Constraint on the EOS of protoneutron stars with asteroseismology

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collaborated with

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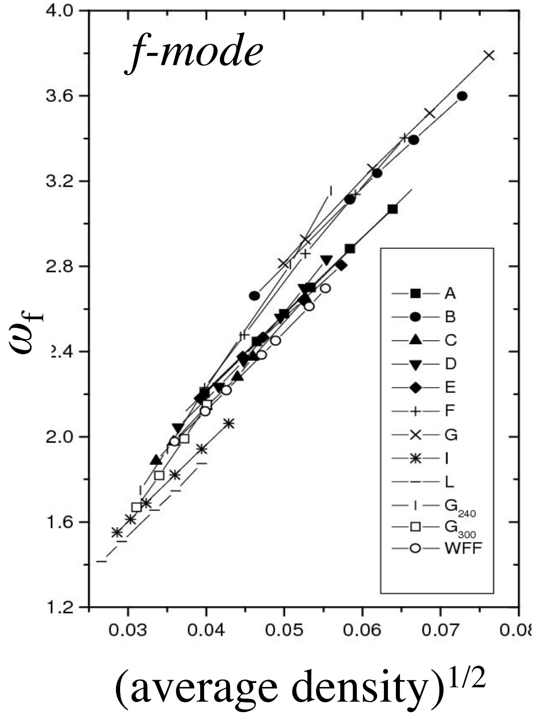
eigenfrequencies

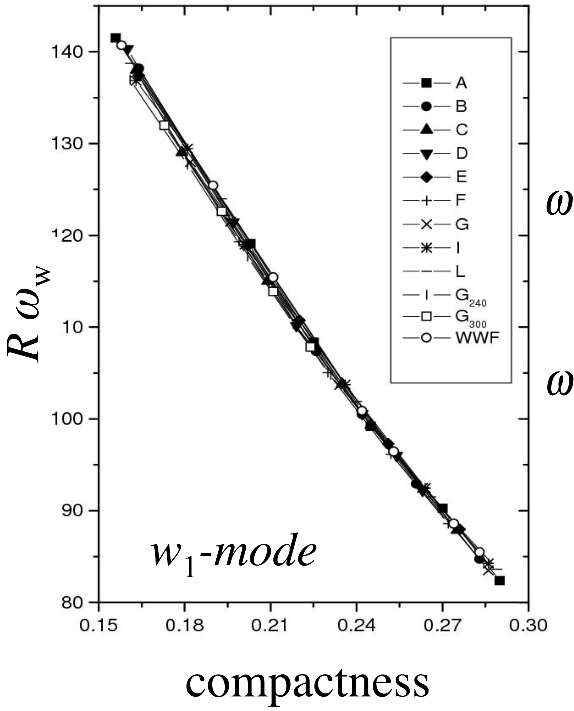
- f- (and p_i-) modes:
  acoustic (pressure) waves ~ density

- w_i-modes:
  spacetime oscillations ~ M/R

Asteroseismology on Cold NSs

- via the observations of GW frequencies, one might be able to see the properties of NSs ---> GW asteroseismology





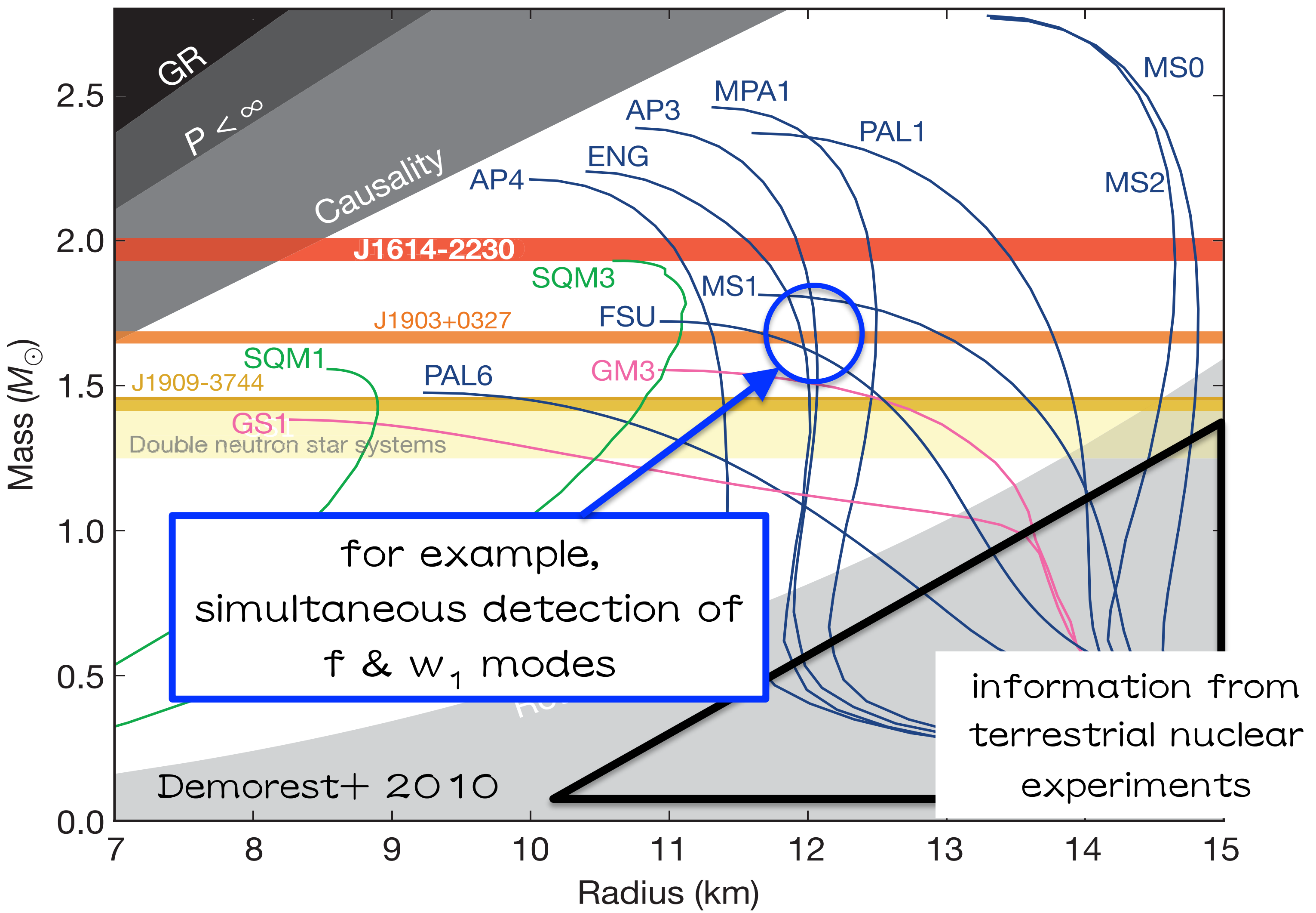
Andersson & Kokkotas (1998)

