Equations of State of Compact Star Matter with Clusters and Phase Transitions
Stefan Typel

Workshop
“Physics of Core-Collapse Supernovae and Compact Star Formations”

Waseda University, Tokyo, Japan
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X-ray: NASA/CXC/J. Hester (ASU)
Optical: NASA/ESA/J. Hester & A. Loll (ASU)
Outline

▶ Introduction

▶ Part 1:
  Generalized Relativistic Density Functional for Nuclei and Compact Star Matter

▶ Part 2:
  CompStar Online Supernovae Equations of State (CompOSE)

▶ Conclusions
Introduction
Strongly Interacting Matter

Development of a unified phenomenological description of

- atomic nuclei
  - light to (super-) heavy, stable and exotic
- nuclear matter
  - all relevant degrees of freedom
  - with phase transitions
Strongly Interacting Matter

development of a unified phenomenological description of

► atomic nuclei
  ► light to (super-) heavy, stable and exotic

► nuclear matter
  ► all relevant degrees of freedom
  ► with phase transitions

► compact star matter
  ► for all densities, temperatures, and isospin asymmetries
  ► with inhomogeneities, clustering
  ► for neutron stars, their mergers and core-collapse supernovae
Essential ingredient in astrophysical model calculations

- Static properties of neutron stars
- Dynamical evolution of core-collapse supernovae, neutron star mergers
- Conditions for nucleosynthesis
- Energetics, chemical composition, transport properties
essential ingredient in astrophysical model calculations

- static properties of **neutron stars**
- dynamical evolution of **core-collapse supernovae**, neutron star mergers
- conditions for **nucleosynthesis**
- energetics, **chemical composition**, transport properties

timescale of reactions \( \ll \) timescale of system evolution

- **equilibrium** (thermal, chemical, . . .)
- application of **EoS** reasonable
wide range of thermodynamic variables

- temperature \( T \)
- baryon density \( n_b \)
- hadronic charge fraction \( Y_q \) or isospin asymmetry \( \beta = 1 - 2Y_q \)
wide range of thermodynamic variables

- **temperature** $T$
- **baryon density** $n_b$
- **hadronic charge fraction** $Y_q$ or
  isospin asymmetry $\beta = 1 - 2Y_q$

modeling of nuclear matter
and stellar matter

- **different systems and conditions**
wide range of thermodynamic variables

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modeling of nuclear matter and stellar matter
- different systems and conditions

⇒ **global, multi-purpose EoS** required

EoS review: M. Oertel et al., Rev. Mod. Phys. 89 (2017) 015007
Theoretical Approaches

- **hadronic ’ab-initio’ methods with realistic interactions**
  - interactions: potential models, meson-exchange, chiral forces, RG evolved (Argonne, Urbana, Tucson-Melbourne, Nijmegen, Paris, Bonn, ...)
  - two-body NN interaction (in vacuum) well constrained by experiment, three-body forces less, large uncertainties for YN, YY, etc.
  - many-body methods: BHF/DBHF, SCGF, CBF, VMC, GFMC, AFDMC, ...
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- **QCD-based/inspired descriptions**
  - Lattice QCD, pQCD, DS, (P)NJL, bag models, ...

- **effective field theories (EFT)**
  - chiral EFT, nuclear lattice EFT, ...
Theoretical Approaches

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- methods not always applicable (methodological/technical limitations)
- many EoS for neutron matter & neutron star matter, but no global EoS for astrophysical applications available from these approaches
  - only **phenomenological models** for global EoS at present
EoSs for Astrophysical Applications

- **constituents:** mostly considered are nucleons, nuclei (light/heavy/representative), leptons, photons, ...
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  (Skyrme/Gogny/relativistic mean-field models, NSE, virial EoS, density functionals, classical/quantum molecular dynamics, . . .)
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- **global EoSs used in astrophysical simulations:**
  - HS (TM1,TMA,FSUgold,NL3,DD2,IUFSU): M. Hempel, J. Schaffner-Bielich, NPA 837 (2010) 210
  - recently many more, also with additional degrees of freedom (hyperons, quarks)
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challenge:
covering of full range of thermodynamic variables in a unified model
⇒ here: generalized relativistic density functional
Generalized Relativistic Density Functional for Nuclei and Compact Star Matter
Objective: development of improved EoS model with

