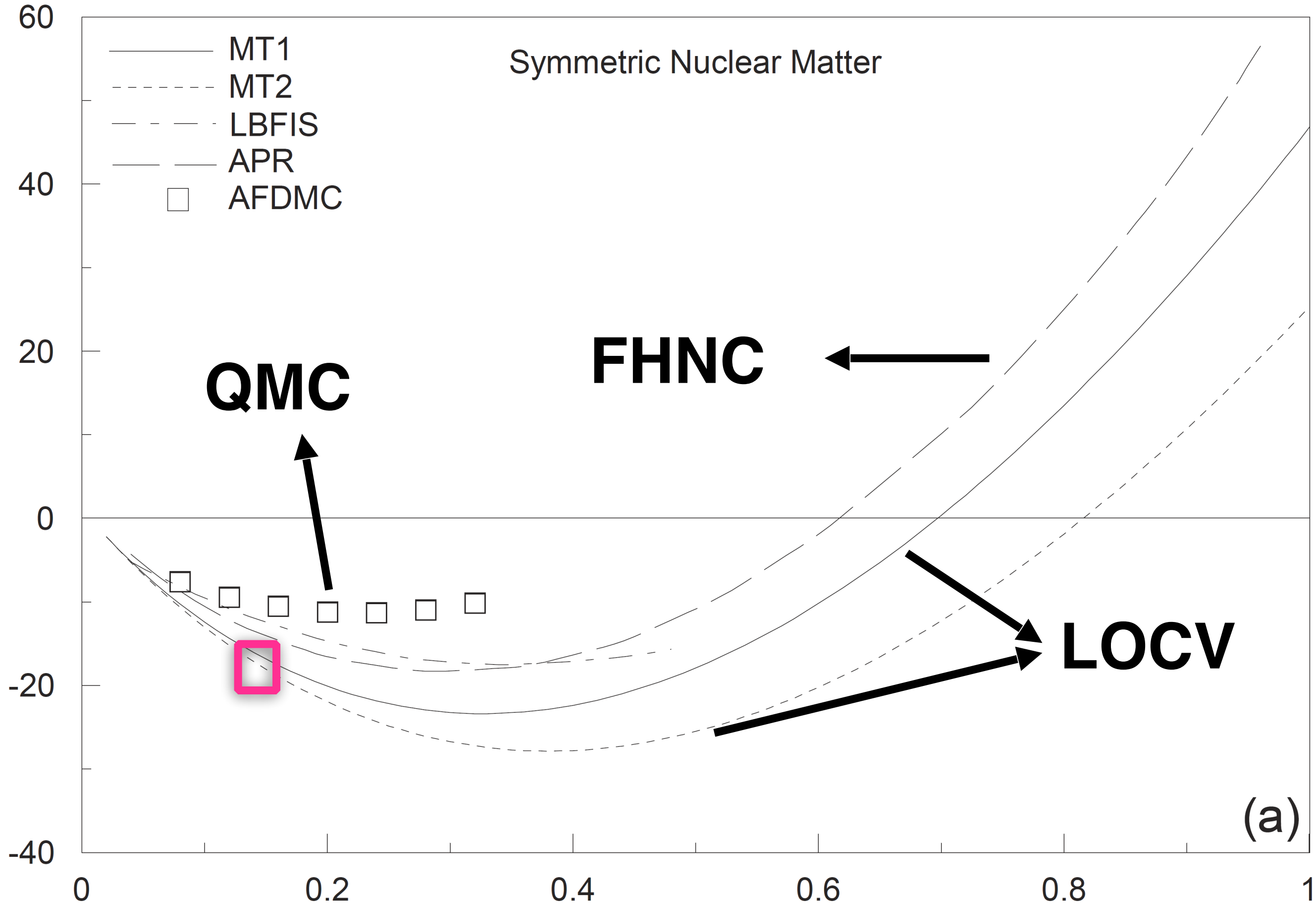
A. Akmal, V.R. Pandharipande, D.G. Ravenhall, Phys. Rev. C 58 (1998) 1804. (APR EOS)