$$\omega_f \approx 0.78 + 1.64\left[\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10\mathrm{km}}{R}\right)^3\right]^{1/2}$$

$$\omega_w \approx \left(\frac{10\mathrm{km}}{R}\right)\left[20.92 - 9.14\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10\mathrm{km}}{R}\right)\right]$$

determination of (M, R)

Cold NS & EOS



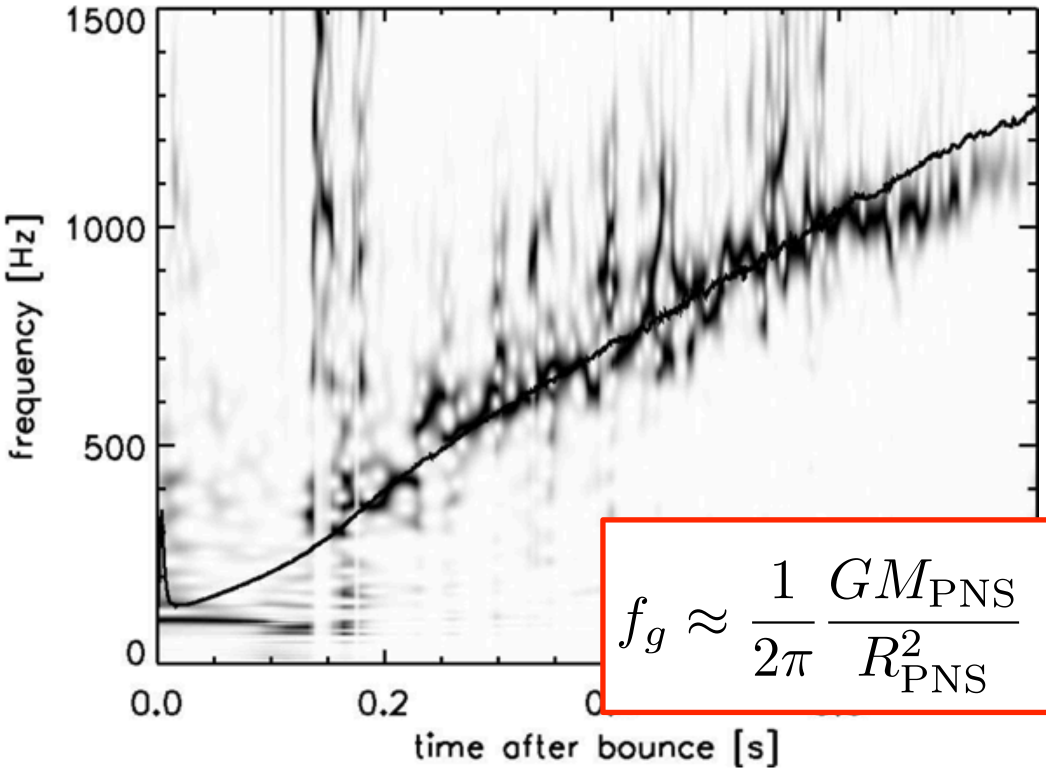
for example, simultaneous detection of f & w_1 modes

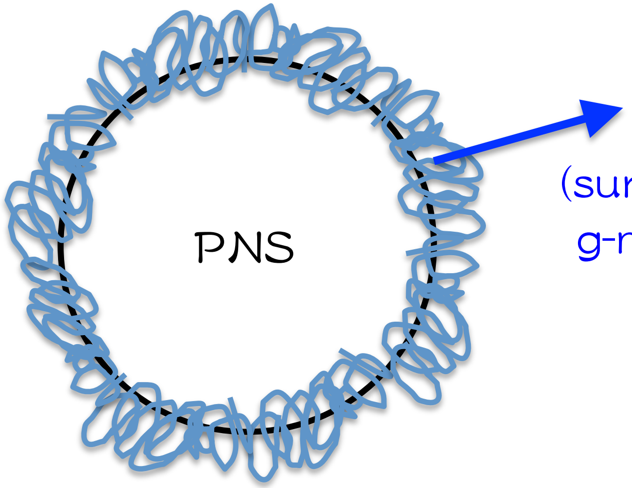
information from terrestrial nuclear experiments

Demorest+ 2010

g-mode oscillations?