- extended set of constituent particles
  - *nuclear matter*: nucleons, nuclei/clusters, ..., mesons, hyperons, ..., quarks
  - *stellar matter*: add electrons, muons, photons
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  - nucleon-nucleon correlations: clustering, pairing
  - Pauli principle: dissolution of composite particles in medium (Mott effect)
  - electromagnetic correlations: essential for solidification/melting
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  - distinguish nuclear matter and stellar matter
  - “non-congruent” phase transitions, gas/liquid/solid(cry stall) phases
Generalized Relativistic Density Functional

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only a selection from these topics considered here
Description of Nuclear Matter and Finite Nuclei

basic approach: relativistic mean-field (RMF) models

- energy density functional
  - origin: field theoretical description
  - derived from Lagrangian density, mean-field approximation
  - phenomenological description
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▶ energy density functional
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  ▶ phenomenological description

▶ various versions
  ▶ interaction: exchange of scalar and vector mesons (σ, ω, ρ, ...)
    ▶ minimal coupling of mesons to nucleons
    ▶ with nonlinear self-interactions
    ▶ with density dependent couplings
  ▶ without explicit meson fields
    ▶ point-coupling models
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  - without explicit meson fields
    - point-coupling models

- many parametrizations
  - different purposes (finite nuclei, excitations, EoS, \ldots)

(see, e.g., M. Dutra et al., Phys. Rev. C 90 (2014) 055203)
Medium Dependence of Effective Interaction

- interaction contribution to Lagrangian

  - nonlinear (NL) RMF models with meson self-interactions

\[ \mathcal{L}_{\text{int}} = \bar{\psi} g_\sigma \sigma \psi - \frac{A}{3} \sigma^3 - \frac{B}{4} \sigma^4 - \bar{\psi} g_\omega \omega_\mu \gamma^\mu \psi + \frac{C}{4} (\omega_\mu \omega^\mu)^2 - \bar{\psi} g_\rho \vec{\rho}_\mu \cdot \vec{\tau} \gamma^\mu \psi \]

  with constants \( g_\sigma, g_\omega, g_\rho, A, B, C, \ldots \)

  (usually scalar and vector contributions not coupled, cross terms added later)
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    with constants \( g_\sigma, g_\omega, g_\rho, A, B, C, \ldots \)
    (usually scalar and vector contributions not coupled, cross terms added later)
  - density dependent (DD) RMF models
    \[ \mathcal{L}_{\text{int}} = \bar{\psi} \Gamma_\sigma \sigma \psi - \bar{\psi} \Gamma_\omega \omega_\mu \gamma^\mu \psi - \bar{\psi} \Gamma_\rho \bar{\rho}_\mu \cdot \bar{\tau} \gamma^\mu \psi \]
    with functionals \( \Gamma_\sigma, \Gamma_\omega, \Gamma_\rho, \ldots \) depending on Lorentz scalars constructed from nucleon fields \( \bar{\psi}, \psi \)
    (motivated by Dirac-Brueckner calculations, more flexible than NL models)

dependence of couplings \( \Gamma_i \) on

- vector density \( \varrho^{(v)} = \sqrt{j^\mu j_\mu} \) with current \( j^\mu = \bar{\psi} \gamma^\mu \psi \) \( \Rightarrow \) standard choice
- scalar density \( \varrho^{(s)} = \bar{\psi} \psi \) \( \Rightarrow \) not really explored so far
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    - scalar density \( \hat{\rho}^{(s)} = \bar{\psi} \psi \Rightarrow \) not really explored so far

- phenomenological approach \( \Rightarrow \) model parameters determined from fits
  (properties of finite nuclei, characteristic nuclear matter parameters)
Relativistic Density Functionals with Density Dependent Couplings

- First DD-RMF parametrization fitted to energies of selected nuclei:
    - Functional form of couplings: $\Gamma_i(\varrho) = \Gamma_i(\varrho_{ref}) f_i(x)$
      with $f_i(x) = a_i \frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2}$ or $f_i(x) = \exp[-a_i(x - 1)]$  $x = \frac{\varrho}{\varrho_{ref}}$
    - Two parameters for isovector part of effective interaction (only one in standard NL-RMF models)
      - Improved nuclear matter parameters, similar to Skyrme Hartree-Fock models
      - Correlation of neutron skin thickness with slope of neutron matter EoS