Quantum Monte Carlo (QMC) method

S. Gandolfi, et al., Phys. Rev. C 79 (2009) 054005

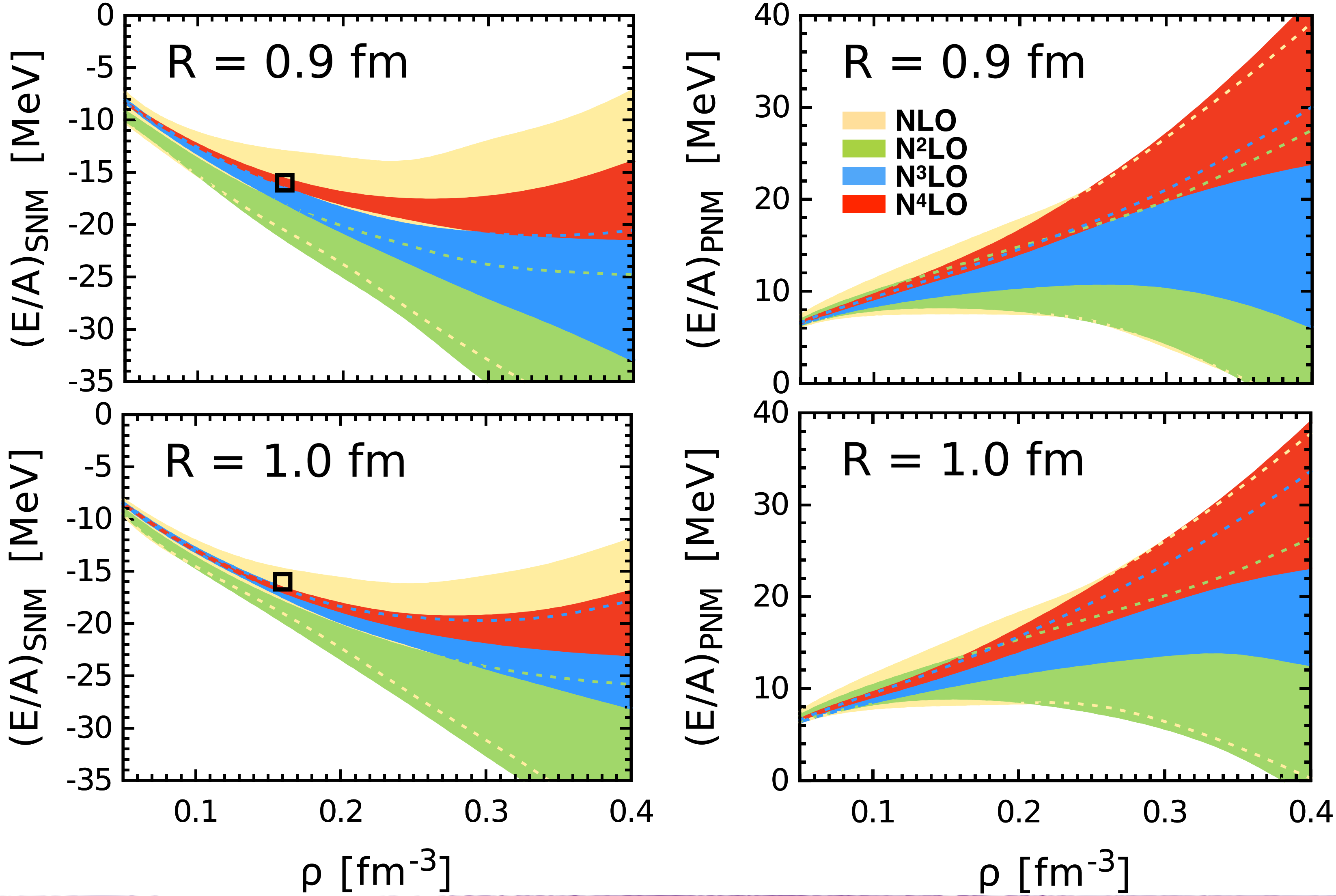
Non-Relativistic calculations

A. Tafrihi, M. Modarres, JPC 702 (2016) 012015.

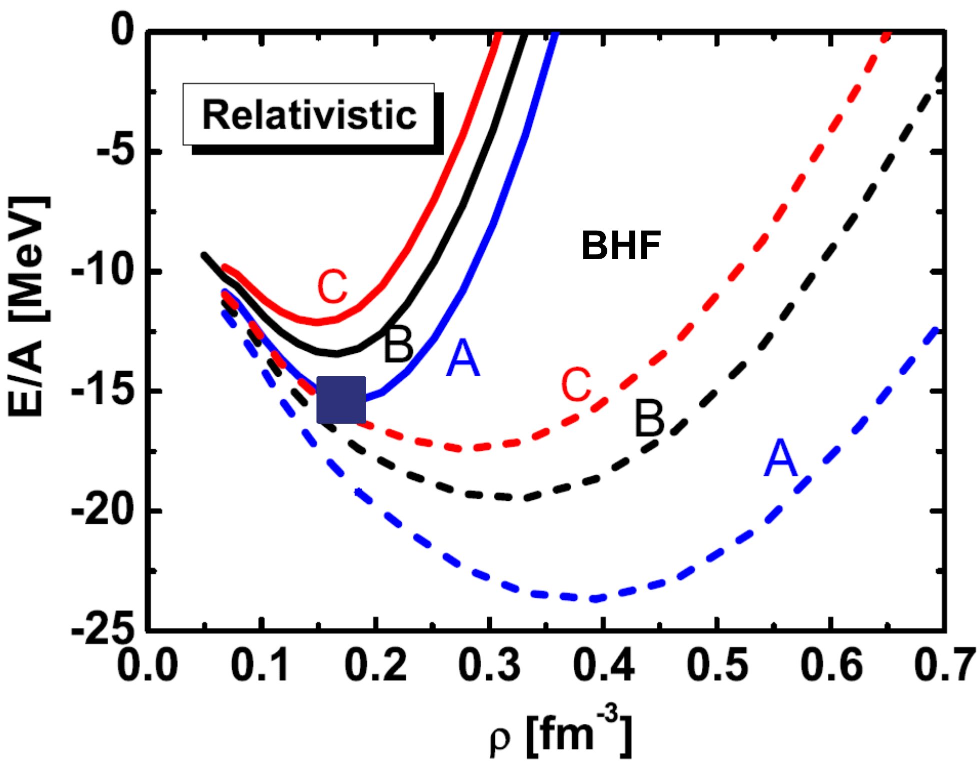


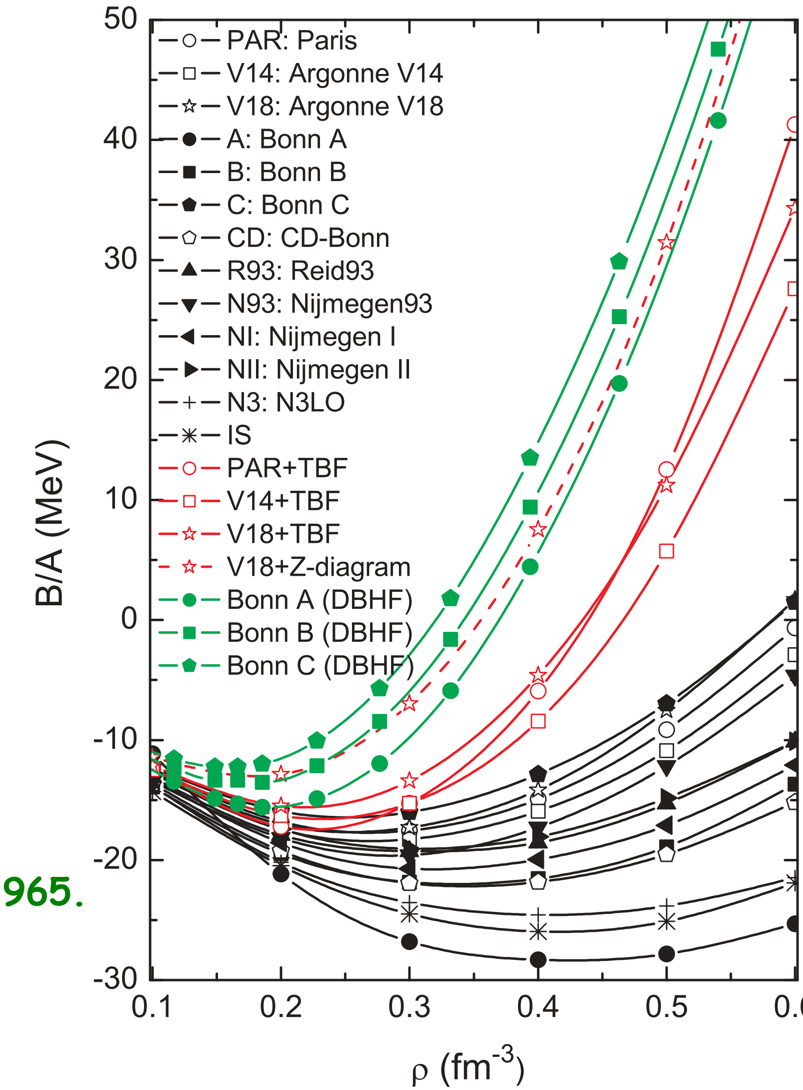
Non-relativistic calculations

JH, Y. Zhang, E. Epebaum, U.-G. Meissner, and J. Meng, Phys. Rev. C 96(2017)034307



Relativistic effect VS. TBF





R.Brockmann and R. Machleidt, Phys. Rev. C 42 (1990)1965.
Z. H. Li, et al., Phys. Rev. C74 (2006) 047304

Outline

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- Summary

Correlation functions

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The key of ab initio calculations in nuclear matter

Real wave function Ψ

Non-interacting wave function $\Psi = F\Phi$

Variational method

$$\frac{\partial}{\partial \Psi} \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0$$

Correlation functions

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Brueckner Hartree-Fock theory

Z. H. Li, et al., Phys. Rev. C 74 (2006)047304

$$\Psi = \left(1 - \frac{Q}{e} G\right) \Phi$$

Variational method

A.Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58 (1998)1804

$$\Psi = \left(S \prod_{i<j} F_{ij}\right) \Phi$$

Green function method

S. Gandolfi, et al., Phys. Rev. C 79 (2009) 054005

$$\Psi = \sum_i c_i e^{-(H-E_0)\tau} \Phi$$

Unitary correlation operator method

H. Feldmeier, T.Neff, R.Roth, and J.Schnack, et al., Nucl. Phys. A632 (1998) 61

$$\Psi = U\Phi$$

The correlation functions in variational method

$$F_{ij} = \sum_p f_{ij}^{p}(r) O^p$$

Spin, isospin (O^p)

Jastrow function (f_{ij}^{p})

For example,

$$f_{ij} = \sum_{t=0}^{1}\sum_{\mu}\sum_{s=0}^{1}\left[f_{\mathrm{C}ts}^{\mu}(r_{ij}) + s f_{\mathrm{T}t}^{\mu}(r_{ij}) S_{\mathrm{T}ij} + s f_{\mathrm{SO}t}^{\mu}(r_{ij})(\boldsymbol{L}_{ij}\cdot\boldsymbol{s})\right] P_{tsij}^{\mu}$$

H. Togashi and M. Takano, Nucl. Phys. A 902(2013)53

Cluster expansion

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The expectation values of Hamiltonian

J. Morales, et al. Phys. Rev. C 66(2002)054308

$$E = T_F + \frac{\left\langle \Phi_A(k_F) \left| \left[\mathcal{S}\prod_{i<j} F_{ij} \right] (H - T_F) \left[\mathcal{S}\prod_{i<j} F_{ij} \right] \right| \Phi_P(k_F) \right\rangle}{\left\langle \Phi_A(k_F) \left| \left[\mathcal{S}\prod_{i<j} F_{ij} \right] \left[\mathcal{S}\prod_{i<j} F_{ij} \right] \right| \Phi_P(k_F) \right\rangle}$$

Cluster expansion in variational energy

$$F_{ij}^{c} = \left[f_{ij}^{c}(r) \right]^2 - 1$$

$$F_{ij}^{p>1} = 2 f_{ij}^{c}(r) f_{ij}^{p}(r) O_{ij}^{p}$$

$$F_{ij}^{p>1,q>1} = f_{ij}^{p}(r) f_{ij}^{q}(r) O_{ij}^{p} O_{ij}^{q}$$

Hamiltonian

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$$H = \int \mathrm{d}^3\mathbf{x}\overline{\psi}(\mathbf{x})(-\mathrm{i}\gamma\cdot\nabla + M_N)\psi(\mathbf{x}) + \frac{1}{2}\sum_{\substack{i=\sigma,\delta,\\ \eta,\pi,\omega,\rho}}\int \mathrm{d}^3\mathbf{x}'\mathrm{d}^3\mathbf{x}\overline{\psi}(\mathbf{x}')\overline{\psi}(\mathbf{x})\frac{\Gamma_i(1,2)}{m_i^2+\mathbf{q}^2}\psi(\mathbf{x})\psi(\mathbf{x}').$$

$$\Gamma_\sigma(1,2) = -g_\sigma^2,$$

$$\Gamma_\delta(1,2) = -g_\delta^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$

$$\Gamma_\eta(1,2) = -\left(\frac{f_\eta}{m_\eta}\right)^2(\not{q}\gamma_5)_1(\not{q}\gamma_5)_2,$$