- 2D non-rotation with convection by Muller et al. (2013)
  → excitations of specific frequency



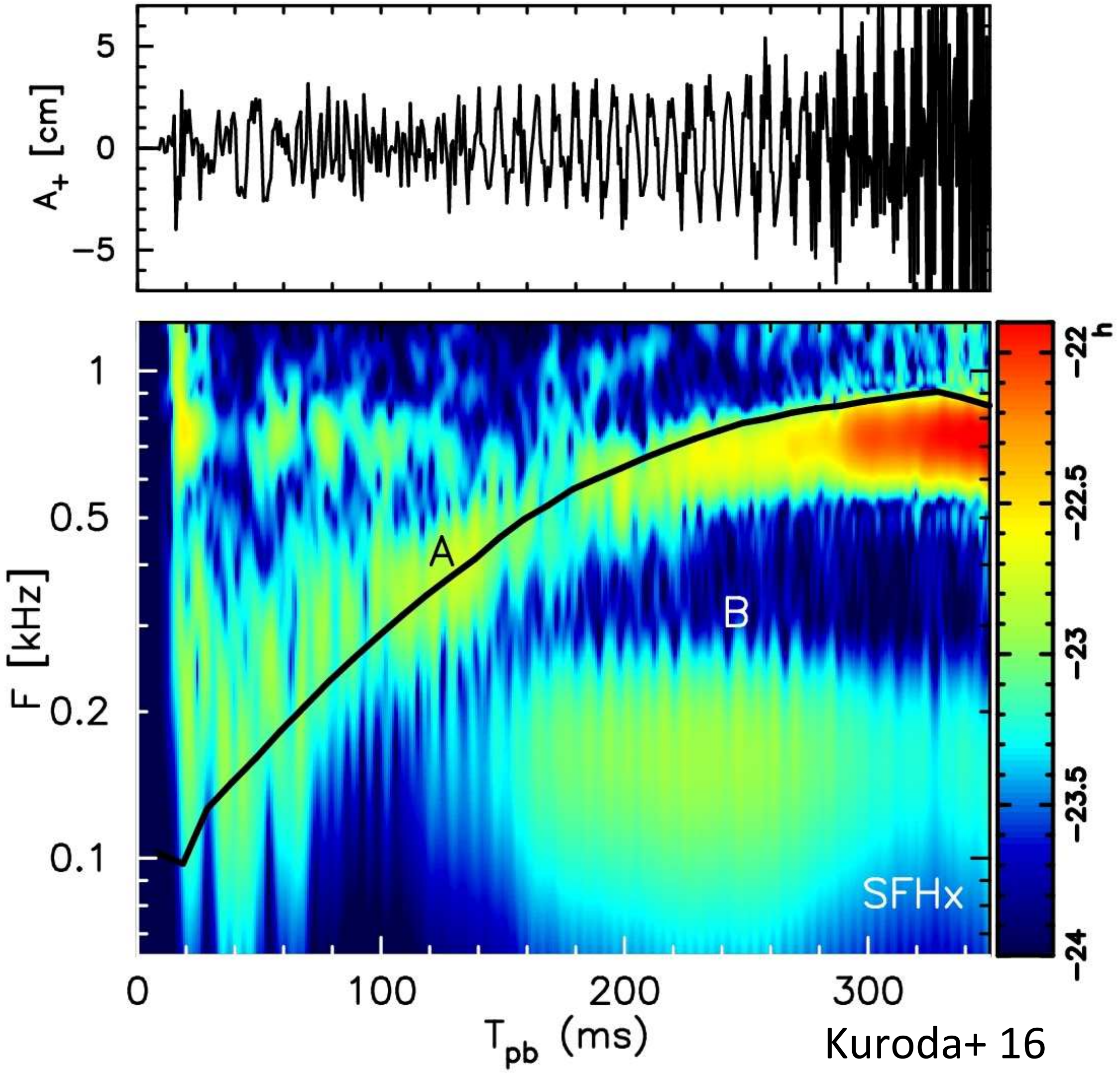


(surface)
g-mode

the convection & the standing accretion-shock instability

$$f_g \approx \frac{1}{2\pi}\frac{GM_{\rm PNS}}{R_{\rm PNS}^2}\left(\frac{1.1m_n}{\langle E_{\bar{\nu}_e}\rangle}\right)^{1/2}\left(1-\frac{GM_{\rm PNS}}{c^2R_{\rm PNS}}\right)^2$$