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S. Typel and B.A. Brown, PRC 64 (2001) 027302
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    ⇒ correlation of neutron skin thickness with slope of neutron matter EoS
- many DD-RMF parametrizations in the following
  - DD-ME1 (T. Nikšić et al., PRC 66 (2002) 024306)
  - DD (S. Typel, PRC 71 (2005) 064301)
  - DD-ME2 (G.A. Lalazissis et al., PRC 71 (2005) 024312)
  - DD-F (T. Klähn et al., PRC 74 (2006) 035802)
  - DD2 (S. Typel et al., PRC 81 (2010) 015803)
  - DD-MEδ (X. Roca-Maza et al., PRC 84 (2011) 054309)
  - DD+++ – DD−− (S. Typel, PRC 89 (2014) 064321)
  - …

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Choice of Functionals

- **density dependence**
  - 'V': dependence of $\Gamma_\omega$, $\Gamma_\sigma$, $\Gamma_\rho$ on vector density $\varrho^{(v)}$
  - 'S': dependence of $\Gamma_\omega$, $\Gamma_\sigma$, $\Gamma_\rho$ on scalar density $\varrho^{(s)}$
  - 'M': dependence of $\Gamma_\omega$, $\Gamma_\rho$ ($\Gamma_\sigma$) on vector (scalar) density $\varrho^{(v)}$ ($\varrho^{(s)}$)
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- **functional form for $\omega$ and $\sigma$ mesons**
  - rational function $f_i(x) = a_i \frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2}$ with conditions $f_i(1) = 1$ and
  - 'P': $f_i''(0) = 0$, $d_i > 0$ (positive)
  - 'Z': $f_i'(0) = 0$, $d_i = 0$ (zero)
  - 'N': $f_i''(0) = 0$, $d_i < 0$ (negative)
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- **functional form for $\rho$ meson**
  - 'E': exponential function $f_i(x) = \exp[-a_i(x - 1)]$
  - 'R': rational function $f_i(x) = a_i \frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2}$ with conditions $f_i(1) = 1$, $f_i'(0) = 0$, $d_i = 0$, $f_i''(1)/f_i'(1) = f_i'''(1)/f_i''(1)$
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  $\Rightarrow$ 18 models with 9 parameters (including $\varrho_{\text{ref}}$ and $m_\sigma$), fitted to properties of nuclei, similar quality in description, details: S. Typel, Particles 1 (2018) 2
Coupling Functions

- similar smooth functions for ‘P’ and ‘Z’ parametrisations
- minimum in functions for ‘N’ parametrisations (ω and σ mesons)
- only small differences between ‘E’ and ‘R’ parametrisations (ρ meson)
Equations of State at Zero Temperature

- very similar below saturation density
- divergence above saturation density
- strong stiffening for 'N' parametrisations
DD-RMF Parametrization DD2

- fitted to properties of finite nuclei
  (S. Typel et al., PRC 81 (2010) 015803)

- very reasonable nuclear matter parameters
  \( n_{\text{sat}} = 0.149 \text{ fm}^{-3}, \ a_V = 16.02 \text{ MeV}, K = 242.7 \text{ MeV}, J = 31.67 \text{ MeV}, L = 55.04 \text{ MeV} \)
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- neutron matter EoS consistent with \( \chi \text{EFT(N}^3\text{LO)} \)

- consistent with unitary gas constraint
Extension of DD-RMF Model

- generalized relativistic density functional (gRDF)
- extended set of particle species
  - nucleons, electrons, muons, photons, hyperons (optional), . . .
  - light nuclei ($^2$H, $^3$H, $^3$He, $^4$He) and heavy nuclei ($A > 4$)
  - shell effects included, full distribution, not only average heavy nucleus
- two-nucleon scattering states
  - consistency with virial EoS at low densities
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- **excited states of nuclei**
  - temperature dependent degeneracy factors with density of states

- **medium dependence of particle properties**
  - quasiparticle picture, mass shifts
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Mass Shifts I