$$\Gamma_\pi(1,2) = -\left(\frac{f_\pi}{m_\pi}\right)^2(\not{q}\gamma_5)_1(\not{q}\gamma_5)_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$

$$\Gamma_\omega(1,2) = g_\omega^2\gamma_\mu(1)\gamma^\mu(2),$$

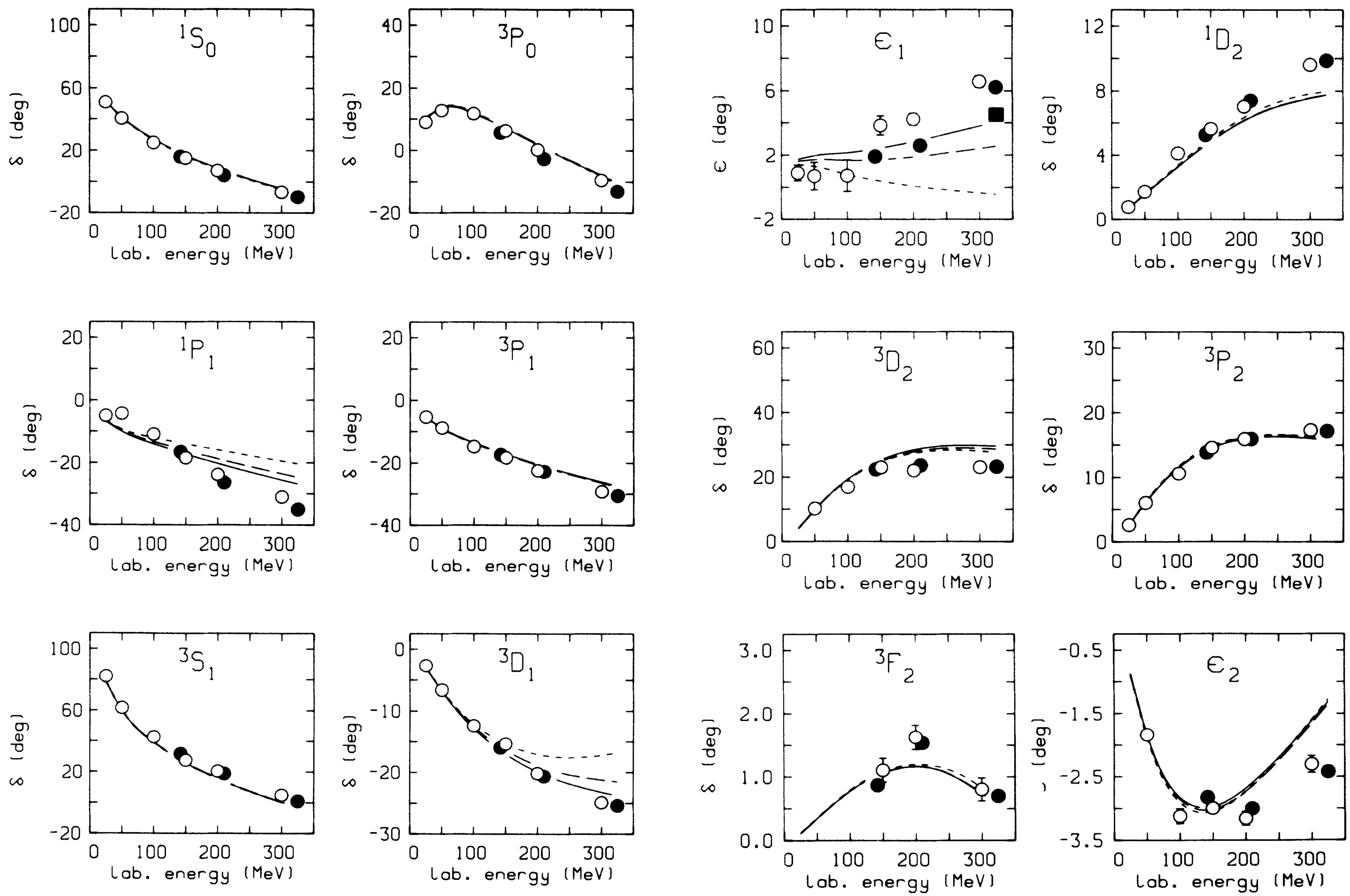
$$\Gamma_\rho^V(1,2) = g_\rho^2\gamma_\mu(1)\gamma^\mu(2)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$

$$\Gamma_\rho^T(1,2) = \left(\frac{f_\rho}{2M_N}\right)^2 q_\nu\sigma^{\mu\nu}(1)q^\lambda\sigma_{\mu\lambda}(2)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$

$$\Gamma_\rho^{VT}(1,2) = \mathrm{i}\left(\frac{g_\rho f_\rho}{M_N}\right)\gamma_\mu(2)\sigma^{\mu\nu}q_\nu(1)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2,$$

Bonn potentials

R. Brockmann, R. Machleidt, Phys. Rev. C 42 (1990)1965.



Variational method

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The energy with correlation function

$$E_c = \frac{\langle \Phi | F^\dagger H F | \Phi \rangle}{\langle \Phi | F^\dagger F | \Phi \rangle}$$

Relativistic plane wave function

Central correlation function

$$F = \prod_{i<j}^{A} f(r_{ij})$$

Constraint of correlation function

$$\int \mathrm{d}^3 \mathbf{r}_{ij} [f^2(r_{ij}) - 1] = 0$$

Variational method

Correlated energy density

$$\mathcal{E}_c = \frac{E_c}{\Omega} = \frac{1}{\Omega}\langle\Phi|\widetilde{H}|\Phi\rangle$$

$$= \langle T\rangle + \langle T_c\rangle + \langle V\rangle,$$

Correlated Hamiltonian

$$\widetilde{H} = \sum_{i}^{A} T_i + \frac{1}{2}\sum_{i,j}^{A}\widetilde{V}_{ij}$$

$$= \sum_{i}^{A} T_i + \frac{1}{2}\sum_{i,j}^{A}\{f^{\dagger}(r_{ij})[T_i + T_j + V_{ij}]f(r_{ij}) - (T_i + T_j)\}.$$