GW from PSNs

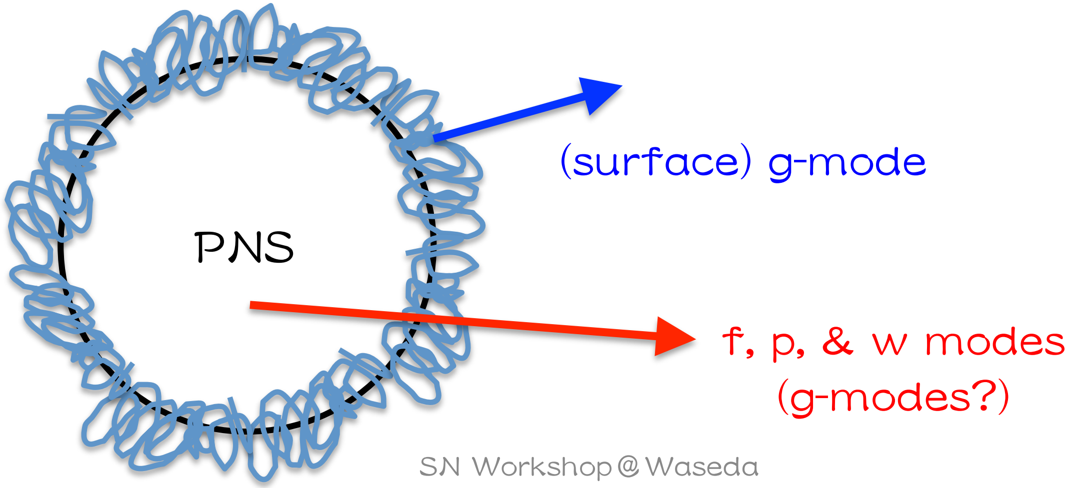


Kuroda+ 16

- Numerical simulations tell us the GW spectra.
- difficult
  - to extract physics of PNS and/or SN mechanism
  - to make a long-term numerical calculations
- We adopt the perturbation approach to determine the freq. from PNS.

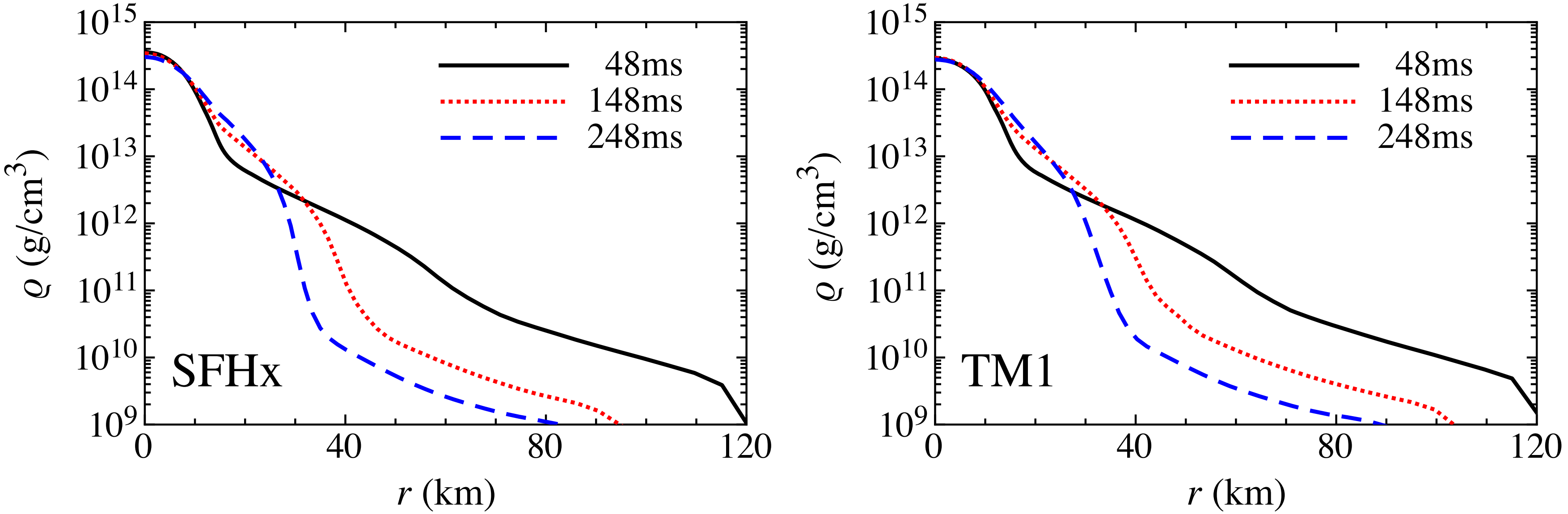
asteroseismology in PNS

- background PNS models:
  - assuming that the PNS models are static spherically symmetric at each time step
  - adopting the numerical results of GR3D by Kuroda et al.
- add perturbations:
  - we particularly focus on
    - f, p-modes : with relativistic Cowling approximation, i.e., $\delta g_{\mu\nu} = 0$
    - w-modes : axial type oscillations with metric perturbation
- solve the eigenvalue problem → eigenfrequency at each time