- Concept applies to composite particles: clusters
  - Light and heavy nuclei
  - Nucleon-nucleon correlations in continuum
- Effective change of binding energies
Mass Shifts I

- concept applies to composite particles: clusters
  - light and heavy nuclei
  - nucleon-nucleon correlations in continuum
- effective change of binding energies
- two major contributions $\Delta m_i = \Delta m_i^{\text{strong}} + \Delta m_i^{\text{Coul}}$
  - strong shift $\Delta m_i^{\text{strong}}$
    - effects of strong interaction (coupling to mesons)
    - Pauli exclusion principle: blocking of states in the medium
      $\Rightarrow$ reduction of binding energies
      $\Rightarrow$ cluster dissolution at high densities: Mott effect
      $\Rightarrow$ replaces traditional excluded-volume mechanism
  - electromagnetic shift $\Delta m_i^{\text{Coul}}$ (in stellar matter)
    - electron screening of Coulomb field
      $\Rightarrow$ increase of binding energies
Mass Shifts II

- **light nuclei**
  parametrization from G. Röpke, simplified and modified for high densities and temperatures

- **NN scattering states**
  mass shifts as for deuteron

effective binding energies $B_i^{(\text{eff})} = B_i^{(0)} - \Delta m_i$
 Mass Shifts III

- **light nuclei**
  parametrization from G. Röpke, simplified and modified for high densities and temperatures

- **NN scattering states**
  mass shifts as for deuteron

- **heavy nuclei**
  simple parametrization
Light Clusters in Heavy-Ion Collisions

emission of light nuclei

- determination of density and temperature of source
  - S. Kowalski et al. PRC 75 (2007) 014601
  - J. Natowitz et al. PRL 104 (2010) 202501
  - R. Wada et al. PRC 85 (2012) 064618

- thermodynamic conditions as in neutrinosphere of core-collapse supernovae
emission of light nuclei

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- thermodynamic conditions as in neutrinosphere of core-collapse supernovae

- particle yields $\Rightarrow$ chemical equilibrium constants
  $$K_c[i] = \frac{n_i}{(n_p^{Z_i} n_n^{N_i})}$$
  - L. Qin et al., PRL 108 (2012) 172701

- mixture of ideal gases not sufficient

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Cluster Correlations at Nuclear Surface

- gRDF with clusters at zero temperature
  - $\alpha$-particles at surface of Sn nuclei
  - reduced probability with increasing neutron excess
  - reduction of neutron skin thickness

S. Typel, PRC 89 (2014) 064321
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S. Typel, PRC 89 (2014) 064321

**Experimental tests**

- quasifree $(p,p\alpha)$ **knockout** reactions,
  experiment with Sn nuclei in February 2018 at RCNP Osaka: successful detection of $\alpha$ particles with expected trend
- $(d,^6\text{Li})$ **pickup** reactions
  $\Rightarrow$ similar trend in reduced widths

Neutron Star Matter
Hadronic Charge Fraction

▶ neutronization with increasing density

March 19, 2018 | Waseda University, Tokyo, Japan | S. Typel | 51
mass fractions of $^2\text{H}$ and $^4\text{He}$
Neutron Star Matter
Heavy Clusters

- mass fraction and average mass number
finite temperatures and very low densities: EoS determined by two-body correlations

theoretical benchmark: virial equation of state
  - expansion of powers of fugacities
  - two-body correlations encoded in second virial coefficient
  - depends only on experimental data (phase shifts, binding energies)

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treatment in generalized relativistic density functional with two-body states as explicit degrees of freedom

(M. D. Voskresenskaya and S. Typel, NPA 887 (2012) 42)
Low-Temperature Limit

- **gap in EoS tables** between $T = 0$ and $T_{\text{min}} > 0$
- **phase transition** from gas/liquid phase to solid phase
- correlations due to Coulomb interaction essential
- lattice-periodic Coulomb potential in crystal
- Wigner-Seitz approximation not sufficient
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- better: effective Coulomb contribution from Monte Carlo simulation (molecular dynamics)
  ⇒ phase transition for plasma parameter

\[ \Gamma = \frac{Z_{\text{ion}}^{5/3} e^2}{a_e T} \approx 175 \quad a_e = \left( \frac{3n_e}{4\pi} \right)^{1/3} \]