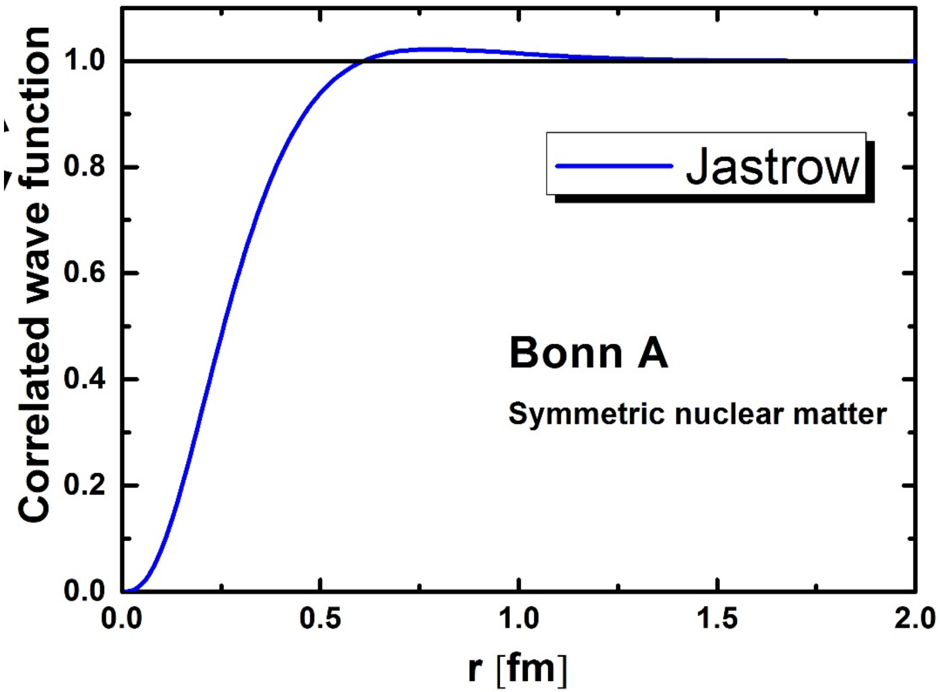
The correlation function

The form of correlation function

$$f(r) = 1 - (c_0 + c_1 r + c_2 r^2 + c_3 r^3)\, e^{-c_4 r}$$

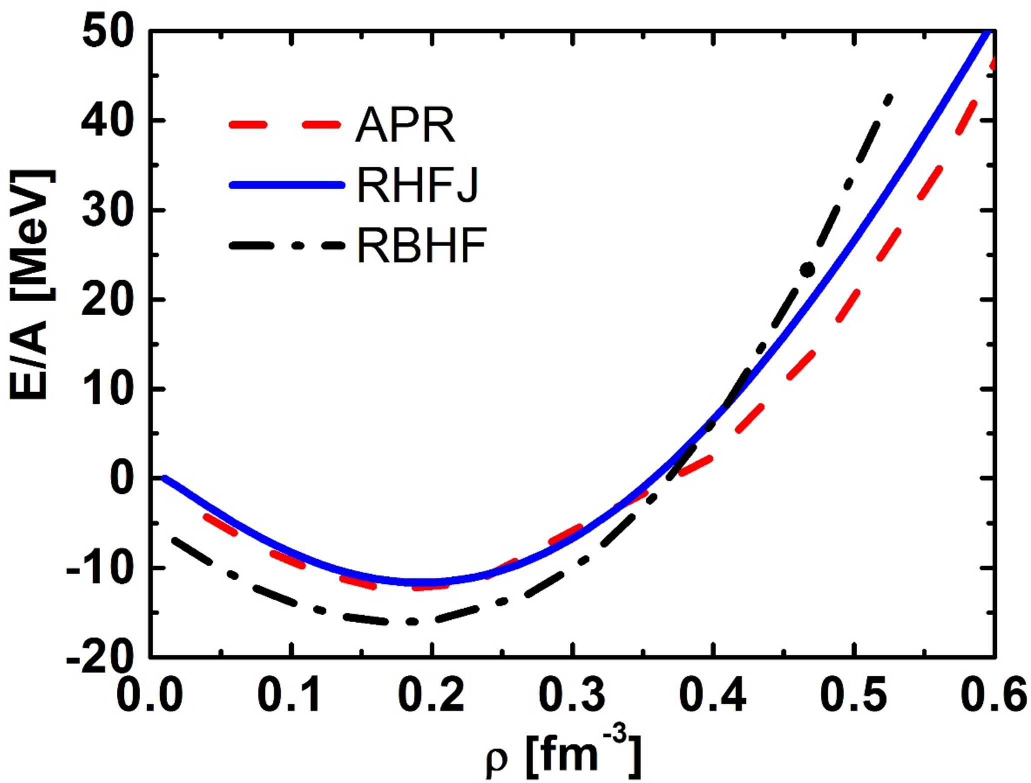
Variational method

$$\frac{\partial \mathcal{E}_c}{\partial c_i} = 0$$



Symmetric nuclear matter

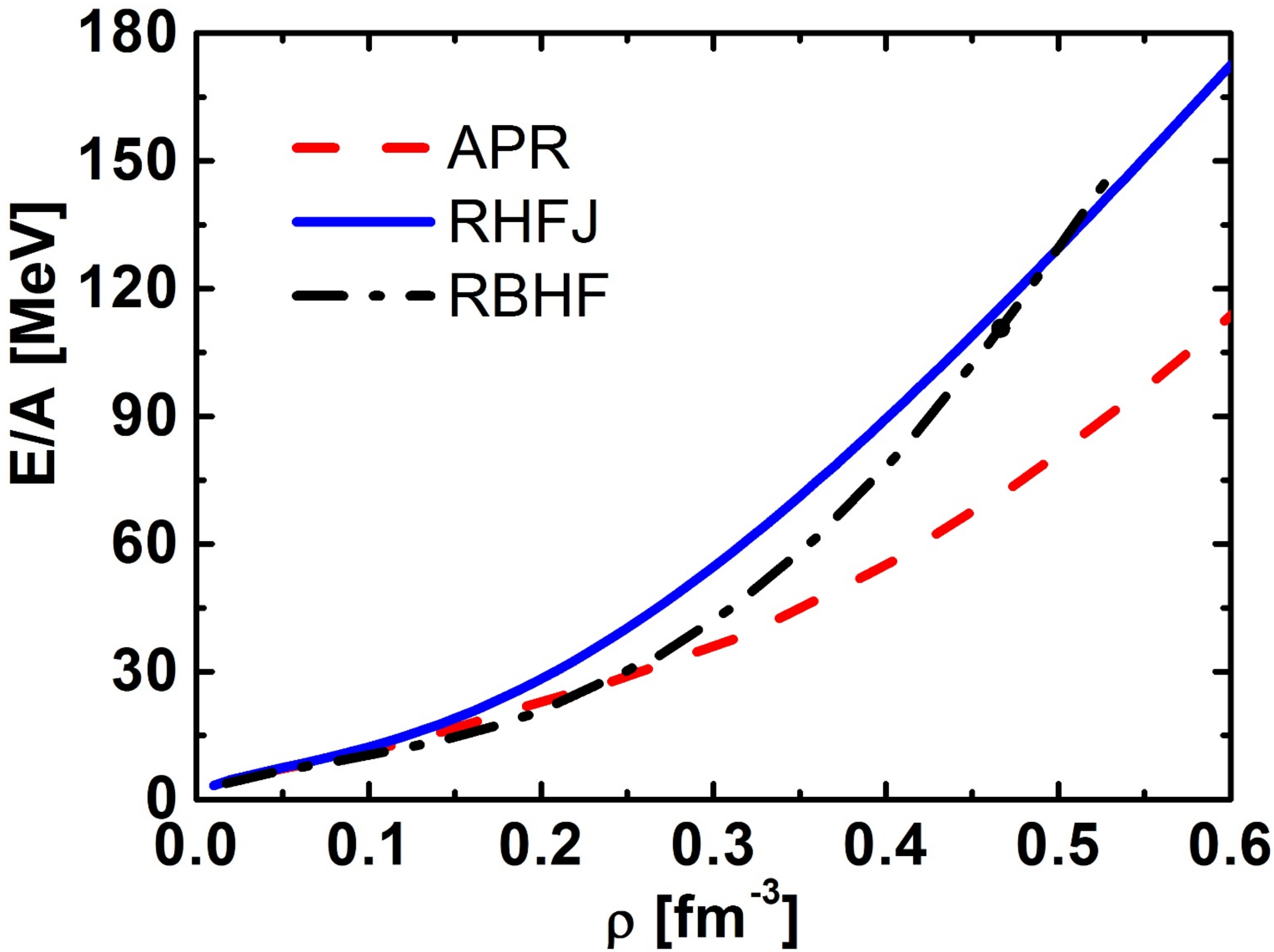
ρ (fm^{-3})	E/A (MeV)	K (MeV)	a_4 (MeV)	T_c/A (MeV)
0.192	−11.66	264	37.88	6.57