PNS models

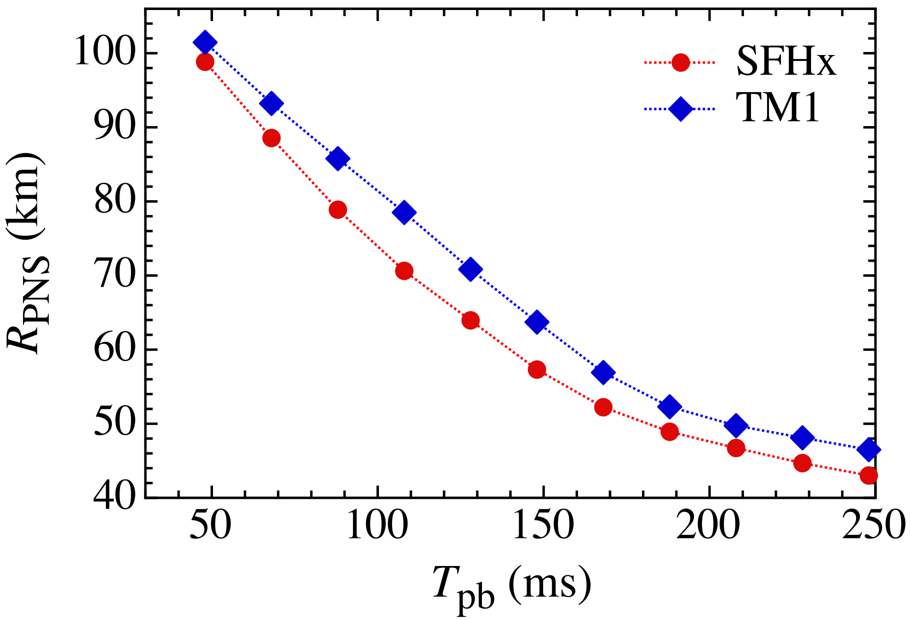
- we adopt the results of 3D-GR simulations of core-collapse supernovae (Kuroda et al. 2016)
  - progenitor mass = $15M_\odot$
  - EOS : SFHx ($2.13M_\odot$) & TM1 ($2.21M_\odot$)

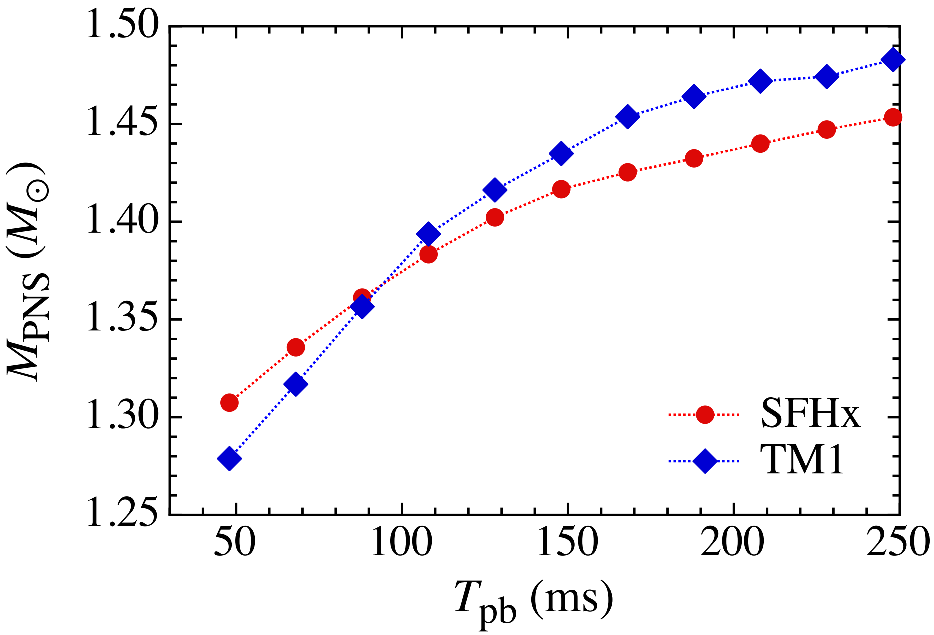


  - R_{PNS} is defined with $\rho_s = 10^{10}$ g/cm^3
  - using the radial profiles as a background PNS model, the eigen-frequencies are determined.