- improved description with model for crystal (in preparation)
example: symmetric nuclear matter

- isothermes in pressure-density diagram
  \[ \Rightarrow \text{critical point} \]
  
  - DD-RMF:
    \[ T_c \approx 13.7 \text{ MeV}, \]
    \[ n_c \approx 0.04515 \text{ fm}^{-3}, \]
    \[ p_c \approx 0.180 \text{ MeV fm}^{-3} \]
    \[ \Rightarrow p_c/(n_c T_c) \approx 0.290 \]
  
  - van-der-Waals gas:
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▶ \( T < T_c \): liquid-gas phase transition
  Maxwell construction of coexisting phases
  ⇒ precursor of clustering
coexistence of phases

- general construction with Gibbs conditions: equal intensive variables
  - temperature
  - pressure
  - chemical potentials

⇒ **binodals**
  (enclose phase coexistence regions)
**Construction of Phase Transitions I**

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  - consider lines of equal charge chemical potential
    \[ \mu_q = \mu_p - \mu_n \]
  ⇒ standard Maxwell construction
  - symmetry with respect to isospin asymmetry
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- **supernova matter**
  - specific condition of charge neutrality
  - consider lines of equal lepton chemical potential
    \[ \mu_l = \mu_e + \mu_q \]
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Phase Transitions in EoS Tables II

- full gRDF supernova EoS table with DD2 parametrization
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- global thermodynamic consistency of other EoS tables?
CompStar Online Supernovae Equations of State (CompOSE)
CompOSE – Main Features

- free-access website (compose.obspm.fr)
  - hosted at LUTH, Observatoire de Paris, Meudon
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- **documentation**
  - manual and ’how-to’ instructions
  - bibliography of EoS publications
  - links to related projects
CompOSE – Team

▶ core team
  ▶ Chikako Ishizuka (Tokyo Institute of Technology, Japan)
  ▶ Thomas Klähn (California State University Long Beach, USA)
  ▶ Micaela Oertel (LUTH, Observatoire de Paris, France)
  ▶ Stefan Typel (Technische Universität Darmstadt and GSI, Germany)

▶ web support
  ▶ Jean-Yves Giot (LUTH, Observatoire de Paris, France)
  ▶ Marco Mancini (LUTH, Observatoire de Paris, France)
presently available types of tables

- 3-dimensional
  - multi-purpose EoS (58 data sets)
- 2-dimensional
  - zero-temperature EoS (5 data sets)
  - neutron matter EoS (26 data sets)
- 1-dimensional
  - cold $\beta$-equilibrated matter EoS (27 data sets)
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EoS files

- parameters (temperature, baryon density and hadronic charge fraction):
  - eos.t, eos.nb, eos.yq
- EoS data: eos.thermo, eos.compo*, eos.micro* (*: optional)
- information on EoS model in data sheet: eos.pdf
- collection of files available as eos.zip
CompOSE – Handling of EoS Data I

software

- FORTRAN code, version 2.16
  (compose.f90, composemodules.f90, out_to_json.f90, Makefile)
- old ‘file version’ (needs input files provided by the user)
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output files

- EoS table: eos.table
- additional information: eos.report
- input for neutron star calculations (if possible): eos.beta
CompOSE – Handling of EoS Data II

▶ web interface
  ▶ access restricted ⇒ registration required
  ▶ generation of EoS tables (in preparation)
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- **LORENE library**
  - cold neutron star EoS can be used as direct input for Nrotstar code
    ⇒ properties of rotating neutron stars
CompOSE – Documentation

- **manual**
  - detailed information on file formats, tabulation scheme, interpolation, ...  
  - version 1.00 published (75 pages)  
  - new version 2.00 (81 pages, available on website)

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  - simple instructions on how to run the **compose** code  
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CompOSE – Future

- extension of EoS tables
  - dependence on other variables?
    (e.g. magnetic field strength, already implemented partly)
  - choice of other primary variables?
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- different representation of data
  - polynomials or other functions?

- additional software
  - conversion of tables?

- extension of data base
  - more EoS tables needed!

- other suggestions?
Conclusions
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  - many aspects:
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Thank You