JH, H. Toki and H. Shen, Jour. Phys G. 38(2011)085105

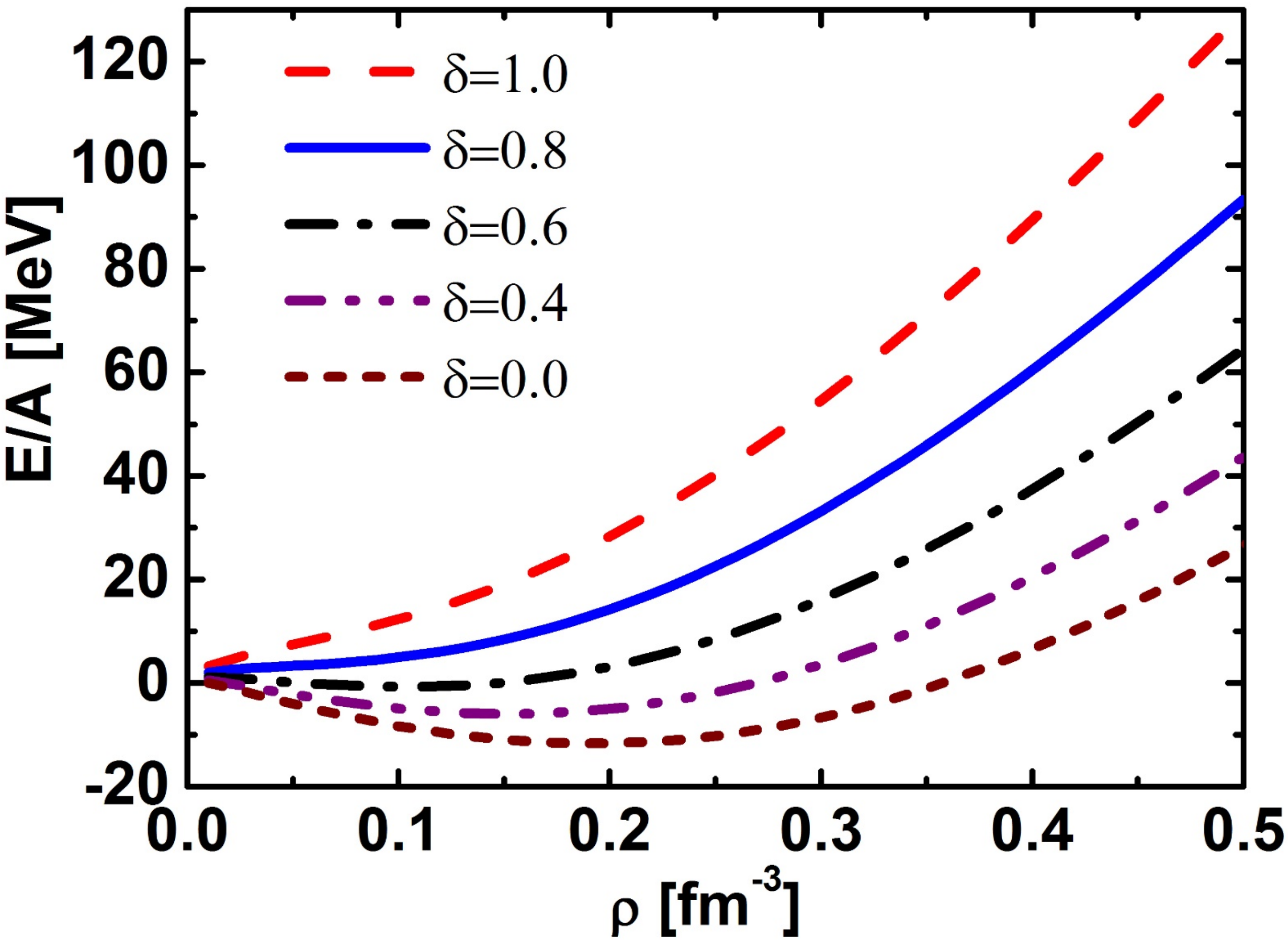
Pure neutron matter

JH, H. Toki and H. Shen, Jour. Phys. G. 38(2011)085105



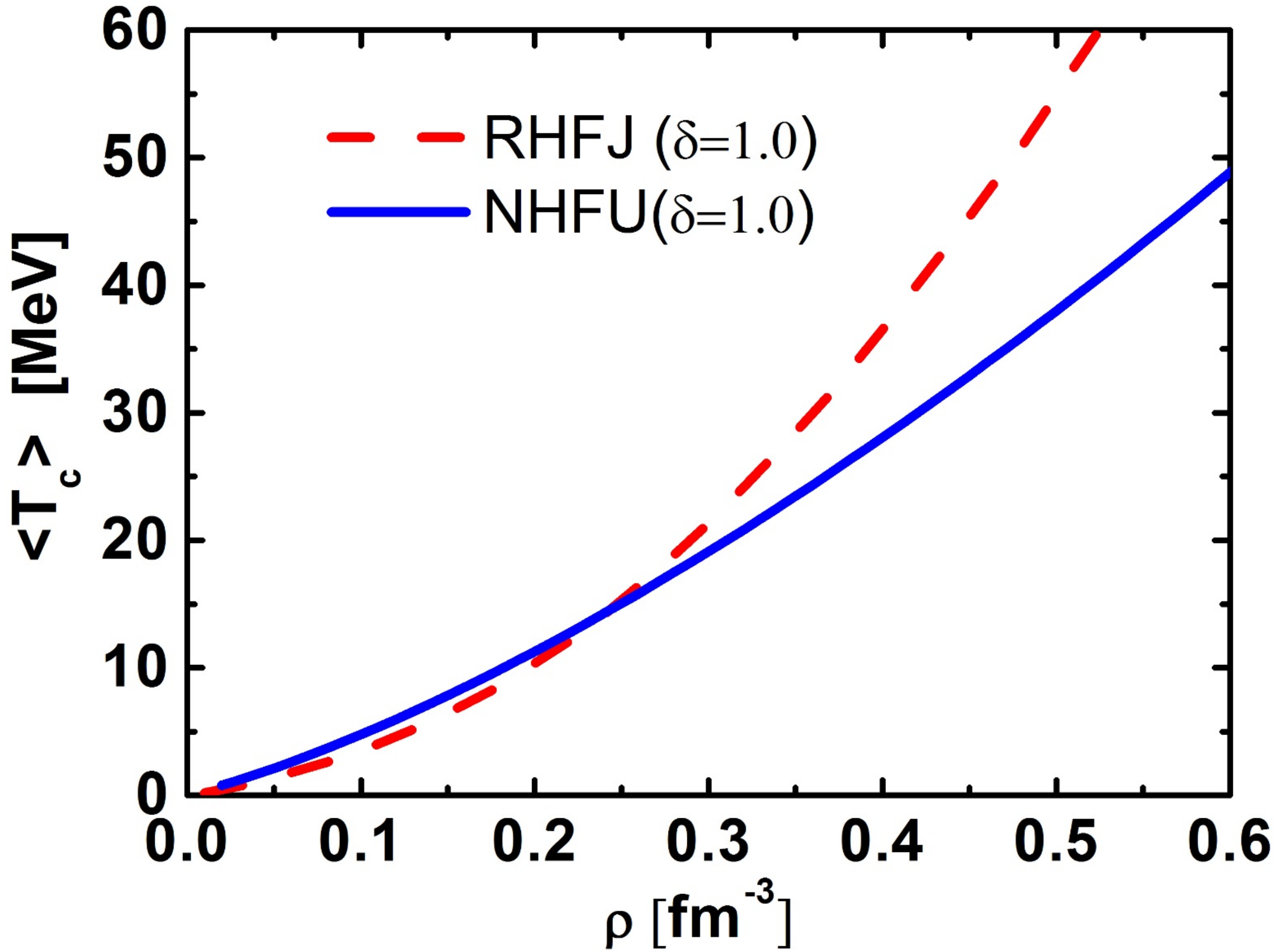
Asymmetric nuclear matter

JH, H. Toki and H. Shen, Jour. Phys. G. 38(2011)085105



Kinetic energy correlation

JH, H. Toki and H. Shen, Jour. Phys. G. 38(2011)085105



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Neutron star matter

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Lagrangian of leptons

$$L = \sum_l \bar{\psi}(i\gamma_\mu \partial^\mu - m_l)\psi_l$$

l: e, m

Beta equilibrium conditions

$$\mu_p = \mu_n - \mu_e$$