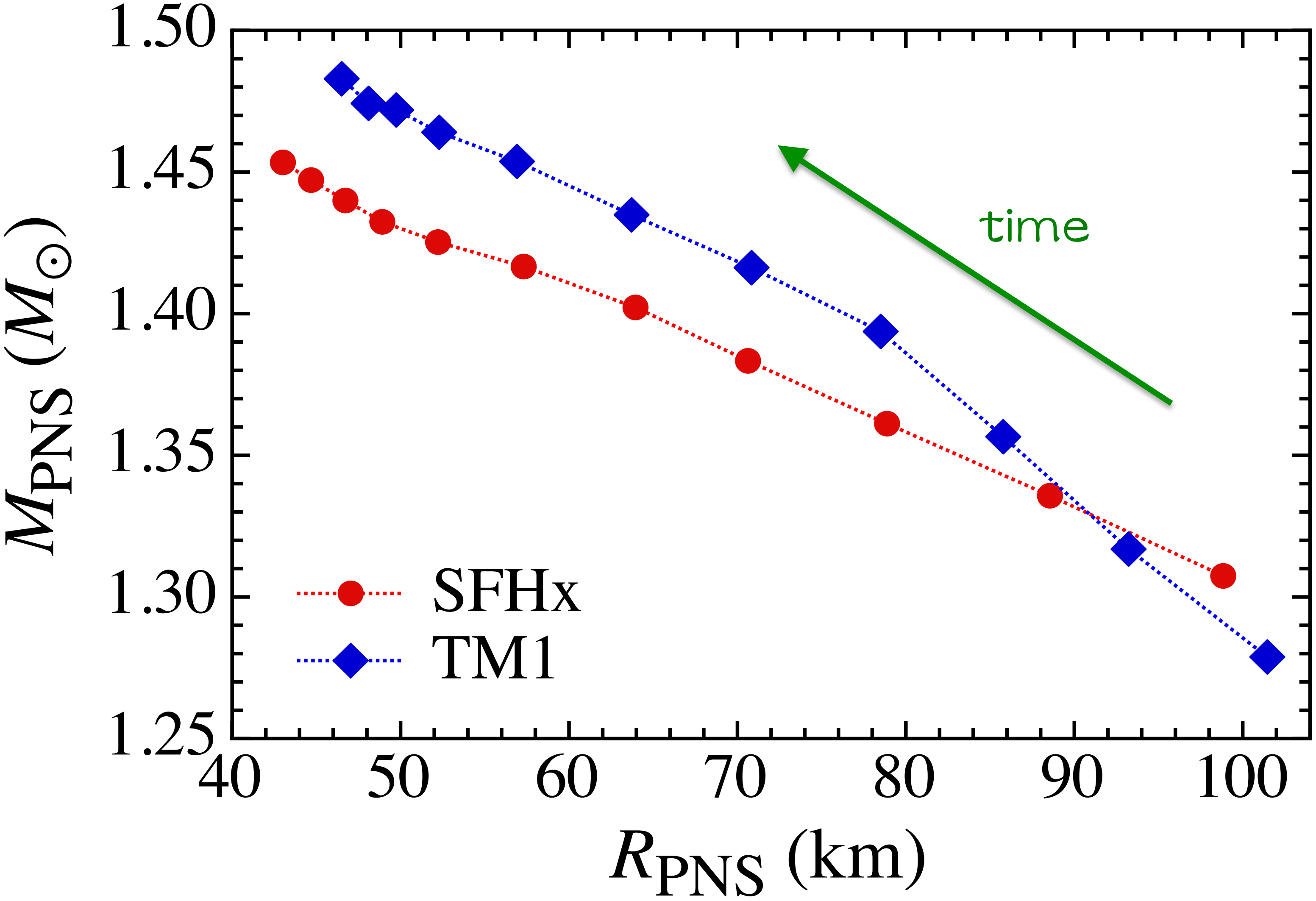
Mass & Radius

- R_{PNS} is decreasing due to the cooling
- M_{PNS} is increasing by mass accretion





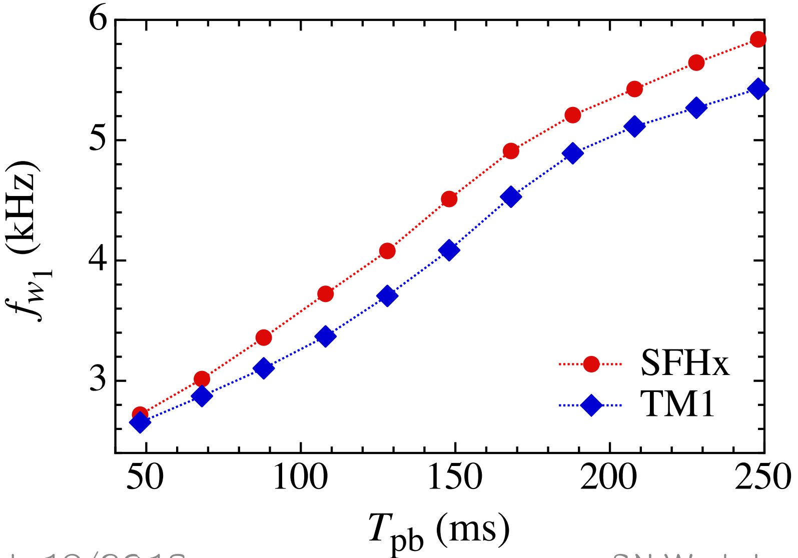
M-R evolution after core-bounce

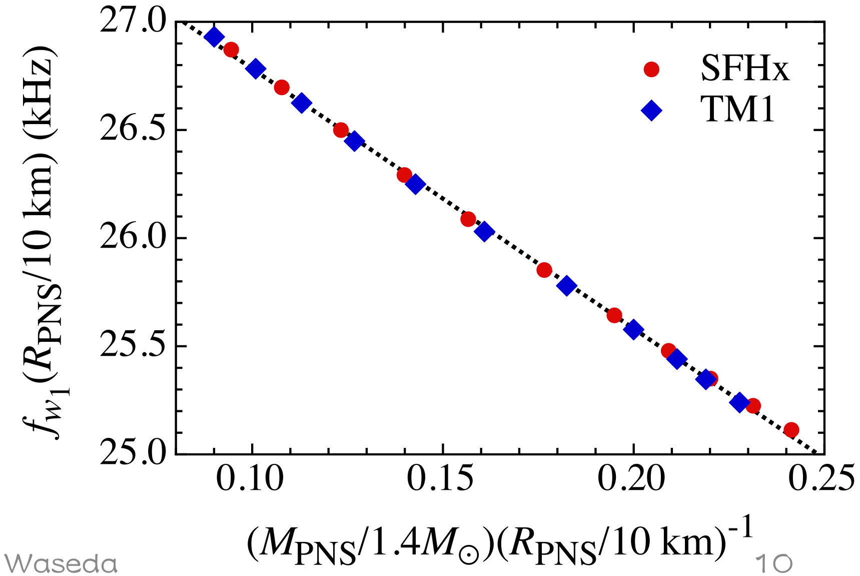


evolution of w_1-modes

- frequencies depend on the EOS.
  - increasing with time
  - can be characterized well by $M_{\mathrm{PNS}}/R_{\mathrm{PNS}}$
- as for cold NS, we can get the fitting formula, almost independent from EOS

$$f_{w_1}^{(\mathrm{PNS})}(\mathrm{kHz}) \approx \left[27.99 - 12.02\left(\frac{M_{\mathrm{PNS}}}{1.4\ M_\odot}\right)\left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-1}\right]\times\left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-1}$$

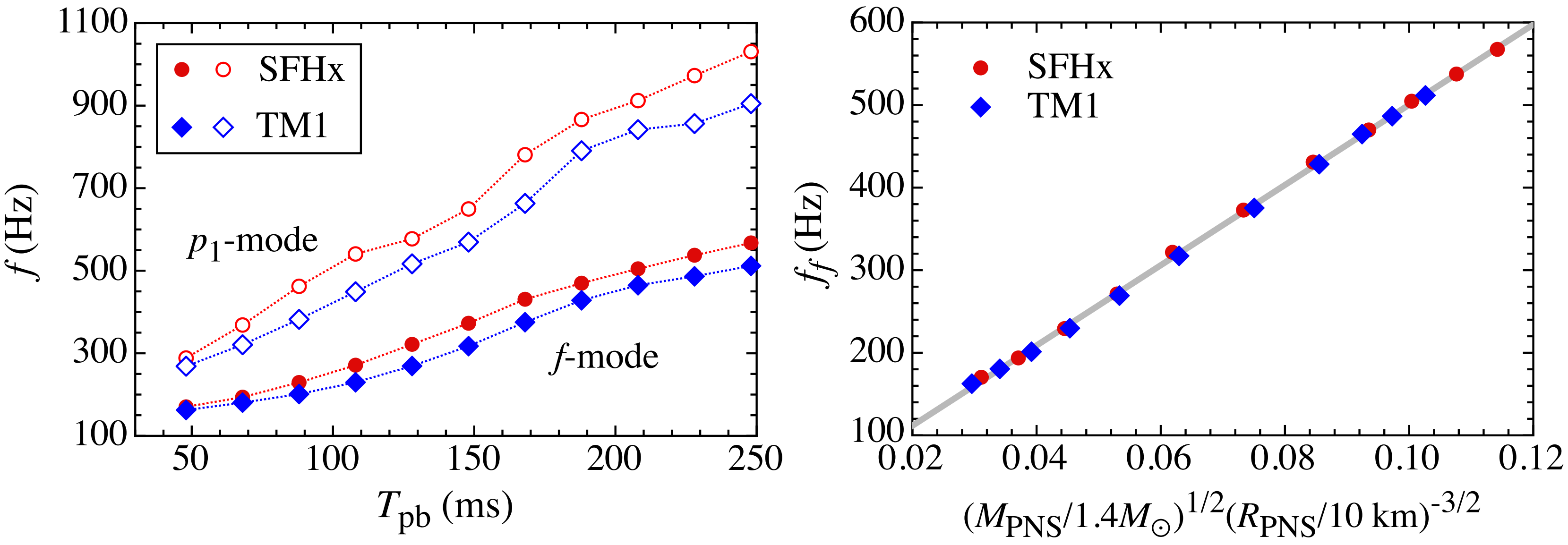




evolution of f-mode

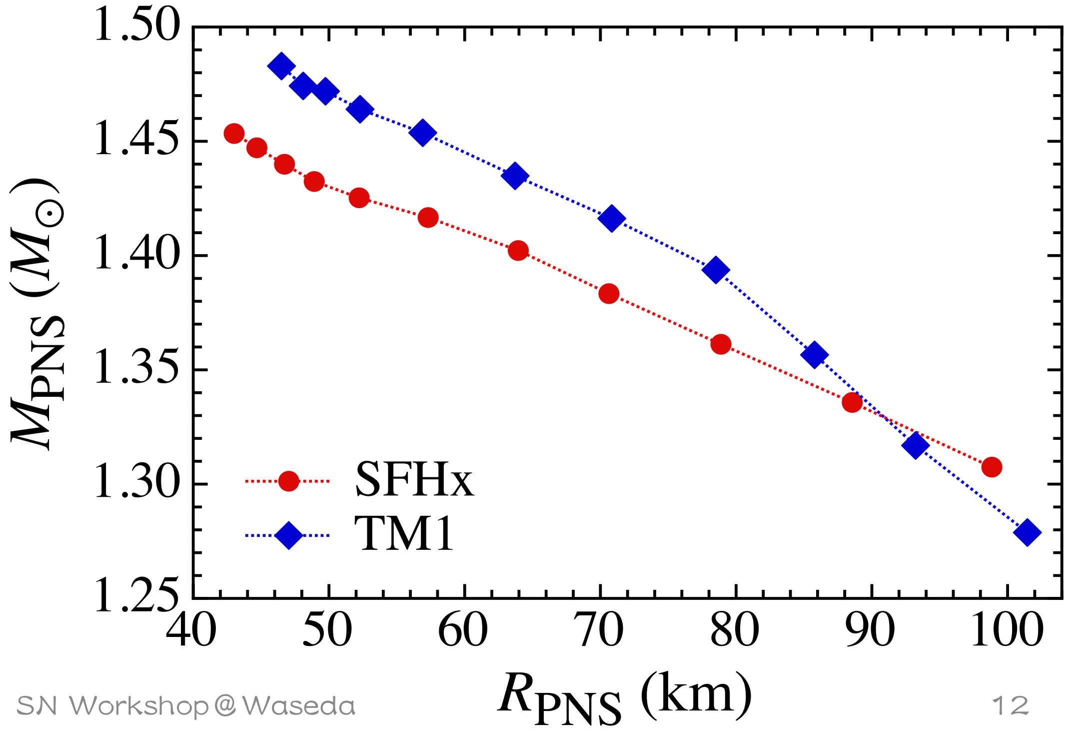
- frequencies can be expressed well by the average density independent of the EOS (and progenitor mass)
- we derive the fitting formula as a function of $M_{PNS}/R_{PNS}{}^3$