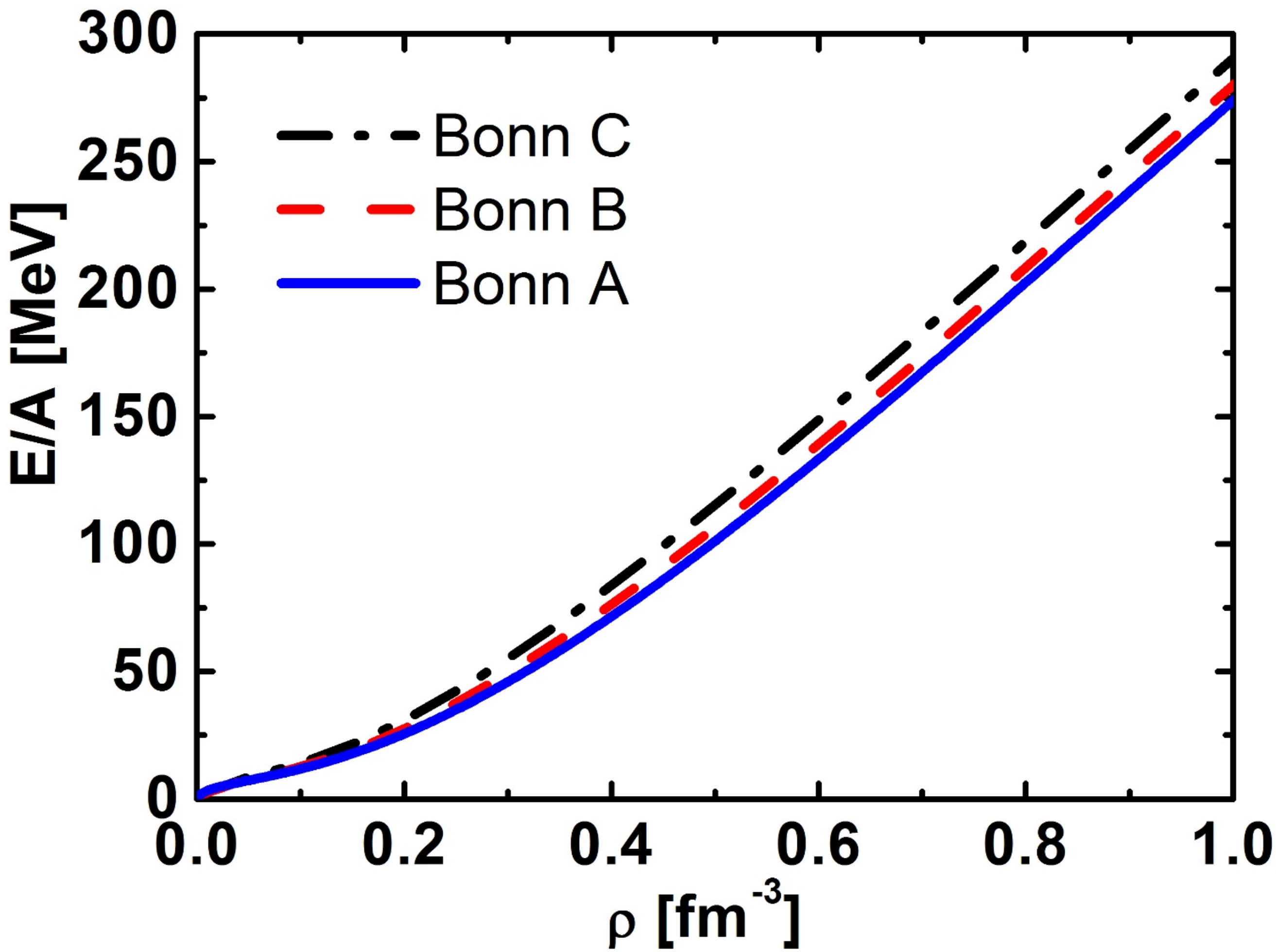
$$\mu_\mu = \mu_e$$

Charge neutrality conditions

$$\rho_p = \rho_e + \rho_\mu$$

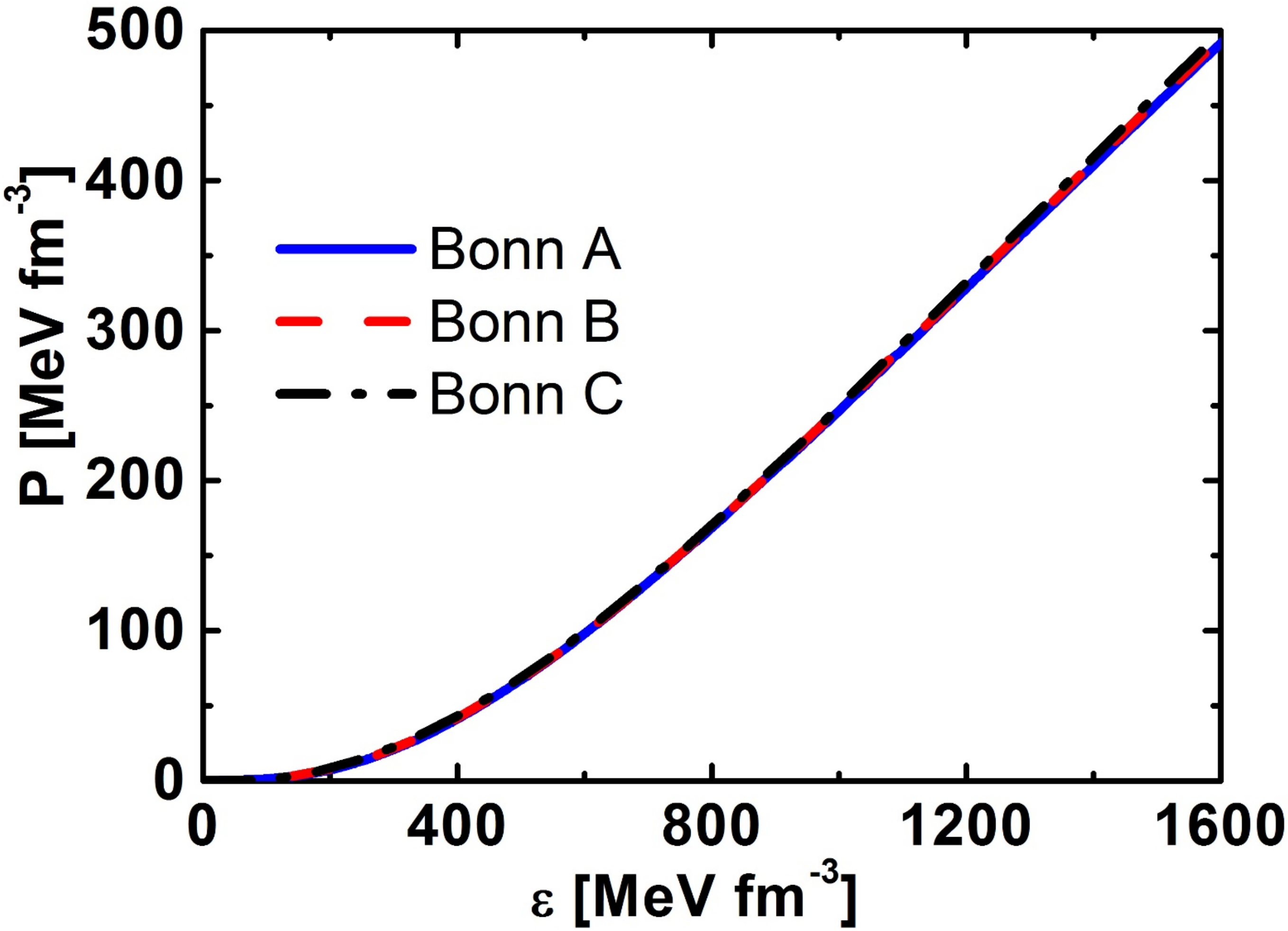
Equation of state

JH, H. Shen, and H. Toki, Phys. Rev. C,95 (2011)025804



Equation of state

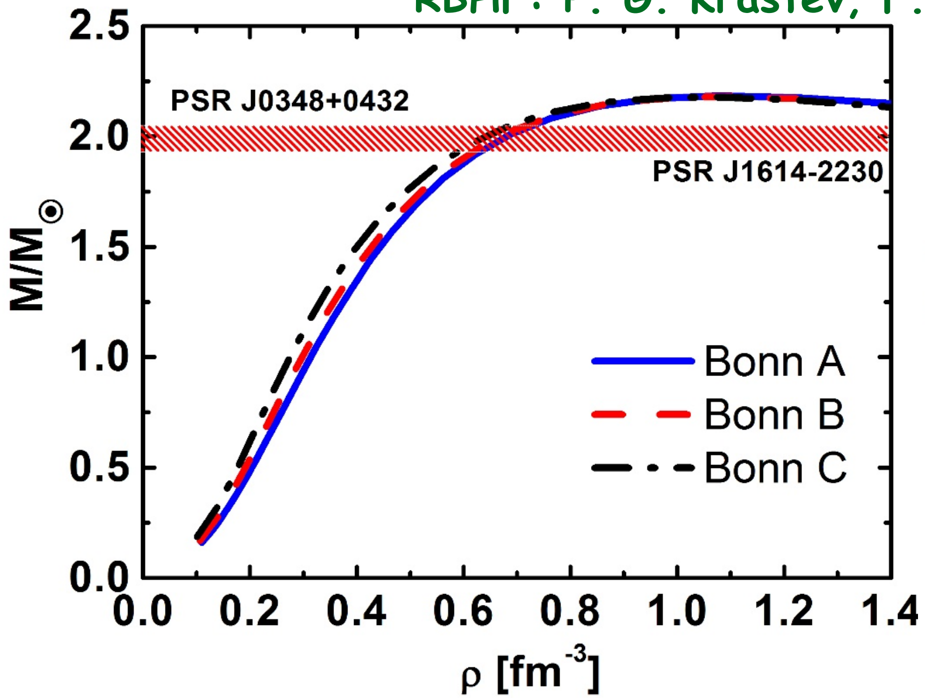
JH, H. Shen, and H. Toki, Phys. Rev. C,95 (2011)025804

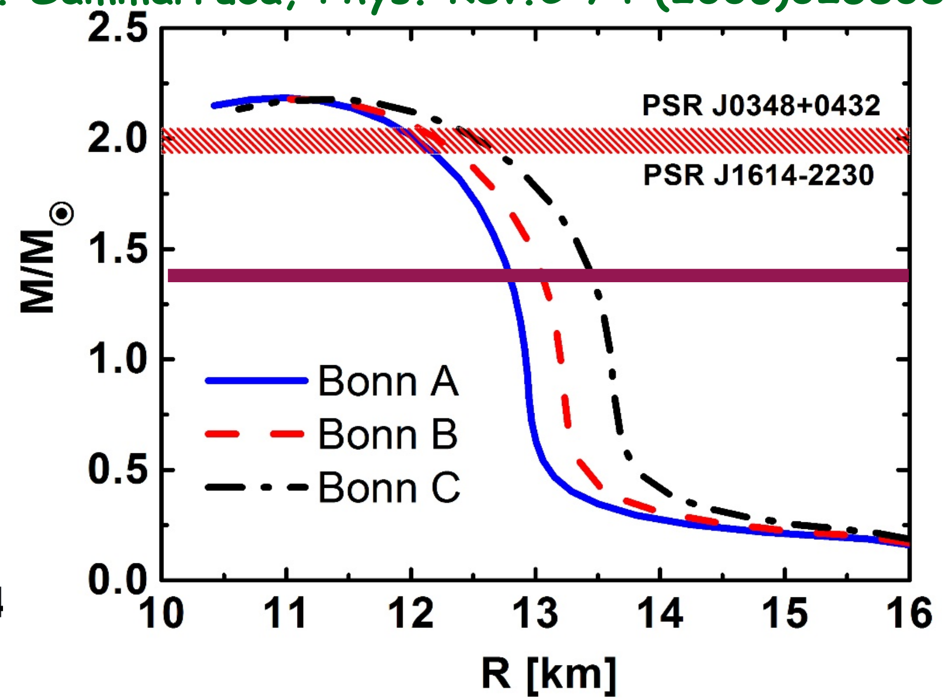


Neutron star properties

Potential	RHFJ			RBHF		
	$M_{max}(M_\odot)$	R(km)	$\rho_c(fm^{-3})$	$M_{max}(M_\odot)$	R(km)	$\rho_c(fm^{-3})$
Bonn A	**2.1841**	10.99	1.078	**2.2401**	10.74	1.013
Bonn B	**2.1805**	11.08	1.078	**2.2399**	10.79	1.008
Bonn C	**2.1786**	11.22	1.076	**2.2384**	10.83	1.003