$$\boxed{f_f^{(PNS)}(\mathrm{Hz}) \approx 14.48 + 4859 \left(\frac{M_{PNS}}{1.4\ M_\odot}\right)^{1/2} \left(\frac{R_{PNS}}{10\ \mathrm{km}}\right)^{-3/2}}$$



determination of EOS

- GW spectra evolutions $f_f(t)$ & $f_{w1}(t)$
  → evolutions of M_{PNS}/R_{PNS}^3 & M_{PNS}/R_{PNS}
- one can determine (M_{PNS}, R_{PNS}) at each time after core bounce
  → determination of the EOS
- unlike cold NS cases, in principle one can determine the EOS even with ONE GW event !



detectability of w_1-modes

- effective amplitude of w_1-modes

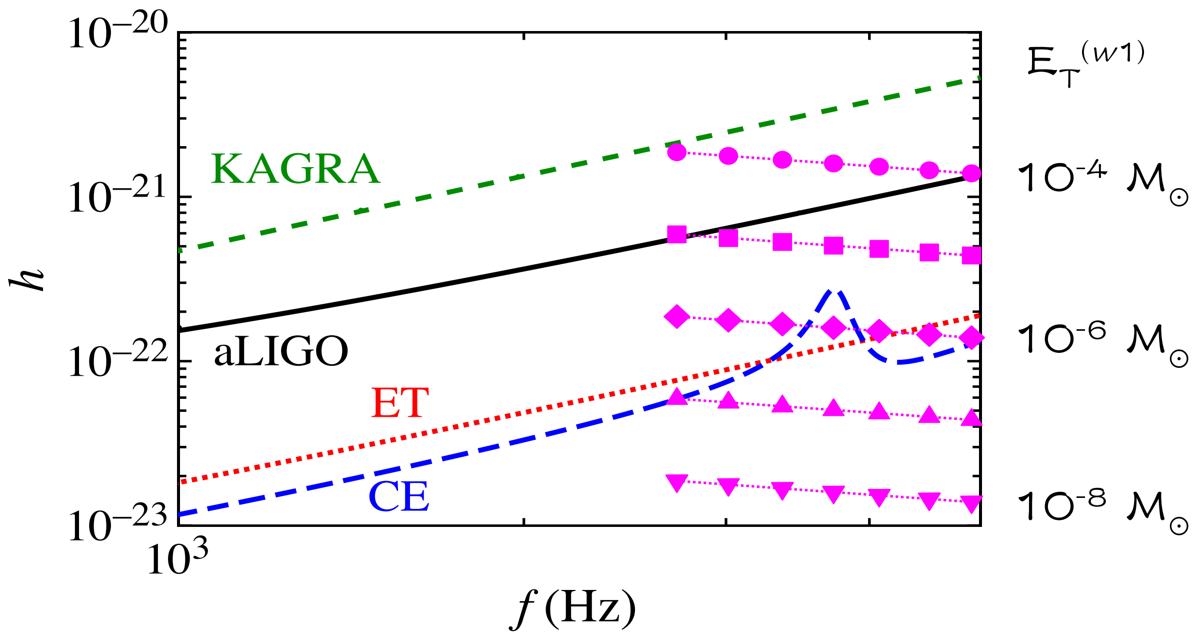
$$h_{\rm eff}^{(w_1)} \sim 7.7 \times 10^{-23} \left(\frac{E_{w_1}}{10^{-10}\, M_\odot} \right)^{1/2} \left(\frac{4\,{\rm kHz}}{f_{w_1}} \right)^{1/2} \left(\frac{10\,{\rm kpc}}{D} \right)$$

Andersson & Kokkotas (1996, 1998)

$$\frac{E_{w1}}{E_T^{(w1)}} \approx \frac{\tau_{w1}}{T_{w1}}$$

E_{w1} : energy for each time step

$E_T^{(w1)}$: total radiation energy in w_1-modes



conclusion

- Asteroseismology could be powerful technique for extracting the interior information
- We examine the frequencies of gravitational waves radiating from PNS after bounce.

$$\boxed{f_{w_1}^{(\mathrm{PNS})}(\mathrm{kHz}) \approx \left[27.99 - 12.02\left(\frac{M_{\mathrm{PNS}}}{1.4\ M_\odot}\right)\left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-1}\right] \times \left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-1}}$$

$$\boxed{f_{f}^{(\mathrm{PNS})}(\mathrm{Hz}) \approx 14.48 + 4859\left(\frac{M_{\mathrm{PNS}}}{1.4\ M_\odot}\right)^{1/2}\left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-3/2}}$$

(M_{PNS}, R_{PNS}) at each time after core bounce

- in principle, even with ONE GW event from supernova, one could determine **the EOS** for high density region.