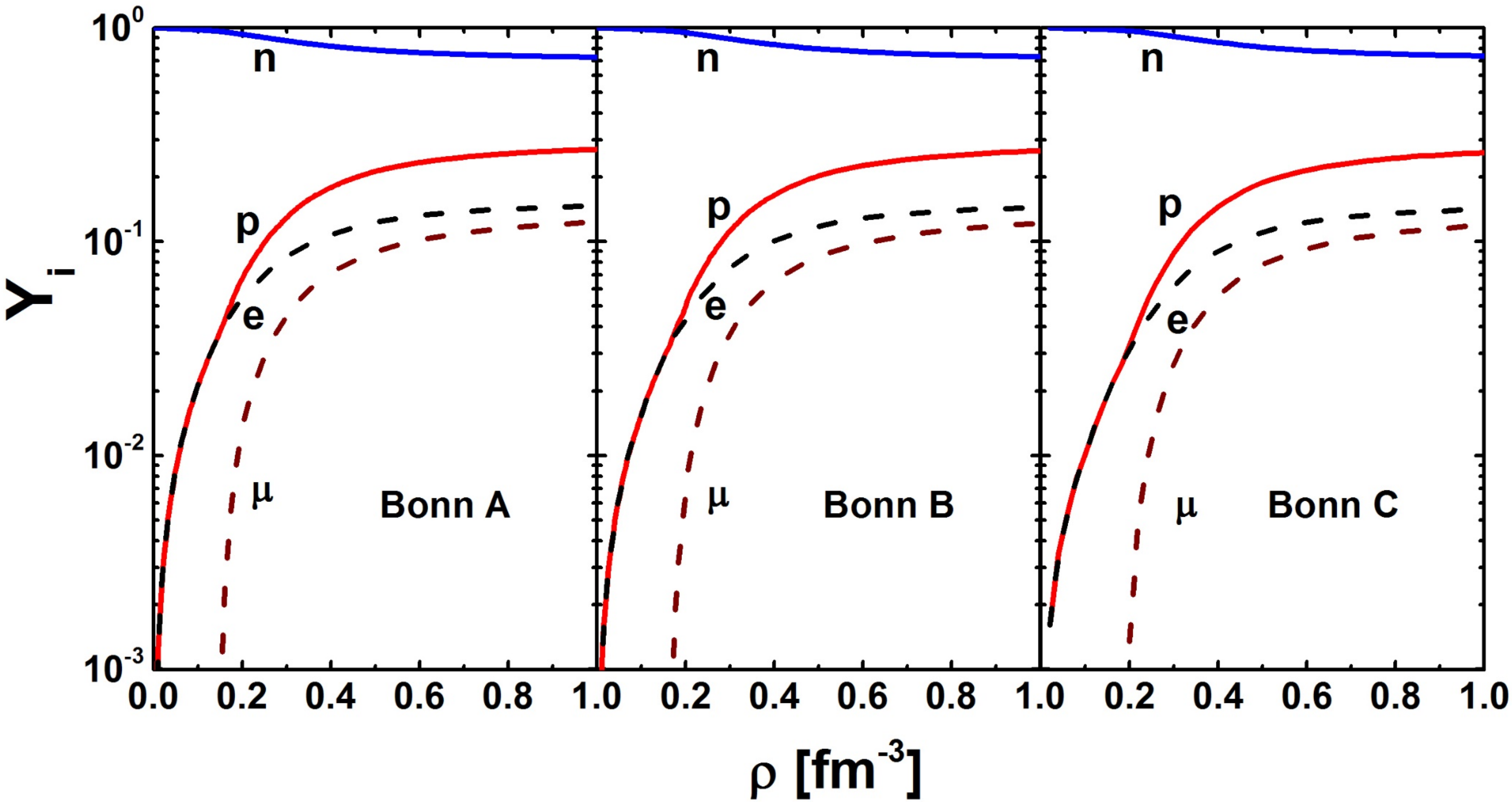
RBHF: P. G. Krastev, F. Sammarruca, Phys. Rev.C 74 (2006)025808.





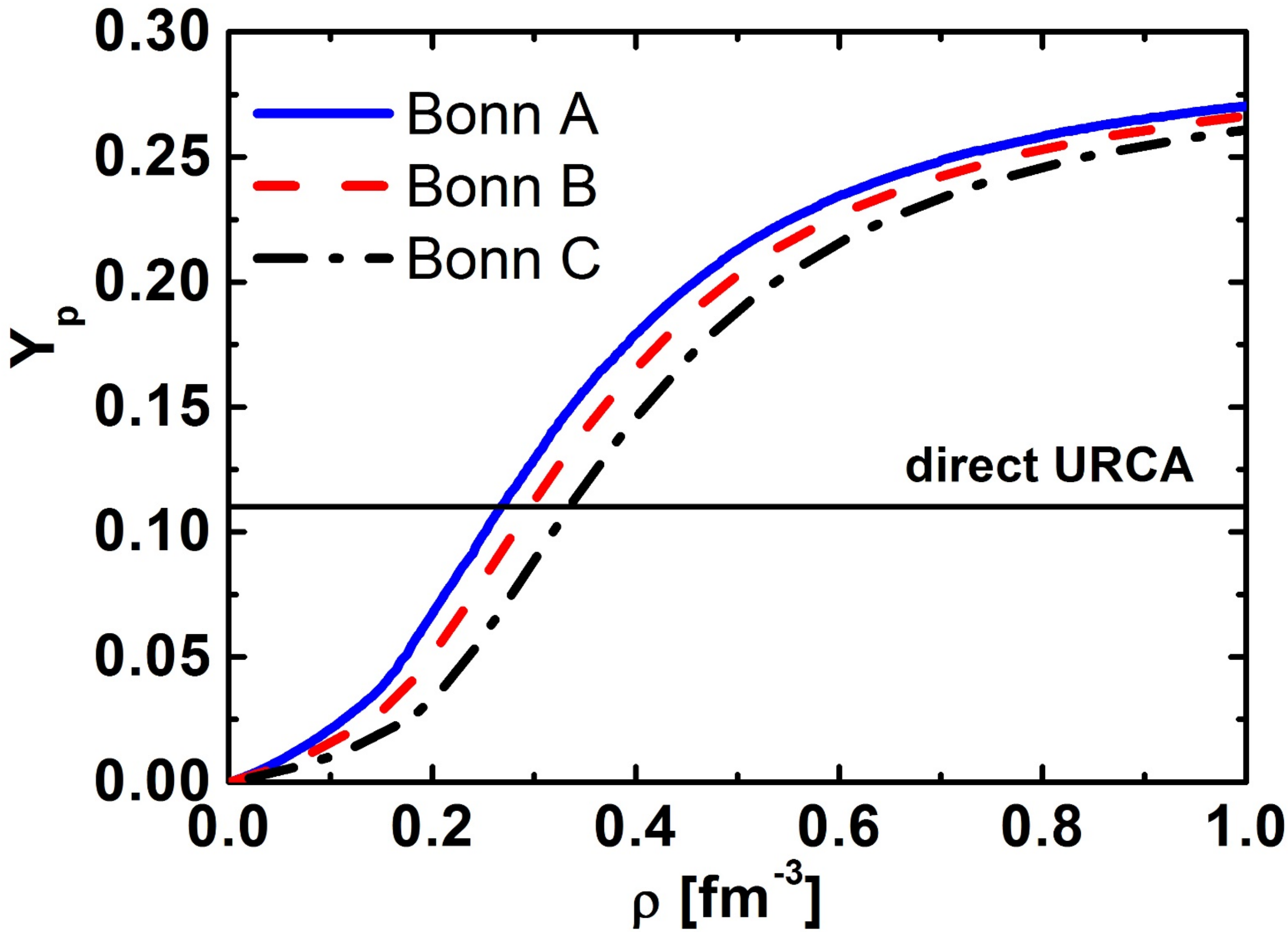
Ratio of particles

JH, H. Shen, and H. Toki, Phys. Rev. C,95 (2011)025804



URCA process

JH, H. Shen, and H. Toki, Phys. Rev. C,95 (2011)025804



Effective masses

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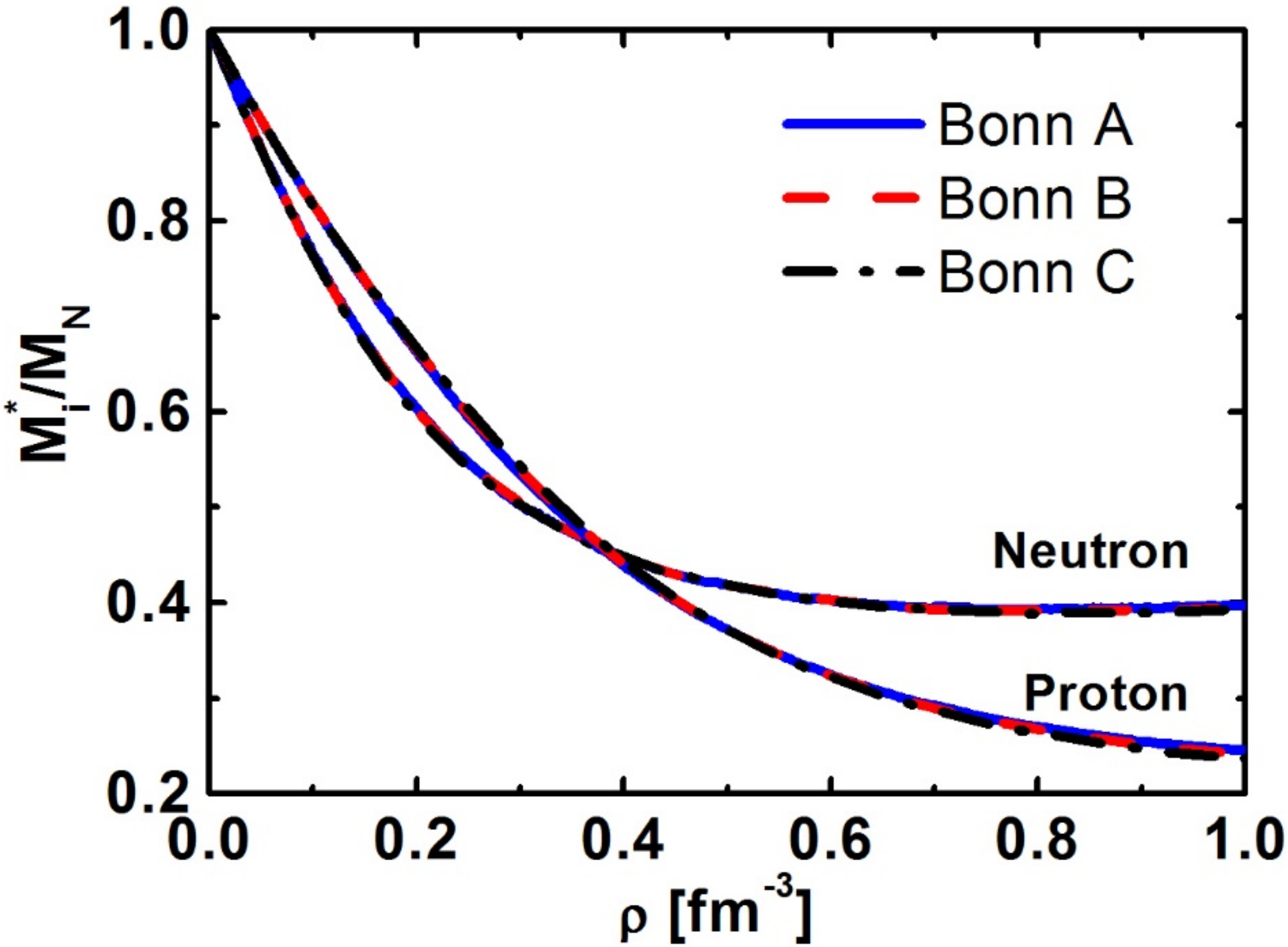
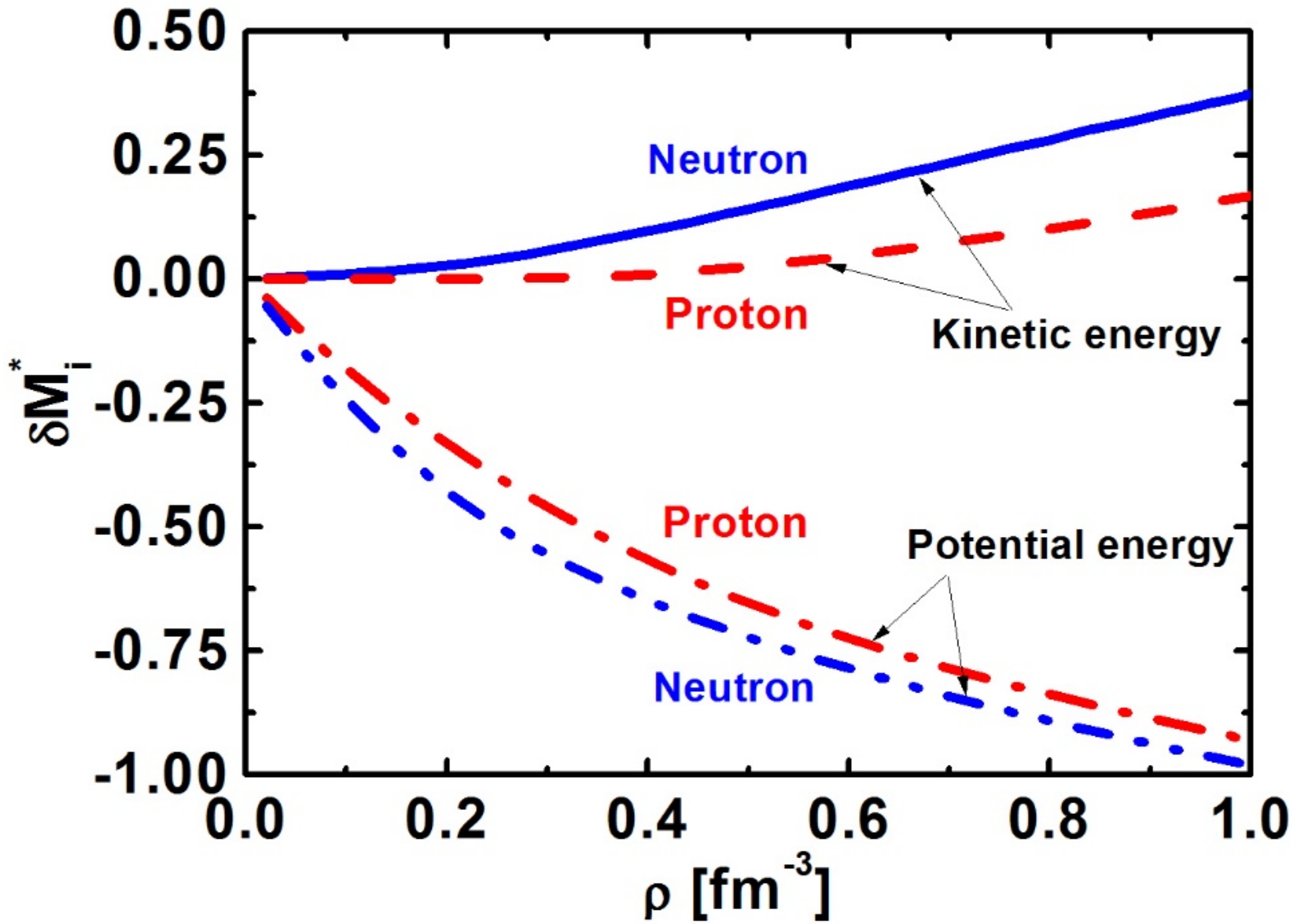
JH, H. Shen, and H. Toki, Phys. Rev. C,95 (2011)025804



The correlation on kinetic energy

JH, H. Shen, and H. Toki, Phys. Rev. C,95 (2011)025804



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Summary and Respectives

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We extended the variational method to relativistic framework with central correlation

The properties of nuclear matter in relativistic variational method could be comparable with RBHF theory

We also applies such methods on the study of neutron star. The maximum mass of neutron star is around 2.18 solar mass.

The more correlation functions will included.